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Very Special Linear Gravity: A Gauge Invariant Graviton Mass

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A linearized theory of gravity is constructed in the framework of Very Special Relativity (VSR), maintaining the usual gauge invariance of linearized General Relativity (GR). After finding the equations of motion of the model, choosing a suitable gauge, we prove that this extension of linearized GR allows for a graviton mass m_g, which could be of extreme interest in different astrophysical scenarios. Furthermore, as expected due to the gauge invariance, we verify the presence of only two physical degrees of freedom in the theory. To start grasping the possible consequences of this modification of linearized GR, we study Gravitational Waves (GW) effects through the geodesic deviation equation: what we find is that VSR signatures would be proportional to the small parameter m_g^2/E^2, with E being the energy of a single graviton in a monochromatic GW. While this parameter is very small ($\sim 10^{-20}$) for GW detected by the interferometers LIGO and VIRGO, it seems to get better ($\sim 10^{-10}$) with the next generation of gravitational interferometers, like LISA. Therefore, this increase plus the anisotropic nature of VSR could lead to observable consequences of the VSR extension in the future.

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1. Introduction

The theory of Very Special Relativity (VSR) was introduced for the first time in 2006 by A. Cohen and L. Glashow [1] as a relativistic theory with a reduced group of spacetime symmetries: spacetime traslations plus a special kind of (Orthochronous and Proper) Lorentz's subgroup. Originally, the main reason for this proposal was the possibility to introduce a new mechanism of mass generation for neutrinos [2], without the need for new particles or extra dimensions. Since then, consequences of VSR have been explored in a wide variety of fields: from General Relativity's extensions to Standard Model's modifications.

1.1 Properties of VSR Subgroups

There exist four different subgroups suitable for VSR, which are

$$\begin{cases} T(2) = \{T_1 := K_1 + J_2, T_2 := K_2 - J_1\}\,; \quad E(2) = \{T_1, T_2, J_3\}\,, \\ HOM(2) = \{T_1, T_2, K_3\}\,; \quad SIM(2) = \{T_1, T_2, J_3, K_3\}\,, \end{cases} \tag{1}$$

where $\vec{J}$ and $\vec{K}$ are the usual rotations and boost generators of the Lorentz group. All of these subgroups have the special property of being enlarged to the full Lorentz group with the addition to them of some discrete transformation, like P, T or CP. Therefore, in this sense, a small violation of discrete symmetries may be interpreted as the origin of small VSR effects, as could be the case in the case of neutrinos' masses [2].
Usually, the two most considered options are $HOM(2)$ and $SIM(2)$, since they do not allow new invariant tensors other than the Minkowsky metric η, implying same kinematical consequences of Special Relativity [1]. Furthermore, being that an invariance under $SIM(2)$ would also ensure CPT invariance in an eventual Quantum Field Theory, that's the subgroup we will focus on and refer to when talking of VSR in this work.
One of the main features of $SIM(2)$ is that, even if it does not allow new invariant tensors, it does allow the existence of a preferred lightlike spacetime direction, labeled by the four-vector n^μ, which under $SIM(2)$ transforms as

$$n^\mu \underset{SIM(2)}{\longrightarrow} e^{\phi} n^\mu\,, \tag{2}$$

implying the possibility of adding new lagrangian terms involving the contraction of the VSR operator $N^\mu \equiv \frac{n^\mu}{n\cdot\partial}$ with other dynamical quantities.

1.2 Why VSR in Linearized Gravity

As already stated above, VSR effects should become relevant only in contexts where some discrete symmetry is broken. Due to Sakharov conditions [3], we know that in cosmology we must have the breaking of CP in order to see the matter-antimatter asymmetry we see today. Therefore, that means VSR could become relevant, for example, in the propagation of gravitational waves in some cosmological background, like the de-Sitter one. From this observation, it shows up the need of writing a field theory for the spacetime perturbation $h_{\mu\nu}$ in the VSR framework, which we name Very Special Linear Gravity (VSLG). In the following, to simplify the treatment, we will work in momentum space and deal with a flat spacetime $\eta_{\mu\nu}$. This way, the comparison with linearized General Relativity (GR) will also be easier.

