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RGEs and positivity bounds of the SMEFT dimension-8 operators

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The Standard Model effective field theory (SMEFT) is a robust model for studying beyond Standard Model physics. The dimension-eight operators are of particular interest to theoretically rule-out regions of Wilson coefficient (WC) space based on positivity bounds derived using properties of scattering amplitudes. The renormalisation group evolution (RGE) of these WCs is necessary to define them consistently at different scales. In this work, we present the renormalisation of the bosonic sector of the SMEFT from tree-level generated dimension-eight operators up to $O(v^4/\Lambda^4)$, with v ∼ 246 GeV as the electroweak symmetry breaking scale and Λ as the cut-off scale of the EFT. Using these RGEs, and the positivity bounds on $X^2\phi^2D^2$ class, we deduce the constraints on WCs of ϕ^4D^4 and $\psi^2\phi^2D^3$ classes. These bounds on $X^2\phi^2D^2$ operators remain valid at sufficiently small scales within the one-loop analysis.

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1. Introduction

The Standard Model effective field theory (SMEFT) is widely studied to understand elementary particles and their interactions. Primarily, it can account for experimental observations which are beyond the Standard Model prediction, and also, it does not conflict with the null results for several resonant searches encountered at different experiments like at the LHC. The SMEFT is based on the same gauge symmetry as that of the SM: $SU(3)_C \times SU(2)_L \times U(1)_Y$, and assumes that the new physics (NP) exists at scale(s) higher than that of the EW symmetry breaking such that the validity of the EFT is preserved.

Naively, low-energy observables receive dominant corrections from dimension-six (dim-6) terms in the SMEFT. Currently, dimension-eight (dim-8) operators are also being studied extensively from both the theory and experimental contexts. In certain observables, the dim-6 contributions vanish, and dim-8 contributes in the leading order. In addition, dim-8 corrections to observables (e.g. EWPO, Higgs) compete with dim-6 squared contributions, and so, studying the dim-8 contributions is necessary from the EFT validity ground [1]. On a separate note, one has to know the relations among the Wilson coefficients (WCs) defined at multiple scales to make consistent predictions from the theory. We present here the renormalisation of SMEFT bosonic operators at 1-loop order upto $O(v^4/\Lambda^4)$ from dim-8 operators that are tree-level generated in weakly coupled UV-complete theories.

A more concrete motivation for studying SMEFT at dim-8 is from 'positivity bounds'. Since SMEFT is an EFT for a UV-complete theory, hence it satisfies the fundamental principles of QFT, such as analyticity, unitarity, crossing symmetry, and Lorentz invariance, and thus, the positivity bounds can be derived, which will constrain the dim-8 WCs space in addition to dim-6 squared WC space [2]. These bounds can be utilised in both ways : (1) Optimise the search strategies for UV-complete models, and (2) Experimentally test the fundamental principles of QFT [3].

The outline of this report is as follows. In section 2, we introduce our convention and methodology. In section 3, we present the dim-8 anomalous dimension matrix. We consider both bosonic and fermionic tree-generated dim-8 that can contribute to the running of SMEFT bosonic operators. We point out the entries which are larger than the naive dimensional analysis estimate. We also show the renormalisation group running of operators with mass dimensions four and six induced from the dim-8 operators. In section 4, we discuss the positivity bounds on $X^2\phi^2D^2$ operator class. Using our RGEs and positivity bounds on $X^2\phi^2D^2$, we show the constraints on classes ϕ^4D^4 and $\psi^2\phi^2D^3$. We find that positivity bounds on $X^2\phi^2D^2$ are valid at sufficiently small scales within the one-loop analysis.

2. Theory

The SMEFT Lagrangian assuming lepton-number conservation reads:

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2}\sum_i c_i^{(6)} O_i^{(6)} + \frac{1}{\Lambda^4}\sum_j c_j^{(8)} O_j^{(8)} + \cdots \quad (1)$$

