

## Cabibbo angle anomalies: indication of new physics at TeV scale?

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Recent high-precision determinations of  $V_{us}$  and  $V_{ud}$  indicate towards anomalies in the first row of the CKM matrix. Namely, the determination of  $V_{ud}$  from beta decays and of  $V_{us}$  from kaon decays imply a violation of first row unitarity at about  $3\sigma$  level. Moreover, there is tension between determinations of  $V_{us}$  obtained from leptonic  $K\mu 2$  and semileptonic  $K\ell 3$  kaon decays. CKM unitarity anomaly can be explained if the Fermi constant  $G_F$  is slightly smaller than the muon decay constant  $G_\mu$ , so that determinations of CKM elements should be rescaled. This situation can be the consequence of the presence of another muon decay channel in interference with the Standard Model one. Horizontal gauge symmetries between lepton families can explain the mass hierarchy between the generations and the relative gauge bosons induce an effective operator in interference with  $W$ -boson exchange in muon decay. Another explanation of the anomalies can be the mixing of SM quarks with vector-like quarks, with mass at the TeV scale, having large enough mixings with the lighter quarks. Mixing with vector-like doublets is the possible explanation to the discrepancy between determination from semileptonic kaon decays and leptonic decays. Although one extra multiplet cannot entirely explain all the discrepancies, a combination such as two species of isodoublet, or one isodoublet and up or/and down type isosinglet is a complete solution. These scenarios are testable with future experiments.

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## 1. The first row of CKM matrix and unitarity

In the Standard Model (SM) three fermion families are in identical representations of the gauge symmetry  $SU(3)_C \times SU(2)_L \times U(1)_Y$ , with left-handed (LH) leptons and quarks,  $\ell_{Li} = (v_i, e_i)_L$  and  $q_{Li} = (u_i, d_i)_L$ , transforming as doublets under  $SU(2)_L$  while right-handed (RH) ones  $e_{Ri}, u_{Ri}, d_{Ri}$  as singlets,  $i = 1, 2, 3$  being the family index. Mixing in charged current interactions in mass basis (interaction with  $W$  boson) are described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix  $V_{\text{CKM}}$ . In the context of the SM,  $V_{\text{CKM}}$  should be unitary, and any deviation from this prediction would point towards new physics. For the first row, unitarity means the equality:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \quad (1)$$

Present data allow enough precision to test the validity of this relation. Using three independent determinations of  $|V_{ud}|$  and  $|V_{us}|$  (the value  $|V_{ub}|^2 \sim 1.6 \times 10^{-5}$  is too small), in ref. [1] about  $4\sigma$  anomaly in first row unitarity was pointed out. After that, new results were released with larger uncertainties. Anyway, the anomalies are still there and about  $3\sigma$  deviation from unitarity emerges.

$|V_{us}|$  can be determined from semileptonic  $K\ell 3$  decays (determination A in [1]) which give  $f_+(0)|V_{us}| = 0.21635(38)$  [3]. Using for the vector form factor  $f_+(0) = 0.9698(17)$  [3], the result is:

$$\text{A : } |V_{us}|_A = 0.22309(55) \quad (2)$$

Leptonic kaon and pion decays  $K\mu 2$  and  $\pi\mu 2$  can independently provide the ratio  $f_K|V_{ud}|/(f_\pi|V_{us}|) = 0.27600(37)$  [3]. Using 4-flavour average lattice QCD calculations for the decay constants ratio  $f_K/f_\pi = 1.1932(21)$ , the result is (determination B):

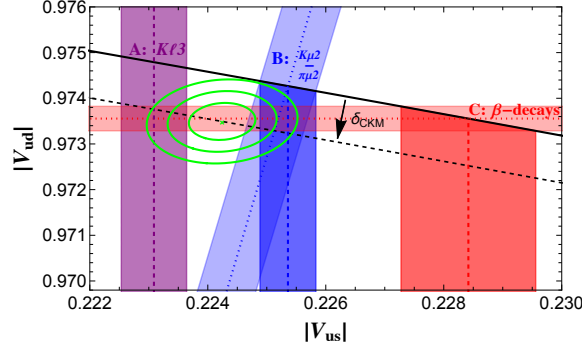
$$\text{B : } |V_{us}|/|V_{ud}| = 0.23131(51) \quad (3)$$

