

## LCDAs of heavy hadrons and their first inverse moments

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Heavy-quark symmetry (HQS), despite being approximate, allows to relate dynamically many hadron systems. In the HQS-limit, heavy mesons and doubly-heavy baryons are very similar as their dynamics is determined by a light quark moving in a color field of a static source. As in the meson case, matrix elements of non-local interpolation currents between the baryon state and vacuum are determined by light-cone distribution amplitudes (LCDAs). The first inverse moment of the leading twist  $B$ -meson distribution amplitude is a hadronic parameter needed for an accurate theoretical description of  $B$ -meson exclusive decays. It is quite natural that a similar moment of doubly-heavy baryon is of importance in exclusive doubly-heavy baryons' decays. In the Heavy Quark Effective Theory, we obtain the sum rules for the first inverse moment based on the correlation functions containing non-local heavy-light operator of the doubly-heavy baryon and its local interpolating current.

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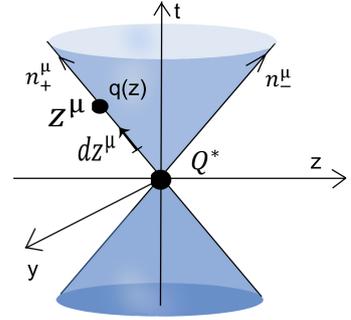
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Colorless bound states — hadrons are the only strongly coupled particles observed experimentally. Among them, mesons — quark-antiquark systems and baryons — three-quark states are the simplest and well studied both theoretically and experimentally, at least, light hadrons formed from the light  $u$ -,  $d$ -, and  $s$ -quarks. Quarkonia —  $\bar{c}c$  and  $\bar{b}b$  states from heavy charm and bottom quarks and heavy mesons with open charm and bottom are known quite well as a lot of experimental data on them is available. Singly-heavy charm and bottom baryons are known experimentally worse because of a lower statistics. Physics of Doubly Heavy Baryons (DHBs), being developed quite a bit theoretically, is an active experimental field of research at the LHC as well as physics of exotic hadrons combining more complicated tetra- and pentaquark states with hidden or open heavy flavors. An experimental searches of triply heavy baryons, whose existence is theoretically natural, is challenging and golden channels for their discovery are unknown yet.

In this paper we discuss a symmetry between heavy mesons and DHBs both of which has one light quark and a heavy source of color force, i. e., heavy antiquark in the meson and Doubly Heavy Diquark (DHD) in the baryon, both of them being a color antitriplet according the  $SU(3)_C$ -group. Heavy mesons like  $D$ - or  $B$ -mesons are studied for more than 40 years while intensive searches of DHBs were one of the experimental goals for a long time but their existence was either contradictory or not confirmed experimentally like, for example, for the  $\Xi_{cc}^+(ccd)$ -baryon. In 2017, the LHCb Collaboration observed its isospin partner,  $\Xi_{cc}^{++}(ccu)$ , in the  $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$  decay [1] and confirmed later in the  $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$  mode [2]. Its averaged mass and mean life time are  $M_{\Xi_{cc}^{++}} = (3621.6 \pm 0.4) \text{ MeV}$  and  $\Gamma_{\Xi_{cc}^{++}} = (2.56 \pm 0.27) \times 10^{-13} \text{ s}$  [3]. The search for its isospin partner  $\Xi_{cc}^+$  still remains controversial and is a priority for the LHCb. The searched processes are  $\Xi_{cc}^+ \rightarrow \Lambda_c^+ K^- \pi^+$  [4] and  $\Xi_{cc}^+ \rightarrow \Xi_c^+ \pi^- \pi^+$  [5] and  $\Xi_{cc}^+$  evidence is a matter of statistics collected by the LHCb as the present combined global significance is 2.9 standard deviations including systematic uncertainties. Searches of bottom-charmed baryons [6–9] is also a LHCb hot topic.

DHB dynamics is similar to a heavy meson as both are determined by a light quark interacting with a field of a heavy color-antitriplet source. In the heavy meson, the source is the heavy antiquark while the DHD produces an external color field in the DHB. Assuming that the DHD is point-like, it is possible to extend the light-cone approach to heavy-mesons [10, 11] into the DHB wave-function. Let the origin of the frame coincides with the heavy source position as exemplified for the case of the heavy meson in Fig. 1. As the light quark is assumed to be massless and situated on the light cone, it is convenient to define two light-like vectors  $n_{\pm}^{\mu} = (1, 0, 0, \mp 1) / \sqrt{2}$  ( $n_{\pm}^2 = 0$  and  $(n_+ n_-) = 1$ ). Any four-vector can be decomposed as follows:  $V^{\mu} = V_+ n_+^{\mu} + V_- n_-^{\mu} + V_{\perp}^{\mu}$ , where  $V_{\pm} = n_{\pm}^{\mu} V_{\mu}$ . The heavy source at the origin is connected with the light quark by the Wilson line  $E(0, z)$ . This line is trivial,  $E(0, z) = 1$ , when the Fock-Schwinger gauge of the gluonic field,  $A_+(z) = 0$ , is used.

