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Theoretical analysis of $\eta^{(\prime)} \to \pi^0 l^+ l^-$ and $\eta' \to \eta l^+ l^-$ decays, and new-physics signatures via CP violation

Rafel Escribano,[a,b] **Emilio Royo**[a,b,*] **and Pablo Sanchez-Puertas**[b]

[a] *Grup de Física Teòrica, Departament de Física, Universitat Autònoma de Barcelona, E-08193 Bellaterra (Barcelona), Spain*

[b] *Institut de Física d'Altes Energies (IFAE), The Barcelona Institute of Science and Technology, Campus UAB, E-08193 Bellaterra (Barcelona), Spain*

E-mail: rescriba@ifae.es, eroyo@ifae.es, psanchez@ifae.es

A complete theoretical analysis of the C-conserving semileptonic decays $\eta^{(\prime)} \to \pi^0 l^+ l^-$ and $\eta' \to \eta l^+ l^-$ ($l = e$ or μ) is carried out within the framework of the Vector Meson Dominance (VMD) model. An existing phenomenological model is used to parametrise the VMD coupling constants and the associated numerical values are obtained from an optimisation fit to $V \to P\gamma$ and $P \to V\gamma$ radiative decays ($V = \rho^0$, ω, ϕ and $P = \pi^0$, η, η'). Theoretical predictions for the six decay widths and associated dilepton energy spectra are calculated and presented. In addition, we investigate the prospect of observing new-physics signatures via CP violation in $\eta^{(\prime)} \to \pi^0 \mu^+ \mu^-$ and $\eta' \to \eta \mu^+ \mu^-$ decays at the REDTOP experiment. We make use of the SMEFT to parametrise the new-physics CP-violating effects and find that the projected REDTOP statistics are not competitive with respect to nEDM experiments for these particular processes.

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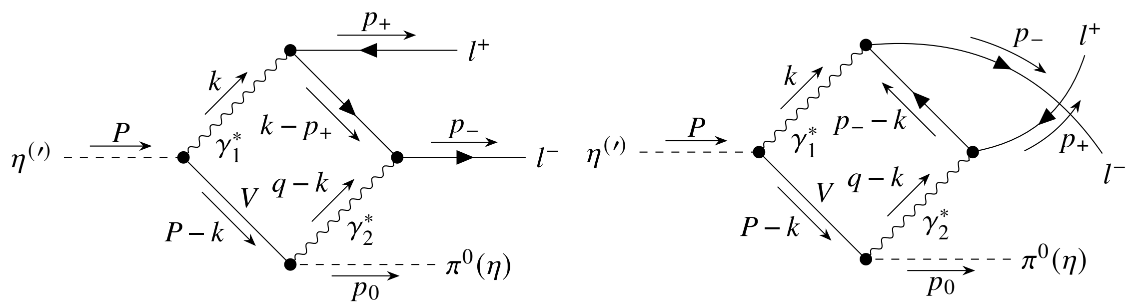
1. Introduction

The electromagnetic and strong interactions conserve parity (P) and charge conjugation (C) within the well-established and well-tested Standard Model of particle physics (SM). In this framework, the η and η' pseudoscalar mesons are specially suited for the study of rare decay processes, for instance, in search of C, P and CP violations, as these mesons are C and P eigenstates of the electromagnetic and strong interactions [1].

Specifically, the semileptonic decays $\eta^{(\prime)} \to \pi^0 l^+ l^-$ and $\eta' \to \eta l^+ l^-$ ($l = e$ or μ) are of special interest given that they can be used as fine probes to assess if new physics beyond the Standard Model (BSM) is at play. This is because these processes are induced at loop level in the SM via the exchange of two photons, with tree-level electroweak-boson exchange forbidden on the basis of C and P, and negligible Higgs contributions. As an example, the $\eta^{(\prime)} \to \pi^0 l^+ l^-$ and $\eta' \to \eta l^+ l^-$ decays could be mediated by a single intermediate virtual photon, but this would entail that the electromagnetic interactions violate C-invariance (e.g. [2, 3]) and, therefore, would represent a departure from the SM.

Early theoretical studies of semileptonic decays of pseudoscalar mesons date back to the late 1960s. A very significant contribution was made by Cheng in Ref. [4] where he analysed the $\eta \to \pi^0 e^+ e^-$ decay mediated by a C-conserving, two-photon intermediate state within the Vector Meson Dominance (VMD) framework. By setting the electron mass to $m_e = 0$ and neglecting in the numerator of the amplitude terms that were second or higher order in the electron or positron 4-momenta, he found theoretical estimations for the decay width $\Gamma(\eta \to \pi^0 e^+ e^-) = 1.3 \times 10^{-5}$ eV, the relative branching ratio $\Gamma(\eta \to \pi^0 e^+ e^-)/\Gamma(\eta \to \pi^0 \gamma\gamma) \approx 10^{-5}$, as well as the associated decay energy spectrum. A different approach was followed by Smith [5] also in the late 1960s, whereby an S-wave $\eta\pi^0\gamma\gamma$ coupling and unitary bounds were used for the calculation of the C-conserving modes associated to both $\eta \to \pi^0 l^+ l^-$ decay processes. By neglecting p-wave contributing terms to simplify the calculations and noting that the unknown $\eta\pi^0\gamma\gamma$ coupling constant cancels out when calculating relative branching ratios, Smith was able to find $\Gamma(\eta \to \pi^0 e^+ e^-)/\Gamma(\eta \to \pi^0 \gamma\gamma) = 3.6 \times 10^{-8}$ and $\Gamma(\eta \to \pi^0 \mu^+ \mu^-)/\Gamma(\eta \to \pi^0 \gamma\gamma) = 6.0 \times 10^{-5}$, after estimating the real part of the matrix element from a single dispersion relation and employing a cut-off $\Lambda = 2m_\eta$.

