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Improved Standard-Model prediction for the dilepton decay of the neutral pion

Bai-Long Hoid,[a,*] Martin Hoferichter,[a] Bastian Kubis,[b] and Jan Lüdtke[b,c]

[a] *Albert Einstein Center for Fundamental Physics, Institute for Theoretical Physics, University of Bern, Sidlerstrasse 5, 3012 Bern, Switzerland*

[b] *Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, 53115 Bonn, Germany*

[c] *Faculty of Physics, University of Vienna, Boltzmanngasse 5, 1090 Vienna, Austria*

E-mail: longbai@itp.unibe.ch, hoferichter@itp.unibe.ch, kubis@hiskp.uni-bonn.de, jan.luedtke@univie.ac.at

We present the recent work on an improved Standard-Model prediction for the rare decay $\pi^0 \to e^+e^-$, which plays a crucial role in the test of the long-distance dynamics of the strong interaction. The reduced amplitude of the decay is determined by the pion transition form factor for $\pi^0 \to \gamma^*\gamma^*$, for which we employ a dispersive representation that incorporates both time-like and space-like data as well as short-distance constraints. The resulting Standard-Model branching fraction, $\text{BR}[\pi^0 \to e^+e^-] = 6.25(3) \times 10^{-8}$, reveals a ten-fold improvement in precision over experiment and sharpens constraints on physics beyond the Standard Model.

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1. Introduction

The main decay modes of the neutral pion are all of electromagnetic nature, in which the two-photon channel yields almost exclusively its total width. This decay is described by the pion transition form factor (TFF) at zero momentum transfer $F_{\pi^0\gamma^*\gamma^*}(0,0)$, which in turn is determined by the chiral anomaly [1–3]

$$F_{\pi\gamma\gamma} \equiv F_{\pi^0\gamma^*\gamma^*}(0,0) = \frac{1}{4\pi^2 F_\pi} = 0.2745(3)\,\text{GeV}^{-1}, \tag{1}$$

where $F_\pi = 92.28(10)$ MeV is the pion decay constant [4]. In addition, the Dalitz decay $\pi^0 \to e^+e^-\gamma$ and the double Dalitz decay $\pi^0 \to 2(e^+e^-)$ also play indispensable roles in examining the neutral pion's properties.

We focus on the rare decay $\pi^0 \to e^+e^-$ in this work [5]. The current best measurement of $\text{BR}[\pi^0 \to e^+e^-]$ is provided by the KTeV experiment at the level of 5% [6]. Performing the extrapolation in line with the latest radiative corrections [7, 8], one finds the full branching fraction

$$\text{BR}[\pi^0 \to e^+e^-]|_{\text{KTeV}} = 6.85(27)(23) \times 10^{-8}, \tag{2}$$

with significant difference to the extrapolation should the previous radiative corrections under a point-like vertex assumption be applied [9] .

Its main contribution in the Standard Model (SM) arises from the loop diagram shown in Fig. 1, where the underlying input becomes the pion TFF. With the apparent loop and chiral suppression [10], an early suggestion of this rare decay to search for physics beyond the SM (BSM) traces back to [11]. More recent theoretical advances concern phenomenological estimates [12–14] and lattice QCD [15]. Here, we present a SM prediction that is based on a dispersive representation of the pion TFF as a dedicated effort to obtain a fully data-driven determination of the pion-pole contribution [16–18] in a dispersive approach to hadronic light-by-light (HLbL) scattering [19–25].

2. Pion transition form factor

The pion TFF is defined by the matrix element of two light-quark currents

$$i\int \text{d}^4x\, e^{iq_1\cdot x}\, \langle 0|T\{j_\mu(x)\, j_\nu(0)\}|\pi^0(q_1+q_2)\rangle = \epsilon_{\mu\nu\alpha\beta}\, q_1^\alpha\, q_2^\beta\, F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2), \tag{3}$$

where the sign convention follows from [26–28] to ensure consistency with the short-distance constraints and the Z-boson contribution. The Primakoff measurement tested the form factor normalization $F_{\pi\gamma\gamma}$ up to 0.8% [29, 30]. Detailed studies of this form factor have been performed in the context of HLbL scattering [17, 18, 31, 32]; see also [33–38] in other contexts.