	ϕ^4D^4	$B\phi^4D^2$	$W\phi^4D^2$	$B^2\phi^4$	$W^2\phi^4$	$WB\phi^4$	$G^2\phi^4$	ϕ^6D^2	ϕ^8	$\psi^2B\phi^3$	$\psi^2W\phi^3$	$\psi^2G\phi^3$	$\psi^2\phi^2D^3$	$\psi^2\phi^5$	$\psi^2\phi^4D$	$\psi^2B\phi^2D$	$\psi^2W\phi^2D$	$\psi^2G\phi^2D$	$\psi^2\phi^3D^2$
$B^2\phi^2D^2$	g_1^2	0	0	0	0	0	0	0	0	0	0	0	g_1^2	0	0	0	0	0	0
$W^2\phi^2D^2$	g_2^2	0	0	0	0	0	0	0	0	0	0	0	g_2^2	0	0	0	0	0	0
$WB\phi^2D^2$	g_1g_2	0	0	0	0	0	0	0	0	0	0	0	g_1g_2	0	0	0	0	0	0
$G^2\phi^2D^2$	0	0	0	0	0	0	0	0	0	0	0	0	g_3^2	0	0	0	0	0	0
$W^3\phi^2$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$W^2B\phi^2$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$G^3\phi^2$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ^4D^4	g_2^2	0	0	0	0	0	0	0	0	0	0	0	$\|y^t\|^2$	0	0	0	0	0	0
$B\phi^4D^2$	$g_1g_2^2$	λ	0	0	0	0	0	0	0	0	0	0	$g_1\|y^t\|^2$	0	0	$\|y^t\|^2$	0	0	g_1y^t
$W\phi^4D^2$	g_2^3	0	g_2^2	0	0	0	0	0	0	0	0	0	$g_2\|y^t\|^2$	0	0	0	$\|y^t\|^2$	0	g_2y^t
$B^2\phi^4$	$g_1^2g_2^2$	$g_1\lambda$	$g_1^2g_2$	λ	0	g_1g_2	0	0	0	g_1y^t	0	0	$g_1^2\|y^t\|^2$	0	0	$g_1\|y^t\|^2$	0	0	$g_1^2y^t$
$W^2\phi^4$	g_2^4	$g_1g_2^2$	g_2^3	0	λ	g_1g_2	0	0	0	0	g_2y^t	0	$g_2^2\|y^t\|^2$	0	g_2^2	0	$g_2\|y^t\|^2$	0	$g_2^2y^t$
$WB\phi^4$	$g_1g_2^3$	$g_2\lambda$	$g_1\lambda$	g_1g_2	g_1g_2	λ	0	0	0	g_2y^t	g_1y^t	0	$g_1g_2\|y^t\|^2$	0	g_1g_2	$g_2\|y^t\|^2$	$g_1\|y^t\|^2$	0	$g_1g_2y^t$
$G^2\phi^4$	0	0	0	0	0	0	g_3^2	0	0	0	0	g_3y^t	0	0	0	0	0	0	0
ϕ^6D^2	g_2^4	$g_1\lambda$	$g_2\lambda$	0	0	0	0	λ	0	0	0	0	$g_2^2\|y^t\|^2$	0	$\|y^t\|^2$	$g_1\|y^t\|^2$	$g_2\|y^t\|^2$	0	$y^t\|y^t\|^2$
ϕ^8	λ^3	$g_1\lambda^2$	$g_2\lambda^2$	$g_1^2\lambda$	$g_2^2\lambda$	$g_1g_2\lambda$	0	λ^2	λ	0	0	0	$\lambda\|y^t\|^4$	$y^t\|y^t\|^2$	$\lambda\|y^t\|^2$	$g_1\lambda\|y^t\|^2$	$g_2\lambda\|y^t\|^2$	0	$\lambda y^t\|y^t\|^2$

Table 1: *Structure of the bosonic-bosonic dim-8 anomalous dimension matrix. The entries indicate the order in SM couplings of the leading contribution. Those in blue represent terms that deviate significantly from naive dimensional analysis. The operators in gray can only arise at loop-level in weakly-coupled UV completions of the SMEFT. The shaded cells indicate those of the latter operators that are renormalised by interactions that can be generated at tree-level.*

where $\mathcal{L}_{\rm SM}$ represents the SM dimension-four Lagrangian, and in our convention, i and j run over the operators in the bases of dim-6 and dim-8 interactions given in Refs. [4] (the "Warsaw" basis) and [5] (dim-8 basis), respectively, and, Λ is the cut-off of the EFT.[1]

To order v^4/Λ^4, and assuming lepton-number conservation:

$$\dot{c}_i^{(8)} \equiv 16\pi^2 \tilde{\mu} \frac{dc_i^{(8)}}{d\tilde{\mu}} = \gamma_{ij} c_j^{(8)} + \gamma'_{ijk} c_j^{(6)} c_k^{(6)} \,. \tag{2}$$

The dim-8 WCs are only renormalised by themselves, proportional to renormalisable couplings, as well as by pairs of dim-6 interactions, which is discussed in Ref. [7]. Here, we analyse γ_{ij} in further detail. The dim-8 classes that are tree-level generated in weakly coupled UV-complete theories are shown in the top row of Tab. 1.

We use a Green's basis which extends the basis of independent physical operators to absorb the divergences generated from computing amplitudes [8]. The onshell basis operators are related to the off-shell basis operators through the equations of motion (EOM). In this procedure, we have to compute only off-shell 1-particle irreducible (1PI) Feynman diagrams. We use `FeynArts` [9], `FormCalc` [10] and `Feynrules` [11] for model creation and amplitude computation. We work in dimensional regularisation with space-time dimension d = $4 - 2\epsilon$, using the background field method and the Feynman gauge. We rely on `matchmakereft` [12] for calculating eight-Higgs processes and extensive cross-checking of RGEs.

