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Results on spin sum rules and polarizabilities at low Q^2

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We report on recently published experimental results on spin sum rules, and particularly on the generalized spin polarizabilities $\gamma_0(Q^2)$ (for both the proton and neutron) and $\delta_{\mathrm{LT}}(Q^2)$ (for the neutron). The data were taken at Jefferson Lab in Hall A by experiment E97110 (neutron) and in Hall B by experiments E03006 and E05111 (proton and deuteron, respectively). The experiments covered the very low Q^2 domain, down to $Q^2 \simeq 0.02$ GeV^2. This is well into the domain where Chiral Effective Field Theory (χEFT) predictions should be valid. Some measured obervables agree with the state-of-the-art χEFT predictions but others are in tension, including $\delta^n_{\mathrm{LT}}(Q^2)$ which χEFT prediction was expected to be robust. This suggests that χEFT does not yet consistently describe nucleon spin observables, even at the very low Q^2 covered by the experiments.

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1. Introduction and background

Chiral effective field theory (χEFT) is the leading effective theory describing the first level of complexity emerging from the Standard Model, *viz* how the fundamental quarks and gluons produce hadronic and nuclear phenomena. As such, χEFT is a crucial component of our global understanding of Nature. It has been very successful in explaining hadronic and nuclear physics [1]. Yet, its description of nucleon spin structure remains imperfect. Table 1 (adapted from [2]) lists nucleon spin observables measured in the late 1990s and early 2000s at Jefferson Lab (JLab) and shows how well the χEFT predictions available at the time described them. (What the observables mean is unimportant here. Their definitions will be given in latter sections.) The table shows that the early χEFT predictions were in tension with the data more often than not. Particularly puzzling was the discrepancy for the spin polarizability δ^n_{LT} because its χEFT prediction was expected to be robust owing to the suppression of the Δ_{1232} resonance contribution to δ_{LT}. This contribution is difficult to account for and was either not included in the early predictions [9, 11], or approximately included phenomenologically [10]. Was the origin of the discrepancy a χEFT calculation problem, maybe with the Δ_{1232}? Or was it because the data were not at low enough Q^2 to reach the χEFT applicability domain? To answer these questions, refined χEFT calculations with improved expansion schemes and including the Δ_{1232} contribution were undertaken [12, 13] and new experiments reaching well into the χEFT applicability domain were performed. Here, we present results from that experimental program [14–18].

	Γ_1^p [3, 4]	Γ_1^n [4, 5]	Γ_1^{p-n} [6, 7]	Γ_1^{p+n} [4, 7]	γ_0^p [4]	γ_0^n [8]	γ_0^{p-n} [7]	γ_0^{p+n} [4, 7]	δ_{LT}^n [8]
Ji *et al.* [9]	**X**	**X**	**A**	**X**	-	-	-	-	-
Bernard *et al.* [10]	**X**	**X**	**A**	**X**	**X**	**A**	**X**	**X**	**X**
Kao *et al.* [11]	-	-	-	**X**	**A**	**X**	**X**	-	**X**

Table 1: Nucleon spin observables measured at JLab by experiments E94010 and EG1 [3–8] compared to early predictions from χEFT [9–11]. The **A** (**X**) letter indicates that data and prediction agree (disagree) over the range $0 \leq Q^2 \lesssim 0.1$ GeV^2. The **-** indicates that no prediction was available at the time.

2. Experimental method

The observables listed in Table 1 are measured with inclusive inelastic lepton scattering in which a lepton (for JLab, an electron) of momentum p scatters off a nucleon or nucleus at rest in the laboratory frame. The lepton transfers a momentum $q = (\nu, \vec{q})$ to the nucleon/nucleus whose fragments are not detected. Here, we will work within the one-photon exchange approximation, where $Q^2 \equiv -q^2 > 0$ quantifies how virtual the exchanged photon is. The experiments discussed here, E97110 [16], E03006 [15] and E05111 [14] were performed at JLab, a facility located in Newport News, Virginia USA, that accelerates a continuous electron beam to energies up to 12 GeV. The beam polarization for an experiment is typically ~85%. Up to 200 μA of beam can be circulated. It supplies four experimental halls, A, B C and D equipped either with high resolution (A and C) or large acceptance (B and D) spectrometers. E97110 occured in Hall A and E03006/E05111 (commonly referred to as Experimental Group EG4) in Hall B during the 6 GeV era of JLab, before its upgrade to 12 GeV.

