

## Matter polarization effect on neutrino spin oscillations

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Neutrino electromagnetic properties are of great importance from the viewpoint of fundamental theory, and applications as well. It is of common knowledge that neutrinos determine the dynamics of supernova explosion to a large extent. In this work we study the effect of matter polarized by an external magnetic field on neutrino spin evolution and propagation inside supernovae. Alternatively, the problem of neutrino interaction with such kind of matter can be treated as the interaction of the induced neutrino magnetic moment (IMM) with the magnetic field. Using the corresponding interaction Lagrangian we obtain the effective evolution equation for a neutrino with IMM and, on its basis, consider neutrino spin oscillations for different cases of Dirac/Majorana neutrino type, absence/presence of neutrino anomalous magnetic moment (AMM). It is shown that due to IMM the neutrino flux from a supernova undergoes additional attenuation. Also, the effects of IMM and AMM when taken together can cancel each other leading to a specific maximum in the neutrino spectrum from supernovae.

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Neutrino induced magnetic moment (IMM) is an electromagnetic characteristic of a neutrino in a dispersive medium that corresponds to the contribution of pseudovector currents of medium particles into the effective vertex of neutrino electromagnetic interaction [1]. It can reach very high values as compared to the value of the conventional (anomalous) neutrino magnetic moment (AMM)  $\mu_\nu \simeq 3.2 \times 10^{-19} \mu_B$  ( $m/1 \text{ eV}$ ) that arises within the framework of the minimally extended Standard Model (SM). Specifically, the IMM for a degenerate electron gas is calculated to be

$$\mu^{\text{ind}} = -\frac{eG_F p_F}{4\sqrt{2}\pi^2} \simeq -2.2 \times 10^{-13} \mu_B \left( \frac{p_F}{1 \text{ MeV}} \right), \quad (1)$$

where  $\mu_B = e/2m_e$  is the Bohr magneton,  $e$  is the absolute value of the electron charge,  $p_F$  is the Fermi momentum

$$p_F \simeq 130 \times \left( \frac{n_e}{10^{37} \text{ cm}^{-3}} \right)^{1/3} \text{ MeV}, \quad (2)$$

and  $n_e$  is the number density of electrons. Instead of treating the interaction of neutrino with external magnetic field  $\mathbf{B}$  as being mediated by the IMM, one may equivalently say that the neutrino in fact interacts with matter polarized by this field [2]. In this work the second picture is adopted.

Our goal here is to study the effect of IMM on neutrino spin oscillations. In [3, 4], neutrino spin oscillations caused solely by IMM were considered using the spin precession treatment. In this paper we use the evolution equation approach and also account for complete matter and  $\nu_\mu$  contributions to the spin evolution. Accordingly, for one Dirac neutrino generation  $\nu_e$ , we write down the evolution equation in the form:

$$i \frac{\partial}{\partial t} \begin{pmatrix} \nu_e^{s=-1} \\ \nu_e^{s=+1} \end{pmatrix} = (\mathbf{H}_{\text{matt}} + \mathbf{H}_M) \begin{pmatrix} \nu_e^{s=-1} \\ \nu_e^{s=+1} \end{pmatrix}, \quad (3)$$

where  $s = \pm 1$  is the neutrino helicity. The effective Hamiltonians for neutrino matter and magnetic moment interactions,  $\mathbf{H}_{\text{matt}}$  and  $\mathbf{H}_M$ , are obtained through corresponding interaction Lagrangians.

Under the assumption that matter is composed by “usual” particles  $e, p, n$  and is at rest, the phenomenological Lagrangian for Dirac neutrino interaction with matter reads as [5]:

$$\Delta L_{\text{eff}} = -f^\mu \left( \bar{\nu} \gamma_\mu \frac{1}{2} (1 + \gamma^5) \nu \right), \quad (4)$$

where in the weak-field limit  $eB \ll p_F^2$  and for two neutrino generations ( $\nu_e$  and  $\nu_\mu$ ) [2]:

$$f_{e,\mu}^\mu = \{V_{e,\mu}, \mp 2\mu^{\text{ind}} \mathbf{B}\}, \quad V_e = \sqrt{2}G_F (n_e - n_n/2), \quad V_\mu = -\sqrt{2}G_F n_n/2. \quad (5)$$