The heavy-antiquark — heavy-diquark symmetry can be demonstrated on hadrons like  $B$ -meson and  $\Xi_{bc}$ -baryon. For the  $B_s$ -meson and  $\Omega_{bc}$ -baryon, the  $s$ -quark mass should be taken into



**Figure 1:** The heavy-quark rest frame used for the heavy meson. The light quark  $q(z)$ , assumed to be a massless particle, is on the light cone.

account but, as far as this mass corrections are neglected, the considered hadrons completely coincide with  $B$ -meson and  $\Xi_{bc}$ -baryon, respectively.

The matrix element of the local interpolation current of the  $B$ -meson has the form:

$$\langle 0 | \bar{b}(0) \gamma^\mu \gamma_5 q(0) | B(p_B) \rangle = i f_B p_B^\mu, \quad (1)$$

where  $q(x)$  and  $b(x)$  are the light and  $b$ -quark fields,  $p_B^\mu$  is the four-momentum of the  $B$ -meson, and  $f_B \simeq 190$  MeV [3] is the  $B$ -meson decay constant. The general structure of the DHB interpolation current is as follows:

$$J_{\Xi_{bc}}(x) = \epsilon_{\alpha\beta\gamma} [b(x)^{\alpha T} C \Gamma c(x)^\beta] \Gamma' q(x)^\gamma, \quad (2)$$

where  $\alpha$ ,  $\beta$ , and  $\gamma$  are color indices of quarks,  $\Gamma^{(\prime)} = \{I, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2\}$ , and  $C$  is the charged conjugation matrix. In the lowest lying  $\Xi_{bc}$ -baryon, the DHD is a scalar state with  $J^P = 0^+$  and, after the light-quark spin is taken into account, the spin-parity of such a baryon is  $J^P = 1/2^+$ . The transition matrix element of the corresponding current between the  $\Xi_{bc}$ -baryon state and vacuum can be written in terms of the bispinor,  $U(v)$ :

$$\langle 0 | J_{\Xi_{bc}}(0) | \Xi_{bc}(v) \rangle = i f_{\Xi_{bc}} U(v), \quad (3)$$

where  $f_{\Xi_{bc}}$  is the  $\Xi_{bc}$ -baryon decay constant and  $v^\mu$  is the DHB velocity. The other possibility for the unexcited DHD is to be in the axial-vector state with  $J^P = 1^+$ . Combining it with the light quark, two baryonic states with  $J^P = 1/2^+$  and  $J^P = 3/2^+$  are possible. After the corresponding interpolation current is projected onto the spinor state, the transition matrix element is parametrized as in (3). For the  $J^P = 3/2^+$  baryon,  $U(v)$  should be replaced by the vector-spinor  $U^\mu(v)$  in (3).

The matrix elements (1) and (3) of local interpolation currents can be generalized to the case of non-local currents following [10]. In the heavy meson, the light quark  $q$  and heavy antiquark  $Q^*$ , being assumed infinitely heavy and static, are separated by  $z$  ( $z^2 = 0$ ), as shown in Fig. 1. So, the meson dynamics is completely determined by the light quark. In the DHB considered the light quark is also at the distance  $z$  from the center of the  $cb$ -DHD which structure is neglected. For point-like diquarks, the non-local interpolation currents are  $\tilde{O}_0(t) = d(0) q(z)$  and  $\tilde{O}_1^\mu(t) = \varepsilon^\mu(0) q(z)$ . Here,  $d(0)$  and  $\varepsilon^\mu(0)$  are spin  $S = 0$  and  $S = 1$  DHD, being at rest,  $t = (vz)$ , and the Wilson line  $E(0, z)$  is suppressed. The scalar diquark,  $d(0)$ , inside the  $\Xi_{bc}$ -baryon is considered in this paper, while its counterpart with the axial-vector diquark,  $\varepsilon^\mu(0)$ , will be discussed elsewhere. Because of the HQS, there are two Light-Cone Distribution Amplitudes (LCDAs),  $\tilde{\varphi}_+(t)$  and  $\tilde{\varphi}_-(t)$ , in the heavy-meson wave-function [10]. The application of the heavy-quark — heavy-diquark symmetry to the DHB gives the same decomposition of the baryon wave-function:

$$\langle 0 | \tilde{O}_0(t) | \Xi_{bc}(v) \rangle = i f_{\Xi_{bc}} \left\{ \tilde{\varphi}_+(t) + [\tilde{\varphi}_-(t) - \tilde{\varphi}_+(t)] \frac{\hat{z}}{2t} \right\} U(v), \quad (4)$$

where  $\hat{z} = z_\mu \gamma^\mu$ . In general, the DHD should be considered as a state of two heavy quarks spatially separated by a distance  $R$ . This makes transition matrix elements dependent on two variables  $t$  and  $R$  which must reduce to (4) in the case of the local diquarks ( $R = 0$ ) as a limiting case. Analysis of baryons with non-local diquarks is left for further publications.