The observables in Table 1 are obtained by integrating over ν the nucleon polarized structure functions g_1 and g_2 [2]. To reach low Q^2 while keeping the wide ν range necessary for the integration, a high-energy beam (here up to 4.4 GeV) is needed and the scattered electrons must be detected at small angles (here, down to about 6°). These angles were reached in Hall A thanks to a new horizontally-bending dipole magnet placed in front of the spectrometer [19]. In Hall B, a dedicated Cherenkov Counter optimized for high efficiency at small angle was added to one of the six sectors (otherwise identical) of the spectrometer [14]. In addition, the Hall B target was moved 1 m upstream of its usual position and the spectrometer magnetic field was set to bent the electrons outward. E97110 studied the spin structures of the neutron and ^{3}He thanks to the Hall A polarized ^{3}He target, using both its longitudinal and transverse polarization capabilities. In particular, the latter allowed to measure g_2, which is crucial to form $\delta_{\rm LT}$. E03006/E05111 studied the proton, deuteron and neutron spin structures with the Hall B longitudinally polarized ammonia (NH_3 or ND_3).

3. Generalized spin polarizabilities

Polarizabilities encode the second order reaction of a body subjected to an electromagnetic field, e.g. the reaction of a nucleon probed by a low energy photon [20]. The complete reaction is described by two Compton scattering amplitudes, f_1 (spin-independent) and f_2 (spin-dependent). Considering for now real photons ($Q^2 = 0$), one can expand f_1 and f_2 in term of ν:

$$f_1(\nu) = -\frac{\alpha}{M} + \left(\alpha_E + \beta_M\right)\nu^2 + O(\nu^4),$$
$$f_2(\nu) = -\frac{\alpha\kappa^2}{2M^2}\nu + \gamma_0\nu^3 + O(\nu^5),$$

where α is the electromagnetic coupling, M is the nucleon mass, κ its anomalous magnetic moment, α_E and β_M are respectively the electric and magnetic polarizabilities, and γ_0 is the forward spin polarizability. The first term in the equations ($\propto \alpha$) represents the purely elastic reaction expected from a perfectly rigid (or pointlike) object. The second term defines the polarizabilities and reflects the deformation of the object, i.e. its internal rearrangement. For virtual photons ($Q^2 \neq 0$), the polarizabilities acquire a Q^2-dependence – they are then named *generalized polarizabilities* – and because virtual photons have a longitudinal spin component, the Longitudinal-Transverse polarizability $\delta_{\rm LT}$ appears.

It is not known how to measure directly generalized spin polarizabilities. Instead, they are measured indirectly using the sum rules [21]:

$$\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1(x,Q^2) - \frac{4M^2}{Q^2}x^2 g_2(x,Q^2)\right]dx, \qquad (1)$$

$$\delta_{\rm LT}(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1(x,Q^2) + g_2(x,Q^2)\right]dx, \qquad (2)$$

where $x = Q^2/2pq$ and x_0 is the inelastic threshold.

The function g_1 (and g_2 for E97-110) is measured from x_0 to a minimum non-zero x, since reaching $x = 0$ with non-zero Q^2 requires infinite beam energy. $\gamma_0(Q^2)$ and $\delta_{\rm LT}(Q^2)$ are then obtained by integrating these measurements according to Eqs. (1) and (2) and using a parameterization [14] to estimate the missing low-x contribution.

4. Experimental results on the generalized spin polarizabilities $\gamma_0(Q^2)$ and $\delta^n_{\rm LT}(Q^2)$

Results on the generalized spin polarizabilities $\gamma_0^p(Q^2)$ [15], $\gamma_0^n(Q^2)$ [17], their isospin decomposition $\gamma_0^{p\pm n}(Q^2)$, and $\delta^n_{\rm LT}(Q^2)$ [17] are shown in Fig. 1.