The corresponding effective Hamiltonian of the evolution equation is then obtained as  $\mathbf{H}_{\text{matt}} = \langle \nu_e^s | \Delta H_{\text{eff}} | \nu_e^{s'} \rangle$ , where  $\Delta H_{\text{eff}}$  is the Hamiltonian, following from (4), and  $|\nu_e^{s'}\rangle$  is the neutrino free-state with definite helicity. Recalling the conventional magnetic moment contribution  $\mathbf{H}_M$  both Hamiltonian parts explicitly read as:

$$\mathbf{H}_{\text{matt}} = \begin{pmatrix} V_e + 2\mu^{\text{ind}} B_\parallel & -\mu^{\text{ind}} \gamma^{-1} B_\perp \\ -\mu^{\text{ind}} \gamma^{-1} B_\perp & 0 \end{pmatrix}, \quad \mathbf{H}_M = \begin{pmatrix} 2\mu_\nu \gamma^{-1} B_\parallel & -\mu_\nu B_\perp \\ -\mu_\nu B_\perp & 0 \end{pmatrix}, \quad (6)$$

where  $B_\parallel$  and  $B_\perp$  are the longitudinal and transverse components of the magnetic field with respect to the neutrino momentum  $\mathbf{p}$ ,  $\gamma = E_\nu/m_\nu$  is the neutrino gamma-factor.

Using the standard technique, from (3) and (6) we obtain the oscillation formula

$$P_{\nu_e^{s=-1} \rightarrow \nu_e^{s=+1}}(x) = \frac{(2(\mu_\nu^{\text{ind}} \gamma^{-1} + \mu_\nu) B_\perp)^2}{(2(\mu_\nu^{\text{ind}} \gamma^{-1} + \mu_\nu) B_\perp)^2 + (V_e + 2(\mu_\nu^{\text{ind}} + \mu_\nu \gamma^{-1}) B_\parallel)^2} \sin^2 \left\{ \sqrt{D} \frac{x}{2} \right\}, \quad (7)$$

where  $D$  is the denominator of the pre-sine factor. The resonance condition corresponds to the minimum of  $D$ , but in general, the transition probability is very small because in the weak-field limit the matter potential  $V_e$  is much larger than the terms containing  $B$ . However,  $V_e$  depends on matter composition and can turn to zero when the number of electrons per baryon  $Y_e = n_e/(n_p + n_n) \approx 1/3$  [6] so that the resonance can be achieved in the vicinity of this condition.

For effective conversion of neutrinos at the resonance point, the adiabatic condition must be also satisfied, requiring that the resonance width is of the order of or greater than half of the oscillation length. In our case it reads as:

$$\kappa = \frac{2(2(\mu_\nu^{\text{ind}} \gamma^{-1} + \mu_\nu) B_\perp)^2}{|dV_e/dr|} \gtrsim 1. \quad (8)$$

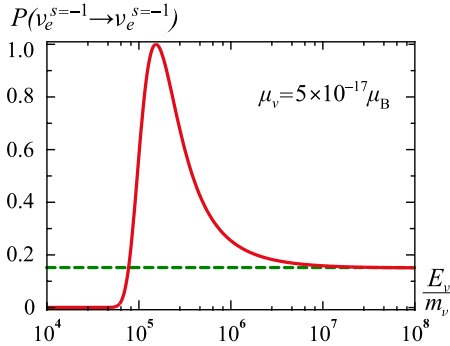
If the adiabaticity parameter  $\kappa \gg 1$ , then a complete conversion of left-handed neutrinos into right-handed takes place. When the adiabaticity condition is violated the survival probability can be estimated by the Landau-Zener formula

$$P_{\nu_e^{s=-1} \rightarrow \nu_e^{s=-1}} = \exp\left(-\frac{\pi}{2}\kappa\right). \quad (9)$$

Neglecting  $\mu_\nu$ , the condition (8) leads to the following restriction on the magnetic field:

$$B \gtrsim 1.2 \times 10^{12} \gamma \left( \frac{\rho_B}{10^{12} \text{ g/cm}^3} \right)^{1/6} \left( \frac{dY_e/dr}{10^{-8} \text{ cm}^{-1}} \right)^{1/2} \left( \frac{Y_e}{1/3} \right)^{-1/3} \text{ G}. \quad (10)$$