$|V_{ud}|$  is obtained from superallowed  $0^+ - 0^+$  nuclear  $\beta$ -decays and free neutron decays. Using in the master formulas  $\Delta_R^V = 0.02467(22)$  [5] as the transition independent short-distance (inner) radiative correction,  $\mathcal{F}t = 3072.24(1.85)$  s [4] for superallowed  $\beta$  decays,  $g_A = 1.27625(50)$  (average of the the three latest precise experiments measuring  $\beta$  asymmetry  $A$  [6–8]) and the “bottle” lifetime  $\tau_n^{\text{bottle}} = 879.4(0.6)$  s [3] (average of the the eight latest experiments using neutron trap method) for free neutron decay and combining the results we have (determination C):

$$\text{C : } |V_{ud}| = 0.97355(27) \quad (4)$$

As shown in figure 1, there is tension between these three different determinations. There is  $3.1\sigma$  deviation from unitarity. Using the unitarity relation, three different values of  $V_{us}$  are obtained. There is  $4\sigma$  tension between determination A from  $K\ell 3$  decays and determination C from  $\beta$ -decays and about  $3.2\sigma$  tension between determination C and a conservative average between determination from kaon decays A and B. In fact, there is  $3\sigma$  tension also between determination A from  $K\ell 3$  and B from  $K\mu 2/\pi\mu 2$  decays.

In ref. [1] two different kind of explanations were proposed.  $G_F$  can be different from the muon decay constant  $G_\mu$ , so that determinations of CKM elements should be rescaled. Another explanation can be the mixing of SM quarks with vector-like quarks. In ref. [2] the scenario with vector-like quarks was examined in detail.



**Figure 1:** Updated situation of independent determinations  $V_{us}$ - $V_{ud}$ .  $1\sigma$ ,  $2\sigma$  and  $3\sigma$  contours of the fit are shown. The black solid curve is the CKM first row unitarity condition, which is  $3.1\sigma$  away from the best fit result. Projections on the  $V_{us}$  axis using unitarity condition (and similarly on  $V_{ud}$ ) give three different determinations  $|V_{us}|_A = 0.22309(55)$ ,  $|V_{us}|_B = 0.22536(47)$ ,  $|V_{us}|_C = 0.2284(11)$ .

	A: $ V_{us} =0.22309(55)$	B: $ V_{us} =0.23131(51) V_{ud} $	Average*
C: $ V_{ud} =0.97355(27)$	$2.4 \cdot 10^{-3}$	$1.5 \cdot 10^{-3}$	$1.9 \cdot 10^{-3}$

**Table 1:** Deviation from unitarity can be parameterized by  $\delta_{\text{CKM}}$ :  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta_{\text{CKM}}$ . In the table values of  $\delta_{\text{CKM}}$  obtained with different determinations of  $V_{us}$  are shown.

\* Average of the values of  $V_{us}$  given in the columns A and B, conservatively taking as error bar the arithmetical average of their uncertainties.

## 2. $G_F$ vs $G_\mu$ with family symmetries

In the SM, in the limit of vanishing Yukawa couplings, the Lagrangian acquires global chiral symmetry  $U(3)_q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_e$ . We can consider to gauge the  $SU(3)$  factors. Then, the form of the Yukawa matrices, and consequently the mass hierarchies between families and the mixing angles, are determined by the breaking pattern of the symmetry. In particular, we can consider the lepton sector and the gauge symmetry  $SU(3)_\ell \times SU(3)_e$  between lepton families, with  $SU(3)_\ell$  acting on LH states  $\ell_{Li}$  transforming as triplets under this symmetry, and  $SU(3)_e$  on RH states  $e_{Ri}$ . Gauge bosons of  $SU(3)_\ell$  symmetry induce an effective operator:  $-\frac{4G_{\mathcal{F}}}{\sqrt{2}}(\bar{e}_L\gamma^\alpha\mu_L)(\bar{\nu}_\mu\gamma_\alpha\nu_e)$  which is in positive interference with the SM contribution to muon decay mediated by the  $W$ -boson. After Fierz transformation, the sum of the diagrams gives effectively the operator:  $-\frac{4G_\mu}{\sqrt{2}}(\bar{\nu}_\mu\gamma^\alpha\mu_L)(\bar{e}_L\gamma_\alpha\nu_e)$ , so that in this scenario, the muon decay constant  $G_\mu$  becomes different from the Fermi constant  $G_F$ :

$$G_\mu = G_F + G_{\mathcal{F}} = G_F(1 + \delta_\mu) = G_F\left(1 + \frac{v_w^2}{v_{\mathcal{F}}^2}\right) \quad (5)$$