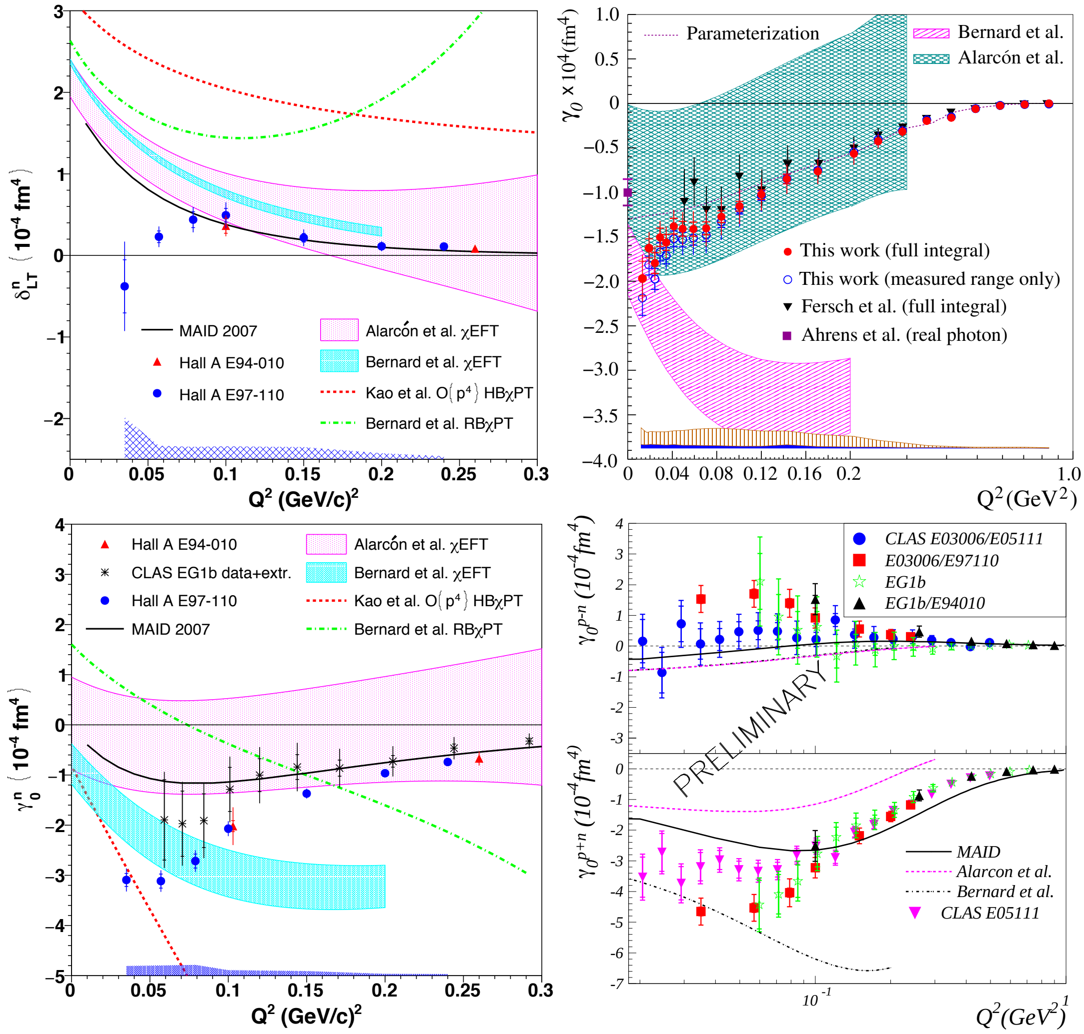

Figure 1: Data on the generalized spin polarizabilities $\delta^n_{\rm LT}(Q^2)$ (top left), $\gamma_0^p(Q^2)$ (top right), $\gamma_0^n(Q^2)$ (bottom left) and the isospin decomposition of $\gamma_0(Q^2)$ (bottom right). Inner error bars, sometimes too small to see, give the statistical uncertainties. Outer error bars are the quadratic sum of statistical and uncorrelated systematic uncertainties. The horizontal bands give the correlated systematic uncertainties. Also shown are state-of-the-art χEFT calculations [12, 13] (Alarcón *et al.* χEFT, Bernard *et al.* χEFT), earlier calculations [10, 11] (Kao *et al.* HBχPT, Bernard *et al.* RBχPT) and results from the MAID model.