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2. Lagrangian

The first task we have to deal with is to find the Lagrangian of the Field Theory for $h_{\mu\nu}$. Since we are interested in the linearized case, the Lagrangian $\mathcal{L}$ will be quadratic in the perturbation

$$\mathcal{L} = h_{\mu\nu} O^{\mu\nu\alpha\beta} h_{\alpha\beta} \,. \tag{3}$$

The ingredients we have to build the operator O are: the flat metric $\eta_{\mu\nu}$, the four-momentum p_μ and the new VSR object $N_\mu = n_\mu / n \cdot p$. Considering all combinations[1] we obtain simbolically

$$O = 3\,\eta\eta + 9\,pp\eta + 12\,pN\eta + 12\,ppNN \,, \tag{4}$$

where the numbers represent how many different terms we can construct with each set of objects. At this point, we fix a series of conditions for the operator O, like the indices interchange symmetries $\mu \Longleftrightarrow \nu$, $\alpha \Longleftrightarrow \beta$, $\mu\nu \Longleftrightarrow \alpha\beta$, and usual gauge invariance $O_{\mu\nu\alpha\beta} p^\alpha = 0$ of linearized GR. By doing that, we get to an explicit expression of O in momentum space

$$\begin{aligned}
\frac{1}{\chi} O_{\mu\nu\alpha\beta} = {} & p_\mu p_\nu \eta_{\alpha\beta} - \frac{1}{2} p_\mu p_\alpha \eta_{\nu\beta} - \frac{1}{2} p_\mu p_\beta \eta_{\nu\alpha} + p_\alpha p_\beta \eta_{\mu\nu} - \frac{1}{2} p_\nu p_\beta \eta_{\mu\alpha} - \frac{1}{2} p_\nu p_\alpha \eta_{\mu\beta} \\
& - p^2 \eta_{\mu\nu} \eta_{\alpha\beta} + \frac{1}{2} p^2 \eta_{\mu\alpha} \eta_{\nu\beta} + \frac{1}{2} p^2 \eta_{\mu\beta} \eta_{\nu\alpha} - m_g^2 \eta_{\mu\nu} \eta_{\alpha\beta} + \frac{m_g^2}{2} \eta_{\mu\alpha} \eta_{\nu\beta} + \frac{m_g^2}{2} \eta_{\mu\beta} \eta_{\nu\alpha} \\
& + m_g^2 N_\mu N_\nu p_\alpha p_\beta - \frac{m_g^2}{2} N_\mu N_\alpha p_\nu p_\beta - \frac{m_g^2}{2} N_\mu N_\beta p_\nu p_\alpha - \frac{m_g^2}{2} N_\nu N_\alpha p_\mu p_\beta - \frac{m_g^2}{2} N_\nu N_\beta p_\mu p_\alpha \\
& + m_g^2 N_\alpha N_\beta p_\mu p_\nu - m_g^2 p^2 N_\mu N_\nu g_{\alpha\beta} + \frac{m_g^2}{2} p^2 N_\mu N_\alpha g_{\nu\beta} + \frac{m_g^2}{2} p^2 N_\mu N_\beta g_{\nu\alpha} + \frac{m_g^2}{2} p^2 N_\nu N_\beta \eta_{\mu\alpha} \\
& + \frac{m_g^2}{2} p^2 \eta_{\mu\beta} N_\nu N_\alpha - m_g^2 p^2 N_\alpha N_\beta \eta_{\mu\nu} \\
& + m_g^2 \eta_{\mu\nu} N_\alpha p_\beta + m_g^2 \eta_{\mu\nu} p_\alpha N_\beta - \frac{m_g^2}{2} \eta_{\mu\alpha} N_\nu p_\beta - \frac{m_g^2}{2} \eta_{\mu\alpha} p_\nu N_\beta - \frac{m_g^2}{2} \eta_{\mu\beta} N_\nu p_\alpha - \frac{m_g^2}{2} \eta_{\mu\beta} p_\nu N_\alpha \\
& - \frac{m_g^2}{2} \eta_{\nu\alpha} N_\mu p_\beta - \frac{m_g^2}{2} \eta_{\nu\alpha} p_\mu N_\beta - \frac{m_g^2}{2} \eta_{\nu\beta} N_\mu p_\alpha - \frac{m_g^2}{2} \eta_{\nu\beta} p_\mu N_\alpha + m_g^2 \eta_{\alpha\beta} N_\mu p_\nu + m_g^2 \eta_{\alpha\beta} p_\mu N_\nu \,,
\end{aligned} \tag{5}$$

depending on only two parameters: an overall constant χ, that we can identify with the Einstein-Hilbert constant $\chi = \frac{1}{2\kappa} = \frac{c^4}{16\pi G}$, and m_g^2, that has the dimensions of a mass squared and, as we will see, plays the role of a graviton mass.

