

Deviations from isotropic turbulence of heavy-ion collision plasma

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At high collision energy, turbulence has been seen at relativistic heavy-ion collisions. We examine the turbulence characteristics in this system and found that the initial and pre-equilibrium phases of the collision show considerable departures from isotropic turbulence. Since the anisotropic fluctuations are subleading to the isotropic fluctuations, the Kolmogorov spectrum can be obtained even during the initial phases. However, departures from isotropic turbulence are indicated by the energy spectrum and temperature fluctuations. We analyse the energy density spectrum in the transverse and longitudinal planes by dividing the sphere into multiple planes since there is a significant momentum anisotropy between the two planes. We observe that the scaling exponent of the spectra is different in the two planes. This shows that the geometrical anisotropy is reflected in the anisotropic turbulence produced in the rotating plasma. The temperature spectrum is also obtained in the pre-equilibrium stage. The spectrum is different from what would be expected for isotropic turbulence. All of them seem to suggest that, for relativistic heavy-ion collisions, the large-scale momentum anisotropy exists in the smaller length scales.

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1. Introduction

The collective flow of the produced particles in heavy ion collision (HIC) experiments suggests the fluid dynamic nature of the system. This is also backed by the success of the viscous hydrodynamic models in describing these flows. The fluctuations present in the initial stages of the HIC system can develop irregularities in the flow. The transition from the initial state to the equilibrium state is an intriguing area of study. The system seeks to attain thermal equilibrium as it evolves. The system expands during this process, and the temperature of the system drops over time until equilibrium is established. A lot of energy is dissipated in the system. The flow instabilities may cause the plasma to behave like a turbulent flow. By examining the energy dissipation spectrum, it is possible to comprehend that the energy dissipation may take place at various turbulent flow length scales. By studying the temperature and velocity distribution in the collision region, we are interested in analysing the spectra of the initial instabilities.

2. Turbulence Spectrum for velocity field

In the early stages of plasma, the large angular momentum may cause the formation of vortices [1]. These vortices carry a substantial proportion of the incoming nuclei energy. Thus, a turbulent flow may develop. The fluid in a turbulent flow consists of continuously interacting swirls forming eddies or vortices. Since there is a large momentum anisotropy in different planes, we utilize the same planes to analyse the anisotropy in the turbulence spectra. The turbulent component is obtained by dividing the actual velocity into the laminar flow and the fluctuating component as, $\vec{u}(\vec{x}) = \vec{U}(\vec{x}) + \vec{u}'(\vec{x})$, where the laminar component is given by $\vec{U} = \langle \vec{u} \rangle$ and the turbulent component is \vec{u}' . We calculate the velocity correlation tensor R_{ij} for the turbulent velocity component at two points \vec{x} and $\vec{x} + \vec{r}$,

$$R_{ij}(\vec{r}) = \left\langle u'_i(\vec{x}) u'_j(\vec{x} + \vec{r}) \right\rangle \quad (1)$$

The energy spectrum tensor $E_{ij}(\vec{K})$ and the R_{ij} are related by,

$$E_{ij}(\vec{K}) = \frac{1}{(2\pi)^3} \int \int \int e^{-i\vec{K} \cdot \vec{r}} R_{ij}(\vec{r}) d\vec{r}. \quad (2)$$

When we compute the velocity correlation in the longitudinal plane, we must account for the Lorentz boost effect because the particles are colliding with relativistic velocities along the z axis. We utilised the formula from a recently published research [2] to determine the energy spectrum in our context. This involves the transfer of $u(x)$ to the new reference frame $u(x + \Delta x)$ which is obtained using the boost $\Lambda(\Delta x)$, $\Lambda(\Delta x)u(x + \Delta x) = u(x)$. The correlator is boosted to the local reference frame at the midpoint between the two points.

$$R_{ij} = \Lambda(d/2)\Lambda(-d/2) \langle u'_i(r - d/2), u'_j(r + d/2) \rangle \quad (3)$$

where $\Delta u = u(x + d/2) - u(x)$ and \vec{d} is now the line connecting the two positions. As we are working in a grid-based structure, we accomplished this by assuming an infinitesimal boost. The two-dimensional scalar kinetic energy spectrum $E(k)$ is calculated after we have R_{ij} , which is specified in equations 2 and 3. The turbulence spectrum of the velocity field on the longitudinal plane at a collision energy ($\sqrt{s_{NN}}$) of 200 GeV for the centrality ranges 0 – 10%, 20 – 40%, and

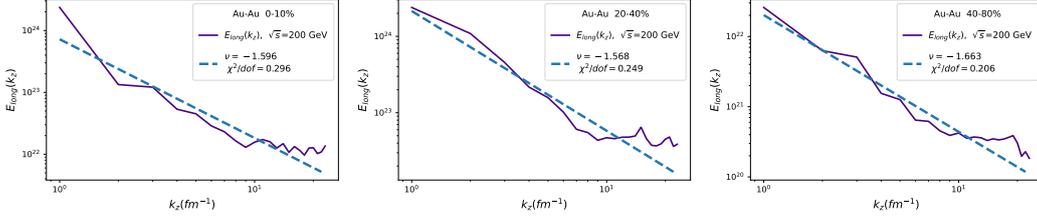


Figure 1: Longitudinal plane turbulence velocity spectra at $\sqrt{s_{NN}} = 200$ GeV. $\nu = -1.596, -1.568, -1.663$ for centrality range 0 – 10%, 20 – 40%, 40 – 80% respectively