The $\delta^n_{\rm LT}$ data from E97110 agree well with the earlier E94010 data. At $Q^2 \gtrsim 0.08$ GeV^2, the data agree with the latest χEFT calculations [12, 13] and the phenomenological MAID model [22]. Data and predictions disagree at lower Q^2 despite the fact that there, χEFT calculations should be most robust. Therefore, while the agreement between the latest χEFT calculations and E94010 data suggested that the $\delta_{\rm LT}$ puzzle may be solved, the new data refute it. The renewed surprise with $\delta^n_{\rm LT}$ makes it interesting to investigate the integral $I_{\rm LT}(Q^2)$ [23] since it involves the same integrand as $\delta_{\rm LT}$ but without the x^2 weighting:

$$I_{\rm LT}(Q^2) = \frac{8M^2}{Q^2}\int_0^{x_0}\Big[g_1(x,Q^2) + g_2(x,Q^2)\Big]dx. \quad (3)$$

The Schwinger sum rule gives $I_{\rm LT}(Q^2) \xrightarrow[Q^2\to 0]{} \kappa e$, with e the target particle electric charge i.e. $e = 0$ for the neutron. $I^n_{\rm LT}$ is shown on the bottom right panel of Fig. 2. The sum rule expectation $I^n_{\rm LT}(0) = 0$ agrees with the E97-110 data once they are guided to $Q^2 = 0$ using the expected behavior of $I_{\rm LT}(Q^2)$ from the Gerasimov-Drell-Hearn (GDH) sum rule [24] and elastic form factors. Since the GDH sum rule is solid, with its validity verified to good accuracy [25], and the form factors are well measured, the agreement strengthens confidence in the data quality. Yet, it cannot entirely rule out possible issues with the low-x extrapolation (important in $I_{\rm LT}$ while suppressed in $\delta_{\rm LT}$) or high-x contamination from elastic/quasi-elastic reactions (enhanced in $\delta_{\rm LT}$ compared to $I_{\rm LT}$). The former is unlikely because the low-x issue would need to conspire with a problem in the data themselves so that $I_{\rm LT}$, Γ_1 and Γ_2 (Fig. 2) still agree with their expectations. A large-x contamination would have to be mild enough so that $I_{\rm LT}$, Γ_1 and Γ_2 still conform to expectations but important enough so that $\delta_{\rm LT}$ does not.

The E03006 data on γ^p_0 agree well with the earlier EG1 data (Fersch *et al.*) and the χEFT result of Alarcón *et al.* They agree with that of Bernard *et al.* only for the lowest Q^2 points. Also shown in the top right panel of Fig. 1 is the datum at $Q^2 = 0$ from MAMI [26]. At first, it may seem incompatible with the E03006 data but their extrapolation to $Q^2 = 0$ assuming either the results of Bernard *et al.* or Alarcón *et al.* shows that under this assumption, the data of JLab and MAMI agree within uncertainties [25].

The Hall A E97110 data on γ^n_0 agree with the earlier EG1 (Hall B) and E94010 (Hall A) data, but not with the predictions except at higher Q^2 for Alarcón *et al.* and MAID. Also shown are the older χEFT calculations [10, 11].

The bottom right panel of Fig. 1 shows the isospin decomposition of γ_0. The new and previous data agree. The E03006/E97110 and E03006/E05111 combinations agree with each other, but there is a tension at lower Q^2. The two combinations differ in the origin of their neutron information (from ^{3}He for E03006/E97110 and D for E03006/E05111) but also in the proton one since the proton presents in D affects both $\gamma^{p\pm n}_0$ quantities: p−n≃2p−D and p+n≃D. The two combinations suggest that γ^{p-n}_0 remains positive in the Q^2 domain experimentally covered, in contrast to the χEFT and MAID predictions. For γ^{p+n}_0, both combinations agree with Bernard *et al.* for the lowest Q^2 points, and disagree with Alarcón *et al.* and MAID.

5. Results on first moments

The first moment $\Gamma_1 \equiv \int g_1 dx$ is shown in Fig. 2 for the proton, neutron, deuteron and the Bjorken sum Γ^{p-n}_1. First moments $I^n_{\rm LT}$ and $\Gamma^n_2 \equiv \int g^n_2 dx$ are also shown. The same observations as for γ^p_0 stand for Γ^p_1: the new data agree well with the earlier EG1 data and with the latest χEFT calculations, albeit only for the lowest Q^2 points for Bernard *et al.* The same holds for Γ^D_1. The