2.1 Gauge Freedom and Equation of Motion

The expression (5) and the related equations of motion (E.o.M) $O_{\mu\nu\alpha\beta} h^{\alpha\beta} = 0$ are quite complicated. However, we fortunately have plenty of gauge freedom to exploit. In the end, due to the residual gauge invariance after imposing the usual Lorentz condition $p^\mu h_{\mu\nu} = 0$, we are able to fix the following set of compatible conditions

$$\begin{cases} p^\mu h_{\mu\nu} = 0 \,, \\ n^\mu h_{\mu\nu} = 0 \,, \\ h \equiv h^\mu_\mu = 0 \,, \end{cases} \tag{6}$$

[1] We are considering only terms with up to two p^μ and parameters with (energy) dimension up to 4

which, first of all, implies that

$$h_{0\beta} = -\frac{n^i}{n^0} h_{i\beta} = -\frac{p^i}{p^0} h_{i\beta} \to (\frac{n^i}{n^0} - \frac{p^i}{p^0}) h_{i\beta} = 0\,. \tag{7}$$

By using the conditions (6) in the E.o.M for the $h-$field, we finally obtain the following fully simplified dispertion equation

$$(p^2 - m_g^2) h_{\mu\nu} = 0\,, \tag{8}$$

from which we see, directly, that the parameter m_g plays for the graviton the role of a mass. This is a very interesting result since we already know that massive gravitons could be of interest in a variety of gravitational applications, like dark matter [4] and Universe's accelerated expansion [5]. Still, the difference with VSLG is that here we are still left with only two physical degrees of freedom, against the five of non-gauge invariant massive gravity, as stated and demonstrated in [8].

3. Gravitational Waves and Geodesic Deviation

Let's introduce, for our solution, the plane wave ansatz for $h_{\mu\nu}$, labeling the axis of propagation as the z-axis: $p^\mu = (E, 0, 0, p)$

$$h_{\mu\nu} = \mathcal{RE}(A_{\mu\nu} e^{ip^\mu x_\mu}) = \mathcal{RE}(A_{\mu\nu} e^{i(Et-pz)})\,, \tag{9}$$

where $A_{\mu\nu}$ is the polarization tensor satisfying the conditions $p^\mu A_{\mu\nu} = 0\,,\; n^\mu A_{\mu\nu} = 0\,,\; A^\mu_\mu = 0$. Note that, by deriving $h_{\mu\nu}$ respect to t, z, we see

$$\begin{cases} \partial_0 h_{\mu\nu} = iE h_{\mu\nu} \\ \partial_3 h_{\mu\nu} = -ip h_{\mu\nu} \end{cases} \to \;\partial_3 h_{\mu\nu} = -\frac{p}{E} \partial_0 h_{\mu\nu}\,. \tag{10}$$

Furthermore, since here $h_{\mu\nu}$ has no dependence on x and y we have $\partial_1 h_{\mu\nu} = \partial_2 h_{\mu\nu} = 0$, and from the Lorentz gauge condition we find $p^\mu h_{\mu\nu} = 0 \to h_{3\nu} = -\frac{E}{p} h_{0\nu}$.

