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Boson stars, primary photons and phase transition

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We analyse the possibility that the dark matter candidate, the dilaton, emerges from the approximate conformal symmetry of the hidden scalar sector. The study includes the warm dark matter scenario and the Bose-Einstein condensation which may lead to massive Boson stars giving rise to detection through the observation of the primary (direct) photons. We study the fluctuations of the scalar particles density under the extreme conditions of phase transition. When the phase transition approaches, the fluctuation of the particle density will increase sharply. Our results suggest that the phase transition in the Boson stars may be identified through the fluctuation in yield of primary photons induced directly by the conformal anomaly. The fluctuation rate of these photons emitted by the hidden scalar particles has an intensive growth and becomes very large at the phase transition. The dynamics and the properties of the dilatons at the temperature and the chemical potential around their critical values may be the keys to understand the evolution of the Universe.

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1. Introduction

The breaking of the scale invariance at high enough temperature (T) and chemical potential (μ) is an important issue in particle physics and cosmology since it is connected to the study of the dark matter (DM) hidden sector. The critical phenomena, if occurred, may be considered through the phase transition (PT) with the Bose-Einstein condensation (BEC) of the scalar fields. In this case, the condensation takes place in a single zero mode that suggests the breaking of conformal symmetry. In the conformal field theory (CFT) at the PT associated with the states which are invariant under the conformal group, the restoration of the symmetry relevant to the Higgs boson mass may be a cross-over [1]. The latter can influence the formation of the gravitationally bound system, so-called "Boson stars" (BS), composed of the DM scalar fields under the repulsive forces between them [2]. The gravitational instability of a spatially uniform state of the scalar fields realised by the self-interactions of these fields, has been investigated in [3]. The dilaton in the form of the scalar "glueball" field χ, the state of the gluon degrees of freedom (see, e.g., [4] and the refs. therein) with the mass m_χ of the order of the strong coupling scale Λ, is of the special interest because of the possible condensation of the glueballs into the BS [5]. The condensation emerges if the approximate conformal symmetry is broken spontaneously and the light pseudo-Goldstone boson, the dilaton, may appear in the spectrum [6]. The χ glueball as the candidate for DM could be cosmologically long lived if $m_\chi < 10^4$ TeV relevant to decay width of χ into two gravitons, $\Gamma_\chi \sim \tau_U^{-1}(m_\chi/10^4\mathrm{TeV})^5$, with respect to the age of the Universe $\tau_U \sim 10^{17}$ sec [7]. All the scales in the particle physics and the cosmology may be the subjects of a light Higgs boson resulting from the approximate conformal invariance of the standard model (SM) where the Higgs particle can be viewed as a dilaton. In the approach to the approximate conformal symmetry, the processes in the gluon and the electromagnetic (EM) sectors are governed by the conformal anomaly (CA). The interactions between the dilaton with the gluon, the photon and the dark photon (DP) fields strength tensors contribute much compared to that of the SM [8,9]. At finite temperatures, the thermal fluctuations and the Bose-Einstein correlations of the direct photons may emerge [10]. The sources of the photons in the vicinity of the PT are the dilatons.

2. The model

The dilaton field $\chi(x)$ can be introduced in the effective theory in terms of the dimensionless field $\sigma(x)$: $\chi(x) = f_\chi\, e^{\sigma(x)}$, where $\sigma(x)$ transforms as $\sigma(x) \to \sigma(xe^\omega)+\omega$ with ω being an arbitrary constant. When the scale symmetry is breaking down, the dilatation current $D^\mu = \theta^{\mu\nu} x_\nu$ is not conserved, and the constant f_χ is defined from the relation $\partial_\mu\langle 0|D^\mu(x)|\chi(p)\rangle = -f_\chi\, m_\chi^2\, e^{-i\,p\,x}$, where $\theta_{\mu\nu}$ is the energy-momentum tensor. The correlation length $\xi \sim m_\chi^{-1}$ is the indicator of the singular behaviour of the observable quantity at the stage close to the critical point (CP), where $\xi(T \to T_c) \to \infty$ at the critical temperature $T = T_c$ when the PT approaches. The χ field may be considered as the mediator between the conformal sector and the SM. At the critical temperature, the role of the mediator disappears, the dilaton becomes massless, the conformal symmetry is recovered, $\theta_\mu^\mu = 0$.