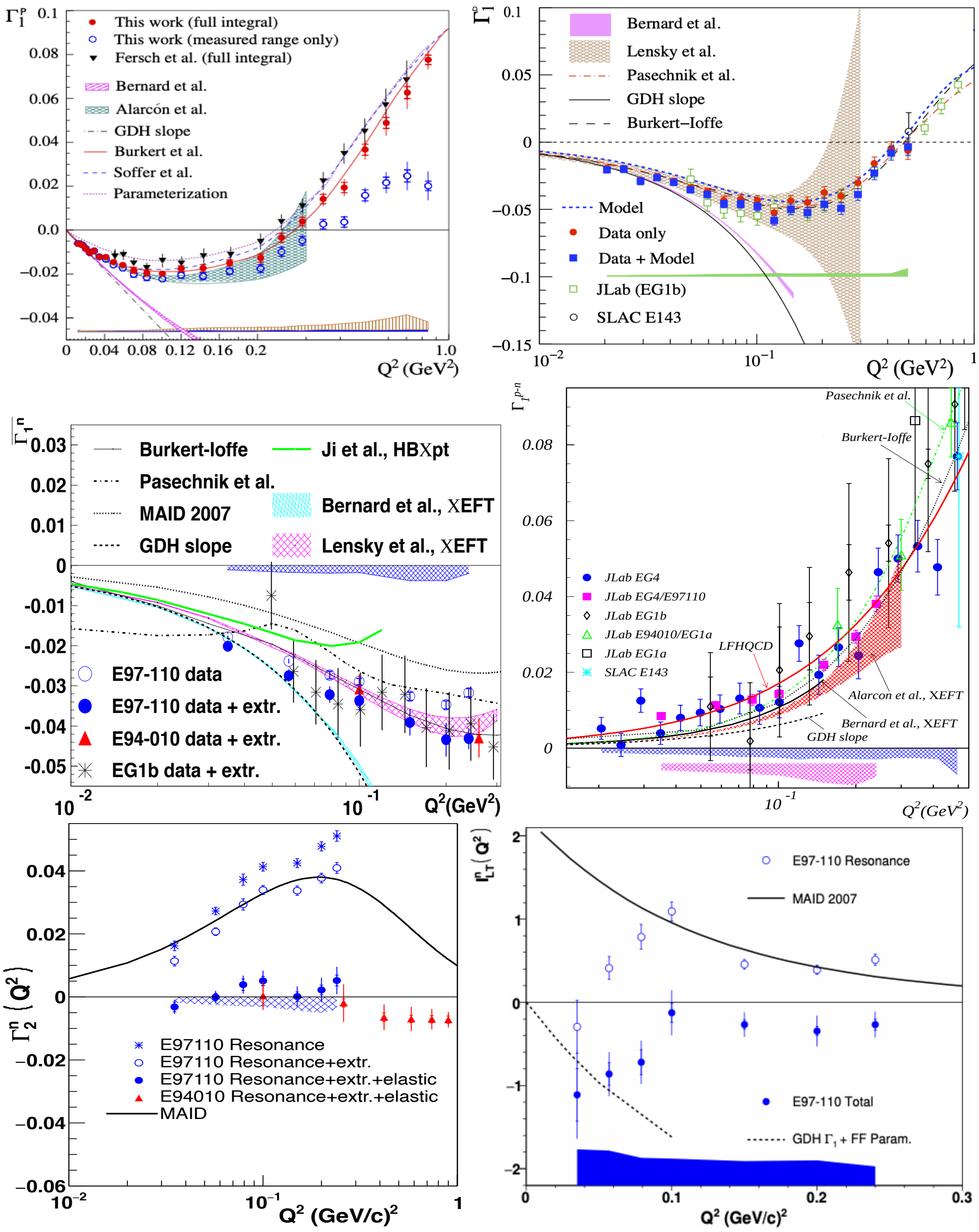

Figure 2: The first moments Γ_1 (first 4 panels), Γ_2^n (bottom left) and I_{LT}^n (bottom right).

E97110 data on Γ_1^n agree reasonably with the latest χEFT calculations, in contrast to γ_0^n. The new Hall A data agree well with the earlier Hall B (EG1) and Hall A (E94010) data.