3. Structure of the anomalous dimension matrix

The RGEs of bosonic interactions of the SMEFT up to $O(v^4/\Lambda^4)$ induced by dim-8 operators that can arise at tree-level upon matching in weakly coupled UV-complete theories, are provided in an auxiliary file on github.com/SMEFT-Dimension8-RGEs [6]. Here we discuss some generic aspects of this result.

[1]More details is available in our article [6].

	ϕ^4D^4	$B\phi^4D^2$	$W\phi^4D^2$	$B^2\phi^4$	$W^2\phi^4$	$WB\phi^4$	$G^2\phi^4$	ϕ^6D^2	ϕ^8	$\psi^2B\phi^3$	$\psi^2W\phi^3$	$\psi^2G\phi^3$	$\psi^2\phi^2D^3$	$\psi^2\phi^5$	$\psi^2\phi^4D$	$\psi^2B\phi^2D$	$\psi^2W\phi^2D$	$\psi^2G\phi^2D$	$\psi^2\phi^3D^2$
ϕ^2	μ^6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ^4	$\lambda\mu^4$	$g_1\mu^4$	$g_2\mu^4$	0	0	0	0	μ^4	0	0	0	0	$\mu^4\|y^t\|^2$	0	0	0	0	0	μ^4y^t
$B^2\phi^2$	$g_1^2\mu^2$	$g_1\mu^2$	0	μ^2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$W^2\phi^2$	$g_2^2\mu^2$	0	$g_2\mu^2$	0	μ^2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$WB\phi^2$	$g_1g_2\mu^2$	$g_2\mu^2$	$g_1\mu^2$	0	0	μ^2	0	0	0	0	0	0	0	0	0	0	0	0	0
$G^2\phi^2$	0	0	0	0	0	0	μ^2	0	0	0	0	0	0	0	0	0	0	0	0
ϕ^4D^2	$\lambda\mu^2$	$g_1\mu^2$	$g_2\mu^2$	0	0	0	0	μ^2	0	0	0	0	$\mu^2\|y^t\|^2$	0	0	0	0	0	μ^2y^t
ϕ^6	$\lambda^2\mu^2$	$\lambda g_1\mu^2$	$\lambda g_2\mu^2$	$g_1^2\mu^2$	$g_2^2\mu^2$	$g_1g_2\mu^2$	0	$\lambda\mu^2$	μ^2	0	0	0	$\lambda\mu^2\|y^t\|^2$	μ^2y^t	$\mu^2\|y^t\|^2$	$\mu^2\|y^t\|^2$	$\mu^2\|y^t\|^2$	0	$\mu^2y^t\|y^t\|^2$

Table 2: *Same as Tab. 1 but for the renormalisation of the bosonic interactions of dimensions two, four and six.*

The structure of the anomalous dimension matrix γ of dim-8 bosonic operators is shown in Tab. 1 for the contributions from both bosonic and fermionic dim-8 operators. In the tables, we show the SM couplings which have the leading contributions to the renormalisation of the operators in the rows by insertions of the dim-8 operators present in the columns. Note that we compute the RGEs at one-loop, and hence two fields from the inserted operator form the loop propagators, and others fields acts as external legs. Thus, for instance, four-Higgs operators will not renormalise operators that contain only gauge bosons, and hence not shown in the tables. Following the same argument, the fermionic operators with more than two fermions are also not relevant at one-loop order.

Some mixing terms (highlighted in blue in the tables those that fulfil $\gamma_{ij}/g \gtrsim 10$) are significantly larger from naive dimensional analysis estimate, $\gamma_{ij}/g \sim 1$; where g represents (products of) SM couplings. In the shaded cells, we observe that operators of class $X^2\phi^2D^2$ that arise at loop-level, are renormalised by tree-level generated operators, namely ϕ^4D^4 and $\psi^2\phi^2D^3$ classes, which is not the case in the dim-6 bosonic sector of the SMEFT [13]. The zeros in Tabs. 1 can be explained following the results in Ref. [14]. It is worth mentioning that some anomalous dimensions arise only from redundant operators. One example is that ϕ^4D^4 that do not renormalise the operator O_{ϕ^8} from an off-shell diagram, because divergences of the latter are momentum-less, and loops of the former always contain external momenta. In Tabs. 2 we present the same information as in Tabs. 1 but for the renormalisation of the bosonic SM Lagrangian terms and dim-6 interactions. Notice that none of the dim-8 fermionic interactions considered here renormalise lower-dimensional bosonic terms.