Let us consider the system of almost ideal scalar (e.g., the glueball) gas. The partition function for N scalar particles in the volume Ω at $\beta = T^{-1}$ is $Z_N = \sum_{\dots n_f \dots} e^{-\beta\sum_f F(f) n_f}$, where

$F(f) = E(f) - \mu\, Q(f)$ in the f-representation. Here, $E(f)$ is the energy, μ is the chemical potential associated with $Q(f)$, the conserved charge with an average density $\langle q \rangle = \Omega^{-1}\langle Q \rangle$, n_f is the occupation number related to the quantum operators of the creation and the annihilation. The thermal equilibrium in the volume Ω where the weak interplays between particles are supported by large values of n_f, may lead to BEC and the formation of the BS. At energies below the scale at which the scale symmetry is breaking, the action for the fields $\chi(x)$ organised in the BS and coupled to gravity is

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - K_\beta(\chi; m_\chi, \lambda)\right],$$

where R is the Ricci scalar, G is the gravitational constant; $K_\beta(\chi; m_\chi, \lambda)$ is the scalar potential. The repulsive self-interaction with λ in the scalar potential can lead to the compact dense BS. Let us consider the function $P(\bar\mu) = \exp\left[-N\Phi(\bar\mu)\right]$ which is the consequence of the power series $\sim \sum_{N=1}^{\infty} Z_N \bar\mu^N = \Pi_f[1 - \bar\mu e^{-F(f)\beta}]^{-1}$ at fixed N which may be very large and where $F(f) \geq 0$. Here, $\bar\mu = \mu/\mu_c$, μ_c is the critical chemical potential. Below T_c, the temperature-dependent potential

$$\Phi(\bar\mu) \sim \int \ln\left[1 - \bar\mu\, e^{-F(f)\beta}\right] df \tag{1}$$

gives the contribution from the glueballs to the total potential $K_\beta(\chi; m_\chi, \lambda)$ running with $\bar\mu$

$$K_\beta = \beta^{-1}\int\left[1 - \bar\mu\, e^{-F(f)\beta}\right]\frac{d^3\vec{p}}{(2\pi)^3} - \frac{1}{2}m_\chi^2\chi^2 + \frac{\lambda}{4}\chi^4\left(\ln\frac{\chi}{f_\chi} - \frac{1}{4}\right) + \lambda\left(\frac{f_\chi}{4}\right)^2.$$

At $T > T_c$, there is the gluon degrees of freedom contribution

$$K_g \simeq \frac{m_g^2}{\pi^2\beta^2}\sum_{n=1}^{\infty}\frac{C_n}{n}K_2(n\beta m_g), \tag{2}$$

where the temperature-dependent phenomenological "quasi-gluon" mass $m_g = m_g(\beta) = g(\beta)/\beta$, $g(\beta)$ is the effective gauge coupling; the colour coefficients C_n are given in [11]; $K_2(x)$ is the Bessel function. The quark contribution to the potential K_β at $T < T_c$ has the negative sign compared to that of the gluon part. The physical result can restrict the number of quark flavours. At zero chemical potential, the K_β will be positive up to 3 light quark flavours if the "constituent" mass of the "quasi-gluon" $\sim 0.5\, m_\chi \simeq 0.85$ GeV [12] and the "constituent" mass of the quark ~ 0.3 GeV are used. The gluons degrees of freedom contribution yields the first-order PT at the CP as found in the SU(3) lattice calculations [13,14]. The gluons are forbidden below the critical temperature as the coloured objects. The PT is related to the well-defined singularity with T and μ in the asymptotic form $\ln Z_N \simeq -N^{-1}\,\Phi(\bar\mu_0) - N\,\ln\bar\mu_0 - \ln 2\sqrt{\kappa\,\pi\,N}$ with $\kappa = -\Phi'(\bar\mu_0) - \Phi''(\bar\mu_0)$. Here, $\bar\mu_0$ is the point at which the distribution $P(\bar\mu)/\bar\mu^N$ has a single minimum, that means $\bar\mu_0$ corresponds to the ground state at given $\bar\mu$. The $\bar\mu_0$ can be estimated from the calculation of the sum