To get the QCD Sum Rules (QCD-SRs) for LCDAs, vacuum average of the local and non-local interpolation currents is calculated [10]. For the  $\Xi_{bc}$ -baryon, one should consider the following correlator:  $\langle 0 | \tilde{O}_0(t) J_{\Xi_{bc}}(-x) | \Xi_{bc} \rangle$ . The procedure of its calculation is same as for the  $B$ -meson [10],

and resulting QCD-SRs for the leading twist LCDA,  $\varphi_+(\omega)$ , which is the Fourier transform of  $\tilde{\varphi}_+(t)$  is as follows:

$$f_{\Xi_{bc}}^2 \varphi_+(\omega) = \frac{3\omega}{8\pi^2\tau} e^{(\bar{\Lambda}-\omega/2)\tau} \left[ 1 - e^{-(\varepsilon_c-\omega/2)\tau} \right] - \frac{\langle \bar{q}q \rangle}{8\tau} \tilde{f}_S\left(\frac{\omega}{2\tau}\right) e^{(\bar{\Lambda}-\omega/2)\tau}, \quad (5)$$

where  $\bar{\Lambda} = M_{\Xi_{bc}} - m_b - m_c$  is the effective baryon mass,  $m_b$  and  $m_c$  are the  $b$ - and  $c$ -quark masses,  $\langle \bar{q}q \rangle$  is the local condensate of light quarks,  $\tau$  is the Borel parameter,  $\varepsilon_c$  is an effective QCD-SRs threshold. The function  $\tilde{f}_S(v)$ , which is a shape of the non-local quark condensate, has the definition [10]:

$$f_S(x^2) = \int dv \tilde{f}_S(v) e^{vx^2}. \quad (6)$$

Among two known models for the shape function [12, 13]:

$$\tilde{f}_S^I(v) = \delta\left(v - m_0^2/4\right), \quad \tilde{f}_S^{II}(v) = \frac{\lambda^{p-2}}{\Gamma(p-2)} v^{1-p} e^{-\lambda/v}, \quad (7)$$

where  $p = 3 + 4\lambda/m_0^2$ ,  $m_0^2 = \langle \bar{q}\sigma_{\mu\nu}G^{\mu\nu}q \rangle / \langle \bar{q}q \rangle$  is the parameter characterized the quark-gluon condensate, and  $\lambda$  is a free parameter, we choose the second one.

The first inverse moment of the leading-twist LCDA:

$$\lambda_{\Xi_{bc}}^{-1} = \int_0^\infty \frac{d\omega}{\omega} \varphi_+(\omega), \quad (8)$$

is of interest in physical applications of the DHB wave-function like in the case of heavy mesons [10]. The sum rules for  $\lambda_{\Xi_{bc}}^{-1}$  can be obtained from the QCD-SRs written for  $\varphi_+(\omega)$  by integrating them over the variable  $\omega$  with the  $1/\omega$  weight factor. Such sum rules for the  $B_{(s)}$ -meson were derived in [14]. We obtain analytically these sum rules for the  $\Xi_{bc}$ -baryon to leading order in  $\alpha_{st}$  for the second type of the non-local quark condensate,  $\tilde{f}_S^{II}(v)$ :

$$\lambda_{\Xi_{bc}}^{-1} = \frac{e^{\bar{\Lambda}\tau}}{4\pi^2 f_{\Xi_{bc}}^2} \left[ 1 - \frac{\pi^2 \langle \bar{q}q \rangle}{\tau \Gamma(p-2)} \lambda^{(p-3)/2} K_{p-1}(2\sqrt{\lambda}) \right], \quad (9)$$

where  $K_\nu(z)$  is the modified Bessel (McDonald) function. Details of the analysis and numerical estimates will be presented in the forthcoming paper.

In summary, the heavy-quark — heavy-diquark symmetry is applied to the DHB wave-function which depends on two LCDAs. Dynamically, these LCDAs are similar to the heavy meson ones as both are determined by one light quark, being on the light cone in the same fields of color external static sources. In the heavy meson, the source is the heavy antiquark while a point-like DHD is the color-field source in the DHB. Among two LCDAs allowed by the heavy-quark symmetry,  $\varphi_+(\omega)$  is the leading one. QCD Sum Rules for the first inverse moment of the DHB leading LCDA are presented.

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