Before turning to the application to $\pi^0 \to e^+e^-$, we first describe the representation we will use for the normalized TFF $\tilde{F}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) = F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)/F_{\pi\gamma\gamma}$ in the following. The constructed form factor decomposition,

$$\tilde{F}_{\pi^0\gamma^*\gamma^*} = \tilde{F}^{\text{disp}}_{\pi^0\gamma^*\gamma^*} + \tilde{F}^{\text{eff}}_{\pi^0\gamma^*\gamma^*} + \tilde{F}^{\text{asym}}_{\pi^0\gamma^*\gamma^*}, \tag{4}$$

has the following remarkable properties: its first dispersive part takes into account all low-energy singularities, extracted from data on $e^+e^- \to 2\pi, 3\pi$ [39–45]; the second (small) contribution from

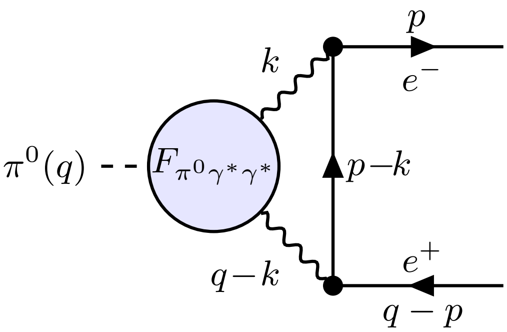

Figure 1: The SM contribution to $\pi^0 \to e^+e^-$: the dominant $\pi^0 \to \gamma^*\gamma^*$ diagram. The gray blob represents the pion TFF.

higher intermediate states incorporates the normalization and space-like high-energy data [46–49]; and the asymptotic constraints for arbitrary virtualities as expected from perturbative QCD [50–54] are implemented via the last term at $O(1/Q^2)$. We have checked that our representation smoothly connects the various constraints on the pion TFF for the kinematic configuration relevant for $\pi^0 \to e^+e^-$. More details of the TFF representation are relegated to [5, 17, 18]; see also [55–57] for further applications to hadronic vacuum polarization.

3. Standard Model prediction for $\pi^0 \to e^+e^-$

The ratio of the two branching fractions,

$$\frac{\mathrm{BR}[\pi^0 \to e^+e^-]}{\mathrm{BR}[\pi^0 \to \gamma\gamma]} = 2\sigma_e(q^2)\Big(\frac{\alpha}{\pi}\Big)^2 \frac{m_e^2}{M_{\pi^0}^2}|\mathcal{A}(q^2)|^2, \tag{5}$$

is typically expressed in terms of the reduced amplitude

$$\mathcal{A}(q^2) = \frac{2i}{\pi^2 q^2}\int \mathrm{d}^4k \frac{q^2k^2-(q\cdot k)^2}{k^2(q-k)^2[(p-k)^2-m_e^2]} \times \tilde{F}_{\pi^0\gamma^*\gamma^*}(k^2,(q-k)^2), \tag{6}$$

where $q^2 = M_{\pi^0}^2$, $\sigma_e(q^2) = \sqrt{1-4m_e^2/q^2}$, and p is the momentum of the outgoing electron. The only relevant imaginary part arises from the $\gamma\gamma$ cut, which reads

$$\mathrm{Im}\,\mathcal{A}(q^2) = \frac{\pi}{2\sigma_e(q^2)}\log\Big[\frac{1-\sigma_e(q^2)}{1+\sigma_e(q^2)}\Big] = -17.52, \tag{7}$$

and leads to the unitarity bound $\mathrm{BR}[\pi^0 \to e^+e^-] > 4.69\times 10^{-8}$ [58, 59].