$$\sum_f \frac{1}{\bar\mu_0^{-1}\, e^{F(f)\beta} - 1} = N \tag{3}$$

up to singularity at $N \to \infty$. At large number of the particles, the sum in (3) is correct in the case of light glueballs with the mass $\sim O(\Lambda)$. It does not concern the phase diagram to scan the critical point in QCD where the position of the CP is not clear.

3. The thermal particle fluctuations

In the late Universe, the observed relic density of the DM can allow the dilaton to be almost warm with the density $\sim exp(-\Lambda\,\beta)$ which follows down very rapidly. The dilaton mass is the temperature - dependent smooth function governed by the integral in

$$m_\chi = \frac{\beta}{2}\left(\frac{2\,\pi^2}{\nu}\right)^{2/3}\left(\int_0^\infty \frac{x^2\,dx}{\bar{\mu}_0^{-1}\,e^{-\mu\,Q\beta}\,e^{x^2}-1}\right)^{-2/3}.$$

At high enough T when $\bar{\mu}_0\,e^{\mu\,Q\beta} < 1$, the singular behaviour of the correlation length comes from $\xi \sim [\mu\,Q/(2\,\pi)]\,(\nu\,B)^{2/3}\,\ln^{-1}\left(\bar{\mu}_0^{-1}\right)$, where $\nu = \Omega/N$ and $B = 2,612...$ is the Riemann's zeta-function, $\zeta(3/2)$. The ξ increases toward infinity when $\bar{\mu}_0 \to 1$, however, at $T << T_c$ the fluctuations are characterised by the small correlation length when $\bar{\mu}_0 < 1$. There is no PT caused by developing of ξ if ν is finite and small enough. Under the thermal influence, the non-monotonic behaviour of ξ is assumed to be as an indicator of the PT (see also [15] for the case to search the CP in QCD). When T lowers down below T_c, the only part of the total number of the dilatons $\sim (N/\Omega)(\beta_c/\beta)^{3/2}$ can be found in Ω. The rest one $\sim [1-(\beta_c/\beta)^{3/2}]$ is the scalar condensate.

The observation of the particles fluctuation is related with the fluctuation of the dilaton mass. For the compact BS of the local volume $V < \Omega$ defined by the geometry of the star, the number n_V of particles is $\sum_{1\leq j\leq N}\hat{n}_V(q_j)$. The weight function $\hat{n}_V(q) = 1$ if $q \in V$, while $\hat{n}_V(q) = 0$ otherwise. One can estimate the particle fluctuations through the standard event-by-event fluctuation of the particle density in the form $\langle(n_V - \langle n_V\rangle)^2\rangle$, where $\langle n_V\rangle = V/\Omega$. In the vicinity of the CP, when $T \to \mu\,Q/\ln(1/\bar{\mu}_0)$ and the PT approaches, we find

$$\frac{\langle(n_V - \langle n_V\rangle)^2\rangle}{\langle n_V\rangle} \simeq 1 + \frac{4}{\sqrt{\pi}\,B}\int_0^\infty \frac{x^2\,dx}{(z_c\,e^{x^2}-1)^2}, \tag{4}$$

where $z_c = \bar{\mu}_0^{-1}e^{-a_c}$, $a_c \simeq \mu_c\,Q\,\Lambda\,(\nu B)^{2/3}/(2\,\pi)$. One can expect the sharp rising of the fluctuation (4) when $\bar{\mu}_0 \to 1$ with $\nu \to 0$. There are no dependence on the dilaton mass related to the critical temperature by means of $T_c = (2\,\pi/m_\chi)(\nu\,B)^{-2/3}$.