Since the values of CKM elements are extracted by assuming  $G_\mu = G_F$ , their determinations should be rescaled:

$$|V_{us}| = 0.22309(55) \times (1 + \delta_\mu), \quad |V_{ud}| = 0.97355(27) \times (1 + \delta_\mu) \quad (6)$$

while the ratio is not affected. Unitarity is recovered with  $\delta_\mu \sim 7 \cdot 10^{-4}$ , corresponding to the symmetry breaking happening at scale  $v_{\mathcal{F}} = 6-7$  TeV. However, gauge bosons also induce other flavour-

changing and flavour-diagonal processes. For example, the operator  $-\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^8 \left( \overline{\mathbf{e}_L} \gamma_{\mu} \frac{\lambda_a}{x_a} \mathbf{e}_L \right)^2$  gives flavour-changing neutral currents processes in the charged lepton sector. The scale  $v_{\mathcal{F}}$  is related to the masses of the lightest gauge bosons acting between the first two families,  $M_{\ell_{1,2}}^2 = \frac{g^2}{2}(w_2^2 + w_1^2) = \frac{g^2}{2}v_{\mathcal{F}}^2$ , where  $w_{1,2,3}$  are the non-zero VEVs of three triplet scalars  $\eta_{i\alpha}$  breaking the symmetry. If  $w_3 = w_2 = w_1$  (e. g. symmetry between  $\eta$ s), gauge bosons have equal masses,  $\lambda_a \rightarrow V^\dagger \lambda_a V$  is simply a basis redetermination of the Gell-Mann matrices. From Fierz identities for  $\lambda$  matrices:  $\mathcal{L}_{eff} = -\frac{1}{4v_{\mathcal{F}}^2} (\overline{\mathbf{e}_L} \lambda^a \gamma^\mu \mathbf{e}_L) (\overline{\mathbf{e}_L} \lambda^a \gamma_\mu \mathbf{e}_L) = -\frac{1}{3v_{\mathcal{F}}^2} (\overline{\mathbf{e}_L} \mathbb{1} \gamma_\mu \mathbf{e}_L)^2$ , so there would be no flavour-changing neutral currents at tree level, since the global  $SO(8)_\ell$  symmetry acts as a custodial symmetry. In the general case, flavour-changing processes ( $\mu \rightarrow 3e$ ,  $\tau \rightarrow 3\mu, \dots$ ) are under control, and  $v_{\mathcal{F}} \simeq 6$  TeV is not contradicting experimental constraints.

### 3. Vector-like quarks

A possible explanation to the lack of unitarity would be the presence of another family mixing with the first three. The existence of a fourth sequential family is excluded by SM precision tests combined with LHC mass limits, Higgs production via gluon fusion and its  $2\gamma$  decay. On the other hand, vector-like quarks can be introduced.

In the case of down-type vector-like quarks  $D_{L,R}$ , with left- (LH) and right-handed (RH) components both singlets of SM symmetry  $SU(2)$ , extra Yukawa couplings and mass term  $h_j \varphi \overline{q_{Lj}} D_R + M_D \overline{D}_L D_R + \text{h.c.}$  should be considered in the Lagrangian. The mass matrix of down type quarks can be diagonalized by bi-unitary transformation  $V_L^{(d)\dagger} \mathbf{m}^{(d)} V_R^{(d)} = \mathbf{m}_{\text{diag}}^{(d)} = \text{diag}(m_d, m_s, m_b, M_{b'})$ , with  $|V_{LD\alpha}| \approx h_\alpha v_w / M_{b'}$ . The CKM matrix in charged currents becomes a  $4 \times 3$  matrix and the mixing  $|V_{ub'}| \approx |V_{LDd}| \approx h_d v_w / M_{b'}$  modifies the unitarity relation as:

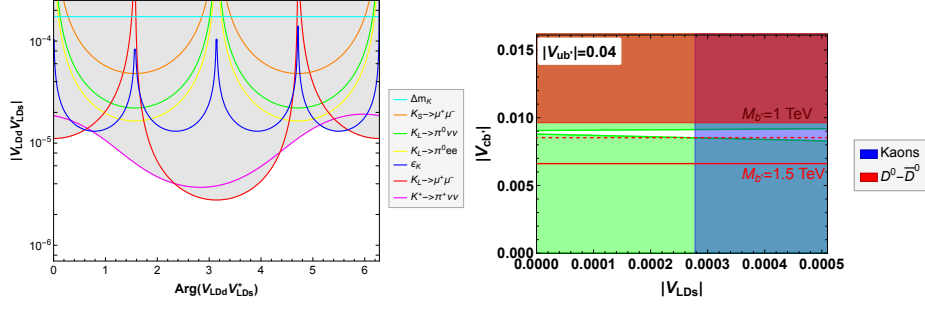
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{ub'}|^2 \quad (7)$$

with the new mixing  $V_{ub'} = 0.045(7)$  comparable with  $V_{cb}$  and ten times bigger than  $V_{ub}$ . However, since normal LH families behave as doublets while  $D_L$  is a singlet, non-diagonal couplings to  $Z$ -boson arise at tree level. Moreover, mass and Yukawa matrices are not proportional anymore and tree (and loop) level flavour-changing couplings with the Higgs boson are generated. Then, the large mixing with the first family  $|V_{ub'}| \approx 0.04$  must confront experimental limits from flavour-changing neutral currents and  $Z$ -boson physics. The result, as shown in fig. 2, is that the mass of the vector-like quark cannot exceed few TeV. In fact, constraints become stronger with increasing mass of the heavy species because the loop contribution increases and becomes relevant. For example, the loop contribution to kaon mixing for  $M_{b'} \gtrsim 3$  TeV grows as  $\propto (V_{LDd}^* V_{LDs})^2 M_{b'}^2$ . Moreover, results on  $Z$  boson decay rate into hadrons imply that  $|V_{ub'}| < 0.050$ .

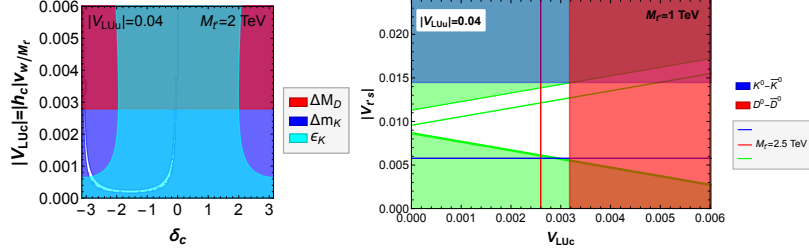
In the case of vector-like up-type singlet  $U_{L,R}$  the extra Yukawa couplings and mass term can be written as:  $h_\alpha \varphi \overline{q_{L\alpha}} U_R + M_U \overline{U}_L U_R + \text{h.c.}$ . Then, the unitarity relation (1) is modified to

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{LUu}|^2 \quad (8)$$

with  $|V_{LUu}| \approx |V_{t'd}| \approx |h_{Uu}| v_w / M_{t'} = 0.045(7)$ . Similarly to the previous case, constraints from flavour-changing neutral currents and  $Z$  physics must be confronted. Even if the window in



**Figure 2:** Constraints in the scenario with vector-like down-type singlet. Left: upper bounds on the product of the mixing elements  $V_{LDb}^* V_{LDb}$  from flavour-changing processes as function of their relative phase. Right: assuming  $|V_{ub'}| \approx 0.04$  and  $M_{b'} = 1$  TeV, the red region is excluded by  $D$  mesons mixing, the blue one by kaons flavour-changing processes and the excluded green area is determined by the relation  $|V_{cb'}| = | -V_{LDb}^* V_{cd} - V_{LDb}^* V_{cs} - V_{LDb}^* V_{cb} |$ , using allowed values. The mixing in charged currents  $V_{cb'}$  remains non-zero because of non-zero  $V_{ub'} \approx -V_{LDb}^*$ . The continuous red line shows where the bound shifts if  $M_{b'} = 1.5$  TeV.