3.1 Geodesic Deviation from Gravitational Waves

As a first application, we studied the modifications produced by VSR to the known geodesic deviation equations for a gravitational wave, represented by the space-time perturbation $h_{\mu\nu}$, in a flat background. The expression of the geodesic deviation equation depends on the linearized Riemann Tensor $R_{\mu\nu\alpha\beta} = \frac{1}{2}(h_{\rho\nu,\mu\kappa} - h_{\mu\nu,\rho\kappa} - h_{\rho\kappa,\mu\nu} + h_{\mu\kappa,\rho\nu})$[2], in the following way

$$\partial_0^2 \delta\xi^\mu = R^\mu_{\;00\gamma} \delta\xi^\gamma = \eta^{\mu\delta} R_{\delta 00\gamma} \delta\xi^\gamma = \eta^{\mu\mu} R_{\mu 00\gamma} \delta\xi^\gamma\,. \tag{11}$$

The case $\mu = 0$ is trivial, since we have already seen that $R_{000\gamma} = 0$, then $\partial_0^2 \xi^0 = 0$, that combined with the initial conditions $\delta\xi^0(t = 0) = \partial_0 \delta\xi^0(t = 0) = 0$, implies $\delta\xi^0 = 0$. So we have no temporal displacement. For the spatial component of the equation, we see that

$$\partial_0^2 \delta\xi^i = \eta^{ii} R_{i00j} \delta\xi^j = -R_{i00j} \delta\xi^j = \frac{1}{2}(h_{00,ij} + h_{ij,00} - h_{0i,0j} - h_{0j,0i}) \delta\xi^j\,. \tag{12}$$

[2]Here we are using the notation $h_{\mu\nu,\alpha} \equiv \partial_\alpha h_{\mu\nu}$

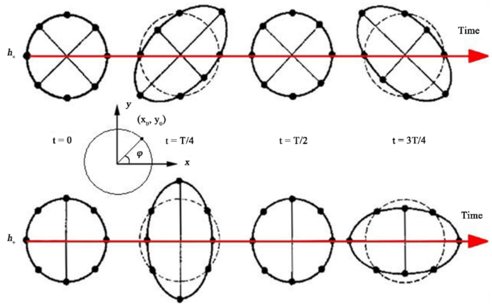

Figure 1: Representation of “+" and “×" polarizations’ effects on a circle of particles in linearized GR [7].

The equation (12) for $i = 1, 2$ becomes

$$\partial_0^2 \delta\xi^i = \frac{1}{2}\partial_0^2 h_{ij}\delta\xi^j - \frac{1}{2}\partial_0\partial_3 h_{0i}\delta\xi^3\,, \tag{13}$$

but, using (7) and (10), we get $\partial_0^2 h_{i3} - \partial_0\partial_3 h_{0i} = \partial_0^2 h_{i3}(1 - \frac{p^2}{E^2}) = \frac{m_g^2}{E^2}\partial_0^2 h_{i3}$, then

$$\partial_0^2 \delta\xi^i = \frac{1}{2}\partial_0^2 h_{i1}\delta\xi^1 + \frac{1}{2}\partial_0^2 h_{i2}\delta\xi^2 + \frac{1}{2}\frac{m_g^2}{E^2}\partial_0^2 h_{i3}\delta\xi^3\,. \tag{14}$$

Since $h_{\mu\nu}$ is a perturbation we can solve the differential equation in a perturbative way by defining $\delta\xi^\mu(t) = \delta\xi_0^\mu + \delta\xi_1^\mu(t)$, where $\delta\xi_1^\mu$ is a small perturbation of $\delta\xi_0^\mu$. The Eq. (14) becomes

$$\partial_0^2 \delta\xi_1^i = \frac{1}{2}\partial_0^2 h_{i1}\delta\xi_0^1 + \frac{1}{2}\partial_0^2 h_{i2}\delta\xi_0^2 + \frac{1}{2}\frac{m_g^2}{E^2}\partial_0^2 h_{i3}\delta\xi_0^3\,,$$

the solution of which, with initial conditions $\delta\xi_1^i(t=0) = \partial_0\delta\xi_1^i(t=0) = 0$, is

$$\delta\xi_1^i = \frac{1}{2}h_{i1}\delta\xi_0^1 + \frac{1}{2}h_{i2}\delta\xi_0^2 + \frac{1}{2}\frac{m_g^2}{E^2}h_{i3}\delta\xi_0^3\,.$$