4. The direct photons

In the effective theory, the interaction of the χ field with the SM is given by the LD $\sim (\chi/f)(\theta^\mu_{\mu_{tree}} + \theta^\mu_{\mu_{anom}})$, where $\theta^\mu_{\mu_{tree}}$ and $\theta^\mu_{\mu_{anom}}$ are the energy-momentum trace contributions to the SM sector ("tree") and the interactions with gluons, photons and DPs ("anom"), respectively. The term $\sim \theta^\mu_{\mu_{anom}}$ is the source of the effective production of the dilatons due to gluon-gluon fusion, and the direct production of the photons because of the dilatons decays. Since the EM and the QCD interactions are embedded in the CFT, the only quarks lighter than the dilaton are included in the calculations because of the conformal relation between the coefficients of the β-function for heavy and light flavours at the scales above Λ. The emission of the photons depends on the anomaly factor F_{anom} in the decay width related to the decay of the dilaton into two photons

$$\Gamma(\chi \to \gamma\gamma) \simeq \left(\frac{\alpha\,F_{anom}}{16\,\pi^{3/2}\,f_\chi}\right)^2 m_\chi^3, \tag{5}$$

where $F_{anom} = -(2n_L/3)(b_{EM}/b_0^{light})$, $b_0^{light} = -11 + (2/3)n_L$ with n_L being the number of light quark flavours in $\beta(g) = g^3 b_0^{light}/(4\pi)^2$, $g^2 = 4\pi\alpha_s$, α_s is the strong coupling constant and $b_{EM} = -8/3$ for 3 quark flavours. The dilaton mass fluctuates with the number of flavours N_f, $m_\chi^2 \simeq (1 - N_f/N_f^c)\Lambda^2$ [16], where the number N_f^c corresponds to the critical phase of the chiral symmetry breaking. In the lattice model [17] for the glueballs with the $SU(\mathcal{N})$ hidden sector at large $\mathcal{N}$, the parametrisation $m_\chi = (a + b/\mathcal{N}^2)\Lambda$ has been used, where $a < 1$ and $b \sim O(1)$. The self-coupling λ in the potential K_β is $\lambda \simeq 112$, if one uses the glueball mass $m_\chi = 1.7$ GeV [11] and the vacuum energy density $\sim \lambda(f_\chi/2)^4 = 0.6\ \mathrm{GeV}\, fm^{-3}$ [18].

Because of the effective couplings of the dilatons with the gluons and the photons, the abundant production of the dilatons due to gluon-gluon fusion and the decays of the dilatons to direct photons are expected. The photons escape is the decisive way to observe and to differentiate the direct photons and the ordinary photons in the decays of the secondary produced light hadrons. The fluctuation rate of the direct photons escape is

$$r_{\gamma\gamma} = 1 + m_\pi^3 \left(\frac{6}{F_{anom}}\right)^2 \xi^3, \tag{6}$$

where m_π is the mass of the π-meson as the example. At the PT, there will be the abundant escape of the photons as $\xi(T \to T_c) \to \infty$, $N_f \to N_f^c$ and $n_L \to 0$. The critical value $N_f = N_f^c$ separates the chiral symmetry breaking phase from that of the conformal one. The method is independent of the values of the model parameters, where for an order of magnitude one can take $f_\chi \simeq \Lambda$ and the pion constant $f_\pi \simeq 0.3\Lambda$. The direct photon fluctuations rate (6) is an indicator whether the hidden sector state is in the vicinity of the PT or not.

Finally, let us note that in the paper [7], the authors presented the direct point-like coupling of the hidden sector associated with the scalar glueball field χ in the BS to the photons within the hidden $SU(\mathcal{N})$ gauge theory, where the value $\mathcal{N}$ is unspecified. The following transformation of the interactions is used

$$\frac{1}{M_{cut}^4} H_{\mu\nu} H^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \to \frac{\mathcal{N}\, m_\chi^3}{M_{cut}^4} \chi\, F_{\alpha\beta} F^{\alpha\beta},$$

where $H_{\mu\nu}$ and $F_{\alpha\beta}$ are the strength tensors of the hidden gauge field of the group $SU(\mathcal{N})$ and the photon, respectively; M_{cut} is the cutoff scale. The decay rate of χ into two photons in the direct interaction in the star is