To obtain $\mathrm{Re}\,\mathcal{A}(q^2)$ we need to evaluate the integral (6) for our representation of the pion TFF. For the dispersive part one may write

$$\mathcal{A}^{\mathrm{disp}}(q^2) = \frac{2}{\pi^2}\int_{4M_\pi^2}^{s_{\mathrm{iv}}} \mathrm{d}x \int_{s_{\mathrm{thr}}}^{s_{\mathrm{is}}} \mathrm{d}y \frac{\tilde{\rho}(x,y)}{xy} K(x,y), \tag{8}$$

where the integration kernel

$$K(x,y) = \frac{2i}{\pi^2q^2}\int \mathrm{d}^4k \frac{q^2k^2-(q\cdot k)^2}{k^2(q-k)^2[(p-k)^2-m_e^2]} \times \frac{xy}{(k^2-x)[(q-k)^2-y]} \tag{9}$$

can be evaluated based on standard loop functions [60–62]. The effective-pole contribution and the asymptotic piece can be evaluated in a similar vein. In practice, the numerical integration over the double-spectral function in (8) requires a stable implementation of $K(x, y)$ over a wide parameter range. We verified the numerical stability by comparing several different methods, including the Gegenbauer-polynomial technique [63–65], semi-analytic expressions in terms of polylogarithms [66], and the implementation from LoopTools [67].

In the end, we find the long-range contribution

$$\operatorname{Re}\mathcal{A}(q^2)|_{\gamma^*\gamma^*} = 10.16(10). \tag{10}$$

At this level of precision, we also need to account for the Z-boson contribution [60]

$$\operatorname{Re}\mathcal{A}(q^2)|_{Z} = -\frac{F_\pi G_F}{\sqrt{2}\,\alpha^2 F_{\pi\gamma\gamma}} = -0.05(0). \tag{11}$$

Both contributions lead to the final SM prediction,

$$\operatorname{Re}\mathcal{A}(q^2)|_{\mathrm{SM}} = 10.11(10), \qquad \mathrm{BR}[\pi^0 \to e^+e^-]|_{\mathrm{SM}} = 6.25(3)\times 10^{-8}, \tag{12}$$

showing a mild 1.8σ tension with the KTeV measurement (2). In particular, the latter implies

$$\operatorname{Re}\mathcal{A}(q^2)|_{\mathrm{KTeV}} = 11.89^{+0.94}_{-1.02}, \tag{13}$$

which, in comparison to (12), can directly be used to constrain BSM effects.

4. Constraints on physics beyond the Standard Model

The comparison between our improved SM prediction (12) and the KTeV measurement (13) sharpens the constraints on BSM scenarios. As a first application, for new interactions of axial-vector and pseudoscalar type,

$$\mathcal{L}^{(1)}_{\mathrm{BSM}} = C_A \bar{q}\frac{\tau^3}{2}\gamma^\mu\gamma_5 q\,\bar{e}\gamma_\mu\gamma_5 e + C_P\bar{q}\frac{\tau^3}{2}i\gamma_5 q\,\bar{e}i\gamma_5 e, \tag{14}$$

with $q = (u, d)^T$, the limits of the couplings derived from $\pi^0 \to e^+e^-$ read

$$C_A = (-280)^{+160}_{-150}\,\mathrm{TeV}^{-2}, \quad C_P = (-0.108)^{+0.062}_{-0.057}\,\mathrm{TeV}^{-2}. \tag{15}$$

Assuming $C_{A,P} \sim 1/\Lambda^2_{A,P}$, the sensitivity of these limits translates to mass scales $\Lambda_A \sim 0.1\,\mathrm{TeV}$, $\Lambda_P \sim 4\,\mathrm{TeV}$, reflecting the chiral enhancement for the latter. Matching onto four-fermion operators in SM effective field theory [68, 69], the constraints translate to

$$C_A = \frac{1}{4}\Big(C_{eu} - C_{ed} - C_{\ell u} + C_{\ell d} - 2C^{(3)}_{\ell q}\Big), \quad C_P = \frac{1}{4}\Big(C^{(1)}_{\ell equ} - C_{\ell edq}\Big). \tag{16}$$

Although the one for C_A is not very stringent, the different combination of Wilson coefficients in contrast to those in parity-violating electron scattering or atomic parity violation may still be helpful to close flat directions in the parameter space [70–72].