The combinations E03006/E05111 and E03006/E97110 used to form the Bjorken sum Γ_1^{p-n} agree with each other and with the earlier data [6, 7, 27]. They also agree qualitatively with the predictions from χEFT and the several available models [28, 29] but the predictions are systematical larger at low Q^2 (except the LFHQCD calculation [29]). This makes the $\Gamma_1^{p-n} = bQ^2 + cQ^4$ fit to the data, performed to provide quantitative comparisons, to yield a parameter b larger than the GDH expectation and a c (the χEFT prediction *per se*) of sign opposite to the χEFT expectations [18].

The E97110 data for $\Gamma_2^n \equiv \int_0^1 g_2^n dx$ (where in contrast to other moments, the $x = 1$ elastic contribution is included in the integral) agree well with the earlier data and with the Burkhardt-Cottingham sum rule expectation that $\Gamma_2 = 0$ [30]. For I_{LT}^n, see discussion in the previous section.

6. Conclusion

We can revisit Table 1, adding to it the refined χEFT calculations and the new data at lower Q^2 and of improved precision. The δ_{LT}^p preliminary data refer to those of JLab Hall A E08027 (see K. Slifer and D. Ruth contributions to these proceedings). An advance is that all the observables in the table are now predicted by χEFT. Furthermore, there is a better agreement between data and predictions than in the past. Yet, puzzles remain. While for the Bjorken sum Γ_1^{p-n} there is qualitative agreement, which was expected since the Δ_{1232} is suppressed in Γ_1^{p-n} [31], there are disagreements for δ_{LT} and γ_0^{p-n} where the Δ_{1232} is also suppressed and have the additional advantage that as higher moments, they have little missing low-x contribution. On the other hand, a complication with γ_0 and δ_{LT} is that their value and slope at $Q^2 = 0$ must be calculated. In contrast, $\Gamma_1(0)$ is known as it *must* vanish, and its slope is given by the GDH sum rule.

	Γ_1^p [3, 4]	Γ_1^n [4, 5]	Γ_1^{p-n} [6, 7]	Γ_1^{p+n} [4, 7]	γ_0^p [4]	γ_0^n [8]	γ_0^{p-n} [7]	γ_0^{p+n} [4, 7]	δ_{LT}^p	δ_{LT}^n [8]
Ji *et al.* [9]	**X**	**X**	**A**	**X**	-	-	-	-	-	-
Bernard *et al.* [10]	**X**	**X**	**A**	**X**	**X**	**A**	**X**	**X**	**X***	**X**
Kao *et al.* [11]	-	-	-	-	**X**	**A**	**X**	**X**	**X***	**X**
Bernard *et al.* [12]	**X**	**X**	~**A**	**X**	**X**	**A**	**X***	**X***	**X***	**X**
Alarcón *et al.* [13]	**A**	**A**	~**A**	**A**	~**A**	**X**	**X***	**X***	**A***	**X**

Table 2: Same as Table 1 but including the latest data and χEFT results. The * denotes preliminary data and ~**A** either an approximate agreement or an agreement over a range significantly smaller than $Q^2 < 0.1$ GeV^2.

Therefore, despite its success in many domains, χEFT remains challenged by nucleon spin observables, the latest data coming from dedicated low Q^2 experiments. Low Q^2 sum rule measurements are undeniably challenging: forward angle detection is difficult to reach and subjected to large backgrounds, a large ν range is needed, there are low-x extrapolations, avoiding high-x contamination requires a careful analysis... An additional challenge for neutron data is that nuclear corrections are needed and, while the general agreement between the neutron data coming from deuteron and ^{3}He is encouraging, one may ask how reliable the corrections are at low Q^2. Yet, the experiments – old and new – provide consistent results and conclusion while being independent and having quite different detectors and methods. One must note the disagreement between the state-of-the-art χEFT predictions. But it does not necessarily indicate an inconsistency. Rather, this seems to mostly arise from including [13] or not [12] phenomenological estimates of higher order terms of the χEFT series. Therefore, it remains unclear what the origin of the experiment/theory discrepancy is. A possibility to advance further, even if no new experiments measuring these observables are foreseen and calculating the next order of the χEFT series is very difficult, is to compute the observables with other non-perturbative approaches, e.g. that based on the Dyson-Schwinger Equations, Lattice QCD, Gauge-Duality (AdS/QCD) or global phenomenological analyses like MAID or SAID [25]. It is important to resolve this issue: it challenges our search for a description of Nature at all level since χEFT is the leading approach to manage the first level of complexity arising above the strong force sector of the Standard Model.

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