$$\Gamma(\chi \to \gamma\gamma) = \frac{1}{4\pi} m_\chi\, \mathcal{N}^2 \left(\frac{m_\chi}{M_{cut}}\right)^8, \tag{7}$$

where for the self-interacting hidden matter χ, the $\mathcal{N}$ takes the maximal value in the interval $\sim [(0.1 GeV/m_\chi)^{3/4}, 2]$. The combined result from the conformal anomaly (5) and the direct interaction (7) gives the strongest constraints on the scale M_{cut} with the scalar mass in the MeV's scale. For $m_\chi \sim O(\Lambda)$ the cutoff M_{cut} has the following lower bounds in the weak scale: $M_{cut} \geq 3.4$ GeV and $M_{cut} \geq 5.2$ GeV for $\Lambda = 330$ MeV and $\Lambda = 500$ MeV, respectively.

5. Conclusions

To conclude, we investigated the possible evidence of the DM candidate from an approximate conformal symmetry. The DM is the lightest hidden scalar field which is likely the dilaton or the glueball. The scalar fields could be warm and may have the feature of the BEC into the compact massive Boson stars. In the vicinity of the PT, the correlation length ξ may be much larger than the size of the interaction region. In this case, ξ should be as large as $\xi \sim N^{2/3} \ln(\bar{\mu}_0^{-1}) \cdot 10^{-20}\, GeV^{-1}$ for N scalar particles with the mass m_χ at $\bar{\mu}_0 \to 1$, where the maximal mass of the BS, $M_{max}^{BS} \sim \sqrt{\lambda}\, G^{-3/2}\, m_\chi^{-2}$, is used [5]. The formation of the BS up the critical size and the maximal mass may explode it into leptons via decays of the dark photons or may emit the direct photons that could contribute to new sources of cosmic rays. The fluctuation rate $r_{\gamma\gamma}$ grows up to become very large at the PT. Both the PT and the CP have the very clear signature: the shower increasing of the photons flow in the detector compared to that produced by light hadrons.

References

[1] M. D'Onofrio, K. Rummukainen, *Phys. Rev. D* **93**, 025003 (2016).

[2] M. Colpi, S.L. Shapiro, and I. Wasserman, *Phys. Rev. Lett.* **57**, 2485 (1986).

[3] M.Yu. Khlopov, B.A. Malomed and Ya.B. Zeldovich, *Mon. Not. R. astr. Soc.* **215**, 575 (1985).

[4] S.R. Choudhury and A.N. Mitra, *Phys. Rev. D* **28**, 2201 (1983).

[5] M.P. Hertzberg, F. Rompineve, and J. Yang, *Phys. Rev. D* **103**, 023536 (2021).

[6] W.A. Bardeen, C.N. Leung, and S.T. Love, *Phys. Rev. Lett.* **56**, 1230 (1986).

[7] A. Soni and Y. Zhang, *Phys. Rev. D* **93**, 115025 (2016).

[8] W.D. Goldberger, B. Grinstein, and W. Skiba, *Phys. Rev. Lett.* **100**, 111802 (2008).

[9] G.A. Kozlov, *Eur. Phys. J. Plus* **136**, 906 (2021).

[10] G. Kozlov, *Mod. Phys. Lett. A* **26**, 1943 (2011).

[11] C. Sasaki and K. Redlich, *Phys. Rev. D* **86**, 014007 (2012).

[12] J. Sexton, A. Vaccarino, and D. Weingarten, *Phys. Rev. Lett.* **75**, 4563 (1995).

[13] G. Boyd et al., *Nucl. Phys. B* **469**, 419 (1996).

[14] O. Kaczmarek, F. Karsch, P. Petreczky, F. Zantow, *Phys. Lett. B* **543**, 41 (2002).

[15] M.A. Stephanov, *Int. J. Mod. Phys. A* **20**, 4387 (2005).

[16] T. Appelquist and Y. Bai, *Phys. Rev. D* **82**, 071701(R) (2010).

[17] B. Lucini and M. Teper, *JHEP* **06**, 050 (2001).

[18] S. Narison, *Nucl. Phys. B Proc. Suppl. A* **54**, 238 (1997).

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