4. Positivity bounds

We can derive positivity bounds for operators of $X^2\phi^2D^2$ class from computing $V_1V_2 \to V_1V_2$ amplitudes [15]. Here we reproduce these bounds for WCs in our operator basis:

$$g_1^2c^{(1)}_{B^2\phi^2D^2} + g_2^2c^{(1)}_{W^2\phi^2D^2} + 2g_1g_2c^{(4)}_{WB\phi^2D^2} \leq 0\,, \tag{3}$$

$$g_1^2c^{(1)}_{B^2\phi^2D^2} + g_2^2c^{(1)}_{W^2\phi^2D^2} - 2g_1g_2c^{(4)}_{WB\phi^2D^2} \leq 0\,, \tag{4}$$

$$c^{(1)}_{W^2\phi^2D^2} \leq 0\,, \tag{5}$$

$$g_1^2c^{(1)}_{W^2\phi^2D^2} + 2g_1g_2c^{(4)}_{WB\phi^2D^2} + g_2^2c^{(1)}_{B^2\phi^2D^2} \leq 0\,, \tag{6}$$

$$g_1^2c^{(1)}_{W^2\phi^2D^2} - 2g_1g_2c^{(4)}_{WB\phi^2D^2} + g_2^2c^{(1)}_{B^2\phi^2D^2} \leq 0\,. \tag{7}$$

$X^2\phi^2D^2$ operators are not generated at tree-level in weakly-coupled UV theories. Thus, at energies $\tilde{\mu} \ll \Lambda$, the running-induced by tree-level operators, namely by ϕ^4D^4 and $\psi^2\phi^2D^3$ will dictate their value. Using the RGEs discussed in the previous section, a set of sufficient conditions for all inequalities above to hold are:

$$2c_{\phi^4}^{(1)} + 3c_{\phi^4}^{(2)} + c_{\phi^4}^{(3)} \geq 0\,, \tag{8}$$

$$c_{\phi^4}^{(1)} + 2c_{\phi^4}^{(2)} + c_{\phi^4}^{(3)} \geq 0\,, \tag{9}$$

$$c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} \geq 0\,, \tag{10}$$

$$\left[c_{\psi_R^2\phi^2D^3}^{(1)} + c_{\psi_R^2\phi^2D^3}^{(2)}\right]_{\alpha_1,\alpha_1} \leq 0\,, \tag{11}$$

$$\left[c_{\psi_L^2\phi^2D^3}^{(1)} + c_{\psi_L^2\phi^2D^3}^{(2)} + c_{\psi_L^2\phi^2D^3}^{(3)} + c_{\psi_L^2\phi^2D^3}^{(4)}\right]_{\alpha_1,\alpha_1} \leq 0\,, \tag{12}$$

$$\left[c_{\psi_L^2\phi^2D^3}^{(1)} + c_{\psi_L^2\phi^2D^3}^{(2)} - c_{\psi_L^2\phi^2D^3}^{(3)} - c_{\psi_L^2\phi^2D^3}^{(4)}\right]_{\alpha_1,\alpha_1} \leq 0\,; \tag{13}$$

for $\psi_L = l, q$ and $\psi_R = e, u, d$. Eqs. 8–10 are always fulfilled because, at tree-level, the four-Higgs operators satisfy these conditions. The remaining inequalities, Eqs. 11–13 are essentially equivalent to those quoted in Eq. 12 of Ref. [16]. We checked explicitly the forward scattering amplitude for $\phi\psi \to \phi\psi$ that these relations hold. **Thus, we show that the positivity bounds on $X^2\phi^2D^2$ class, in contrast to those of ϕ^4D^4 [17], hold at sufficiently small scales within one-loop.**

5. Conclusions

Taking into account the central result presented in this work i.e. the one-loop renormalisation of bosonic operators in the (lepton-number conserving) SMEFT up to $O(v^4/\Lambda^4)$ is complete. We have used the Green's basis and the reduction of redundant operators onto physical ones derived in Ref. [8]. There are several remarkable conclusions that can be drawn from the results above. **Firstly, a number of contributions in anomalous dimensions are significantly larger than expected from naive power counting. These large factors compete partially with the loop suppression. Secondly, we find that there are tree-level generated dim-8 operators that mix into loop-level dim-8 operators (first noticed in Ref. [14]), and also into loop-level dim-6 terms (that we exhibit here for the first time). Finally, we have found the remarkable result that, unlike for ϕ^4D^4 [17], positivity bounds on $X^2\phi^2D^2$ operators, first derived in Ref. [15], hold at all sufficiently small scales at one-loop accuracy.** This supports the proposal that these bounds in Eqs. 3–7 could be used as priors in experimental fits [18].

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