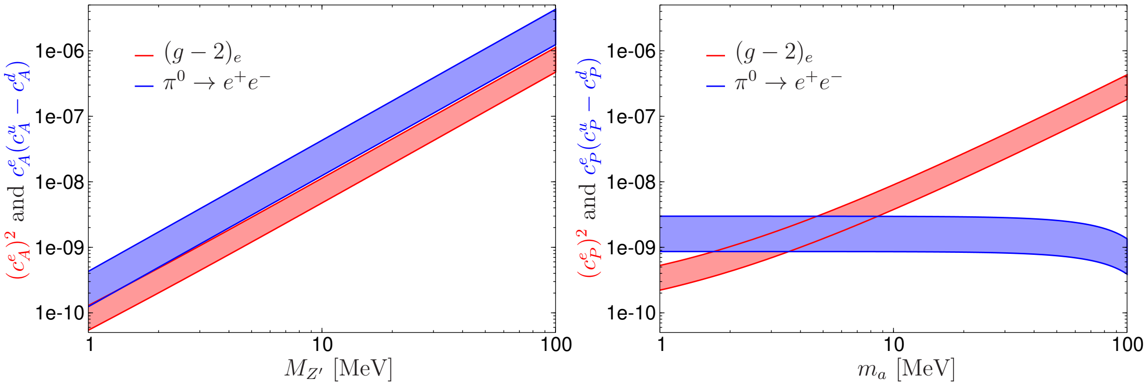

Figure 2: 1σ parameter ranges of light axial-vector (left) and pseudoscalar (right) mediators preferred by $\pi^0 \to e^+e^-$ (blue) and Δa_e[Cs] (red). (We restrict the masses to the parameter region below M_{π^0}, and neglected the potential π^0–a mixing.) Figures taken from [5].

As another application, we consider light axial-vector (Z') or pseudoscalar (a) states:

$$\mathcal{L}_{\text{BSM}}^{(2)} = \sum_{f=e,u,d} \bar{f}\Big(c_A^f \gamma^\mu \gamma_5 Z'_\mu + c_P^f i\gamma_5 a\Big) f, \tag{17}$$

which leads to

$$C_A = -\frac{(c_A^u - c_A^d)c_A^e}{M_{Z'}^2}, \qquad C_P = \frac{(c_P^u - c_P^d)c_P^e}{m_a^2 - q^2}, \tag{18}$$

where the new particles correspond to a Z' or an axion-like particle a, respectively. We consider the interplay with the anomalous magnetic moment of the electron a_e to constrain the parameter space, which is timely given the current tensions between the direct measurement [73] and the SM prediction [74, 75] either based on the fine-structure constant measured with Cs [76] or Rb [77] atom interferometry

$$\begin{aligned}\Delta a_e[\text{Cs}] &= a_e^{\text{exp}} - a_e^{\text{SM}}[\text{Cs}] = -0.88(36) \times 10^{-12},\\ \Delta a_e[\text{Rb}] &= a_e^{\text{exp}} - a_e^{\text{SM}}[\text{Rb}] = 0.48(30) \times 10^{-12},\end{aligned} \tag{19}$$

corresponding to a tension of -2.5σ and $+1.6\sigma$, respectively. With the pending 5.4σ disagreement between [76] and [77], we consider here the case of Δa_e[Cs], as a negative effect can be explained by axial-vector or pseudoscalar mediators [78, 79]. As illustrated in Fig. 2, our improved SM prediction and the radiative corrections [7, 8] applied to the KTeV measurement updates the the previously favored region for the axial-vector case [76, 80]. Figure 2 also shows that if both mild tensions were confirmed at this level, similar regions in parameter space seem to be preferred.

5. Conclusions

We have reported on an improved SM prediction for the decay $\pi^0 \to e^+e^-$, by means of a dispersive representation of the pion transition form factor. This representation implements constraints from all available data and ensures a smooth matching to short-distance constraints. This leads to a SM prediction with a precision of 0.5% (12), whose conceptual advances could also be applied to the dilepton decays of $\eta^{(\prime)}$ [81].

We then calculated the resulting constraints on axial-vector and pseudoscalar operators from the deviation between our SM prediction and the KTeV measurement, both in SM effective field theory and for light mediators. In particular, our calculation of the $\pi^0 \to e^+e^-$ width now exceeds experiment by an order of magnitude, allowing for concurrent advances in BSM constraints should there be an improved measurement. Such efforts are in progress at NA62 [82].

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