**Figure 3:** Constraints in the scenario with vector-like down-type singlet. Left: assuming  $|V_{LUu}| = 0.045(7)$  and  $M_{l'} = 2$  TeV, excluded space for the mixing with the second family, as function of its relative phase  $\delta_c = \text{Arg}(V_{LUu} V_{LUc}^*)$ . The cyan and blue area are excluded by  $K^0$  system, the red area by  $D^0$  mesons mixing. The result is that a phase can be found for which some value of  $|V_{LUc}|$  is allowed, even if different from zero. Right: assuming  $|V_{Uu}| \approx 0.04$  and  $M_{l'} = 1$  TeV, the red region is excluded by  $D$  mesons mixing, the blue one by the mass difference in  $K^0$  system and the excluded green area is determined by the relation  $|V_{rs}| = | -V_{LUu} V_{us} - V_{LUc} V_{cs} - V_{LUt} V_{ts} |$  using allowed values of moduli and phases. The continuous lines show where the bounds shift if  $M_{l'} = 2.5$  TeV.

parameter space is somewhat larger, also in this case the mass of the up-type vector-like singlet cannot exceed few TeV, as shown in fig. 3.

The case of a vector-like doublet  $q_{L4}, q_{R4}$  is different. The new Yukawa couplings and mass term are:  $h_{uj} \overline{q_{L4}} \tilde{\varphi}_{URj} + h_{d\alpha} \overline{q_{L4}} \varphi_{dR\alpha} + M_q \overline{q_{L4}} q_{R4} + \text{h.c.}$ . After diagonalization of up and down mass matrices, the mixings in the right-handed sector can be large,  $V_{R4\alpha} \approx -h_{\alpha} v_w / M_q$  (while LH mixing are proportional to SM yukawas  $s_{Li}^{u,d} \approx y_i |h_i| v_w^2 / M_q^2$  and  $V_{CKM,L}$  is a unitary matrix). Charged right-handed currents coupling with the  $W$  boson are generated. Then, in this scenario, semileptonic kaon decays and beta decays actually determine vector couplings while leptonic decays of kaons and pions determine axial-vector couplings:

$$|V_{Lus} + V_{Rus}| = 0.22309(55), \quad \frac{|V_{Lus} - V_{Rus}|}{|V_{Lud} - V_{Rud}|} = 0.23131(51), \quad |V_{Lud} + V_{Rud}| = 0.97355(27) \quad (9)$$

where  $V_{Rus} = V_{R4u}^* V_{R4s} = -1.27(38) \times 10^{-3}$ ,  $V_{Rud} = V_{R4u}^* V_{R4d} = -0.92(27) \times 10^{-3}$  would explain both the discrepancies between the three different determinations A (2), B (3), C (4). However, also in this case such large mixings should confront limits from flavour-changing neutral currents and from electroweak low-energy observables and Z physics. As a consequence, one vector-like doublet alone cannot account for both the anomalies. In fact, by multiplying the two mixings  $V_{Rus}$  and  $V_{Rud}$  it is obtained that, in order to explain the discrepancies, we need  $|V_{R4u}|^2 V_{R4s}^* V_{R4d} \approx 1.0 \times 10^{-6}$ . However, flavour-changing kaon processes set an upper bound roughly of  $|V_{R4d} V_{R4s}^*| \lesssim 1.0 \times 10^{-5}$ , which would imply  $|V_{R4u}| \gtrsim 0.3$ . However, just only considering Z-boson decay into hadrons, we obtain a limit  $|V_{R4u}|^2 + 0.5(|V_{R4d}|^2 + |V_{R4s}|^2) < 6.8 \times 10^{-3}$ , which clearly excludes values of the mixing with the first up-quark as large as  $|V_{R4u}| \gtrsim 0.3$ . Two vector-like weak doublets (or one vector-like doublet together with isosinglet, up-type and/or down-type) can explain all the discrepancies of the first row of the CKM matrix.

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