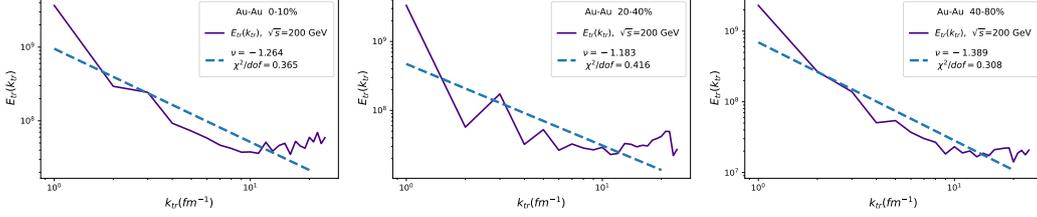


Figure 2: Transverse plane turbulence velocity spectra at $\sqrt{s_{NN}} = 200$ GeV. $\nu = -1.264, -1.183, -1.389$ for centrality range 0 – 10%, 20 – 40%, 40 – 80% respectively

40 – 80% are shown in Fig. 1. The dashed line is the best fit line in the range of k_z considered for our simulation. The range of k is fixed by calculating the range of underlying length scales of the system [3]. These distributions show a power-law behaviour where the exponent ν can be obtained by computing the slope of the fitted straight line. In every occasion, the exponent is around -1.6 for the longitudinal plane spectra, which is close to the Kolmogorov limit of $-5/3$.

The velocity components perpendicular to the z-axis remain unchanged because the system is only boosted along one axis. The transverse plane turbulence spectrum of the velocity field for 0 – 10%, 20 – 40%, and 40 – 80% centrality at $\sqrt{s_{NN}} = 200$ GeV is shown in Fig. 2. It is interesting to note that in none of these instances the power law exponent remains constant. In the presence of significant dissipative forces, a power law exponent of $-4/3$ is attained. These numbers seem to be closer to $-4/3$ than the Kolmogorov limit. The exponent is also obtained for other collision energies of the RHIC-BES program (19.6 – 200 GeV). It is always closer to $-5/3$ for the longitudinal plane and $-4/3$ for the transverse plane.

3. Power spectrum of temperature fluctuations

The temperature gradient in the direction perpendicular to the direction of flow causes the heat flow to develop. This corresponds to the energy conservation which is analogous to the conservation of momentum flow in the viscous diffusive medium. Once we are aware of the particle distribution, we segment the system into smaller cells. These cells each contain a sufficient number of particles to justify the assumption of local thermal equilibrium. Similar to velocity correlation, the temperature correlation $m(r)$ can be obtained as $m(r) = \langle TT' \rangle$. Here T and T' are the temperatures at any two points. If we assume that the temperature variation is a random function of space $\Phi(\mathbf{k})$, we can represent the correlation function as a stochastic Fourier integral.

$$m(r) = \langle TT' \rangle = 4\pi \int_0^\infty k^2 \Phi(k) \frac{\text{Sinkr}}{kr} dr \quad (4)$$

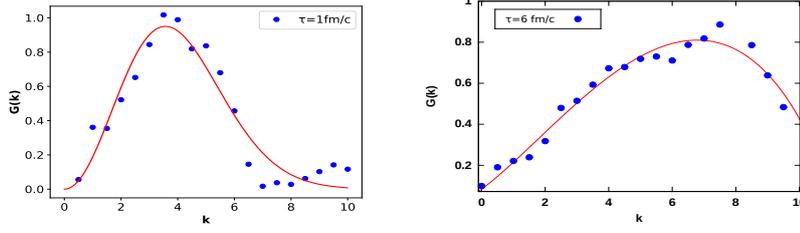


Figure 3: The power spectrum of the temperature fluctuations for $\sqrt{s_{NN}} = 200$ GeV Au-Au central collision events at $\tau = 1$ fm/c (left) and $\tau = 6$ fm/c (right). The units of k is in fm^{-1} . It is fitted with a Gaussian function (left) and q-Gaussian function (right).

Thus, we can obtain the power spectrum as $G(k) = 4\pi k^2 \Phi(k)$. The connection between the temperature correlation and the power spectrum is represented by,

$$G(k) = \frac{2}{\pi} \int_0^{\infty} m(r) k r \text{Sinkr} dr \quad (5)$$

We begin by assuming that the temperature fluctuations are isotropic in 3-dimension. However, we will see from the results that the fluctuation spectrum does not end up being Gaussian contrary to what is predicted for the isotropic case [4]. At two distinct times, we plot the power spectrum of the temperature fluctuations at $\sqrt{s} = 200$ GeV, which is shown in Fig. 3. We observe that the Gaussian fit in $\tau = 1$ fm/c plot is less adequate at higher k and shorter length scales. We found that as time progresses, the peak of the Gaussian shifts to smaller length scales and higher k values, which suggests that the majority of energy dissipation is shifted to smaller eddies. The q-Gaussian distribution provides a better fit to the spectra at later times. This is shown in $\tau = 6$ fm/c plot.

4. Conclusions

To summarise, we obtained the turbulent power spectrum for the HIC system under various initial conditions. We found different spectra coefficients, ν , for the two separate planes, which suggest different energy dissipation in the two planes. We also observed that the power-law coefficient is only affected by the collision's centrality for the transverse plane. The anisotropic pressure gradients produced in this plane result in spatial anisotropy in particle distribution. Further, the viscous nature of the fluid suppresses the collective flow in the transverse plane and also viscosity has a centrality dependence. This may cause asymmetric energy dissipation. The temperature spectrum we obtained also shows the anisotropic behaviour.

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