Doing the same for the case $i = 3$, at the end we obtain the following set of equations

$$\begin{cases} \delta\xi^1 = \delta\xi_0^1 - \frac{1}{2}(h_{11}\delta\xi_0^1 + h_{12}\delta\xi_0^2 + \frac{m_G^2}{E^2}h_{13}\delta\xi_0^3)\,, \\ \delta\xi^2 = \delta\xi_0^2 - \frac{1}{2}(h_{12}\delta\xi_0^1 + h_{22}\delta\xi_0^2 + \frac{m_g^2}{E^2}h_{23}\delta\xi_0^3)\,, \\ \delta\xi^3 = \delta\xi_0^3 - \frac{1}{2}\frac{m_g^2}{E^2}(h_{13}\delta\xi_0^1 + h_{23}\delta\xi_0^2 + \frac{m_g^2}{E^2}h_{33}\delta\xi_0^3)\,. \end{cases} \tag{15}$$

Considering Eq. (15) in the case of a circle of particle, as shown in Fig. 1, we immediately see two new effects affecting the two usual “+" and “×" polarizations: first of all, the presence of a non trivial $\delta\xi^3$ implies a motion also on the propagation direction of the gravitational waves, which is not there in linearized GR. Furthermore, the two oscillation modes also get directly modified in their transversal plane’s motion, while the VSR anisotropic nature is encoded in the h_{ij} components.

3.2 Magnitude of VSR effects

We want to conclude our analysis by studying the magnitude of VSR effects in this framework, which means estimating the perturbative factor $\frac{m_g^2}{E^2}$. To do that, we will use an averaged upper

bound for the graviton mass from literature of $m_g \sim 10^{-24}\, eV$ [6].
With this restriction in mind, we see that in the range of frequencies spanned by the interferometers LIGO and VIRGO, $[10Hz, 10kHz]$, we would get a very small perturbative parameter of $\frac{m_g^2}{E^2} \sim 10^{-20}$. Nevertheless, as expected, we observe that lowering the frequency of the GW gets us a larger factor: for example, for the future generations of interferometers, like LISA, which will explore a lower frequency range $[0.1mHz, 1Hz]$, we get $\frac{m_g^2}{E^2} \sim 10^{-10}$, that combined with larger dimensions of future interferometers and the anisotropic nature of VSR could lead to observable effects.

4. Conclusions

In these few pages, we tried to introduce the reader to a new theory of linearized gravity in the framework of VSR: Very Special Linear Gravity. We have shown that one of the most important features of VSLG is the presence of a massive graviton with still only two physical d.o.f, due to the conservation of gauge invariance. Through the geodesic equation for GW, we tried to understand some of the VSR effects and their magnitude, finding results proportional to the perturbative parameter m_g^2/E^2. Despite the smallness of the new effects, more work should be carried on to give precise numerical predictions on distinct gravitational phenomena, above all in “integrated" ones: a clear example is the binary system’s energy loss over a period of time due to gravitational radiation, on which we are already working. In the end, the presence of a “gauge-invariant" graviton mass may be such an important feature that many gravitational areas of study would result affected, making VSLG worth exploring.

References

[1] A. G. Cohen and S. L. Glashow, Very special relativity, Phys. Rev. Lett., 97, 021601, 2006.

[2] A. G. Cohen and S. L. Glashow, A lorentz-violating origin of neutrino mass?, ArXiv.org, hep-ph/0605036, 2006.

[3] A. D. Sakharov, Violation of CP Invariance, C asymmetry, and baryon asymmetry of the universe, Pisma Zh. Eksp. Teor. Fiz., 5, 32, 1967.

[4] K. Aoki and S. Mukohyama, Massive gravitons as dark matter and gravitational waves, Phys. Rev. D, 94, 024001, 2016.

[5] A. De Felice, A. E. Gümrükçüoğlu, C. Lin, and S. Mukohyama, On the cosmology of massive gravity, Clas. Quant. Grav., 30, 184004, 2013.

[6] C. de Rham, J. T. Deskins, A. J. Tolley, and S. Y. Zhou, Graviton mass bounds, Rev. Mod. Phys., 89, 025004, 2017.

[7] A. V. Lukanenkov, Gravitational Telescope. J. of High Energy Physics, Gravit. Cosmol., 2, 209-225, 2016.

[8] J. Alfaro and A. Santoni, Very special linear gravity: A gauge-invariant graviton mass, Phys. Lett. B, 829, 137080, 2022.