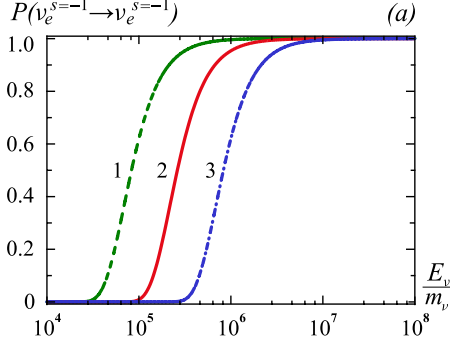
An important new feature is the dependence of Eqs. (8) and (9) on energy. To illustrate its meaning, we plot the dependence of the survival probability amplitude (9) as function of energy in Fig. (1). The relevant parameters chosen are indicated in the legend appended. If IMM is not accounted for, the probability has no dependence on energy (shown by the dashed line in the figure). As it is seen from the figure, the interplay between  $\mu_\nu$  and IMM results in a characteristic maximum in the supernova neutrino spectrum at relatively low energies  $E \lesssim 1$  MeV that in case of observation may signify that neutrino possesses a sufficiently large magnetic moment  $\mu_\nu \gtrsim 10^{-17} \mu_B$ .



**Figure 1:** The survival probability for Dirac neutrino with magnetic moment and with negative helicity depending on the neutrino energy:  $dY_e/dr = 10^{-9} \text{ cm}^{-1}$ ,  $\rho_B = 10^{12} \text{ g/cm}^3$ ,  $B = 6.6 \times 10^{16} \text{ G}$ ,  $Y_e = 1/3$ ,  $\mu_\nu = 5 \times 10^{-17} \mu_B$ . The dashed line corresponds to the conversion due to the magnetic moment only.

If the condition  $|\mu_\nu^{\text{ind}} \gamma^{-1}| \gg |\mu_\nu|$  is fulfilled, i.e. neutrino energy is not very high, then  $\mu_\nu$  can obviously be neglected. In this case instead of the peak, the rise of the survival probability is

expected above a certain energy value (see Fig. (2)). This leads to attenuation of the low-energy part of the neutrino spectrum. The conversion energy range depends on the value of  $dY_e/dr$ , this is shown on Fig. (2) by three different lines. After about 500 ms since the supernova core bounce, the derivative  $dY_e/dr$  grows fast and the flux attenuation terminates. For higher neutrino energies, upon the condition  $\mu_\nu^{\text{ind}}\gamma^{-1} = -\mu_\nu$  (possible for different signs of  $\mu_\nu$  and  $\mu_\nu^{\text{ind}}$  which is the case for  $\nu_e$  in the minimally extended SM), from Eq. (7) it follows that the oscillations disappear. For even higher energies, when  $|\mu_\nu| \gg |\mu_\nu^{\text{ind}}\gamma^{-1}|$ , neutrino spin dynamics is determined by  $\mu_\nu$  only.



**Figure 2:** The survival probability for Dirac neutrino with negative helicity as a function of neutrino energy; Line 1:  $dY_e/dr = 10^{-8} \text{ cm}^{-1}$ , Line 2:  $dY_e/dr = 10^{-9} \text{ cm}^{-1}$ , Line 3:  $dY_e/dr = 10^{-10} \text{ cm}^{-1}$ ;  $\rho_B = 10^{12} \text{ g/cm}^3$ ,  $B = 6.6 \times 10^{16} \text{ G}$ ,  $Y_e = 1/3$ .

If neutrino is of Majorana nature, the non-zero  $\mu_\nu^{\text{ind}}$  is possible in charge-asymmetric matter. In this case the results above are preserved but due to the doubling of entries in the effective neutrino Hamiltonians (7) and the resulting doubled adiabaticity parameter  $\varkappa$  the border of the energy region where conversion takes place (corresponding to  $\gamma \sim 10^5$  on Fig. (2)) shifts to higher energies.

By this means, we predict that account for neutrino IMM (or either matter polarization) makes the spin oscillation pattern energy-dependent. Also, under certain conditions, the ceasing of neutrino oscillations is possible. These features can be important for the formation of properties of the resulting neutrino flux coming from supernova and thus IMM should be taken into account when modelling the neutrino dynamics in supernovae.

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