Ng et al. [6] also found in the early 1990s lower limits for the decay widths of the two $\eta \to \pi^0 l^+ l^-$ processes by making use of unitary bounds and the decay chain $\eta \to \pi^0 \gamma\gamma \to \pi^0 l^+ l^-$. The transition form factors associated to the $\eta \to \pi^0 \gamma\gamma$ decay, which are required to perform the above calculation, were obtained using the VMD model supplemented by the exchange of an a_0 scalar meson. The lower bounds that they found are $\Gamma(\eta \to \pi^0 e^+ e^-)|_{\text{VMD}} = 1.1^{+0.6}_{-0.5}$ μeV and $\Gamma(\eta \to \pi^0 \mu^+ \mu^-)|_{\text{VMD}} = 0.5^{+0.3}_{-0.2}$ μeV, making use of VMD only. By adding the a_0 exchange to the latter process, they obtained $\Gamma(\eta \to \pi^0 \mu^+ \mu^-)|_{\text{constr}} = 0.9^{+0.6}_{-0.5}$ μeV and $\Gamma(\eta \to \pi^0 \mu^+ \mu^-)|_{\text{destr}} = 0.3^{+0.4}_{-0.2}$ μeV for a constructive and destructive interference, respectively. The real parts of the amplitudes were estimated by means of a cut-off dispersive relation and the authors argued that the expected dispersive contribution should be no larger than 30% of the absorptive one. A few months later, Ng and Peters provided in Ref. [7] new estimations for the unitary bounds of the $\eta \to \pi^0 l^+ l^-$ decay widths. This new contribution was two-fold; on one hand, they calculated the $\eta \to \pi^0 \gamma\gamma$ decay width within a constituent quark model framework; on the other hand, they recalculated the VMD transition form factors from Ref. [6] by performing a Taylor expansion and keeping terms linear in



(a) t-channel Feynman diagram

(b) u-channel Feynman diagram

Figure 1: Feynman diagrams contributing to the C-conserving semileptonic decays $\eta^{(\prime)} \to \pi^0 l^+ l^-$ and $\eta' \to \eta l^+ l^-$ ($l = e$ or μ). Note that $q = p_+ + p_-$ and $V = \rho^0, \omega, \phi$.

M_η^2/M_V^2, x_1 and x_2 ($x_i \equiv P_\eta \cdot q_{\gamma_i}/M_\eta^2$), which had been neglected in their previous work. Their new findings were: (i) $\Gamma(\eta \to \pi^0 e^+ e^-)|_{\text{box}} \geq 1.2 \pm 0.2$ μeV and $\Gamma(\eta \to \pi^0 \mu^+ \mu^-)|_{\text{box}} \geq 4.3 \pm 0.7$ μeV for a constituent quark mass $m = 330$ MeV$/c^2$; and (ii) $\Gamma(\eta \to \pi^0 e^+ e^-)|_{\text{VMD}} \geq 3.5 \pm 0.8$ μeV and $\Gamma(\eta \to \pi^0 \mu^+ \mu^-)|_{\text{VMD}} \geq 2.4 \pm 0.8$ μeV. It is important to highlight that their estimations using the quark-box mechanism were strongly dependent on the specific constituent quark mass selected, especially for the electron mode.

On the experimental front, new upper limits have recently been established by the WASA-at-COSY collaboration for the $\eta \to \pi^0 e^+ e^-$ decay width [8]. This is a useful contribution, as the previous available empirical measurements date back to the 1970s. In particular, Adlarson et al. [8] found from the analysis of a total of 3×10^7 events of the reaction pd $\to {}^3$He$+\eta$, with a recorded excess energy of $Q = 59.8$ MeV, that the results are consistent with no C-violating single-photon intermediate state event being recorded. Based on their analysis, the new upper limits $\Gamma(\eta \to \pi^0 e^+ e^-)/\Gamma(\eta \to \pi^+ \pi^- \pi^0) < 3.28 \times 10^{-5}$ and $\Gamma(\eta \to \pi^0 e^+ e^-)/\Gamma(\eta \to \text{all}) < 7.5 \times 10^{-6}$ (CL = 90%) have been established for the C-violating $\eta \to \pi^0 \gamma^* \to \pi^0 e^+ e^-$ decay. The experimental state of play is expected to be further improved in the near future with the advent of new experiments such as REDTOP, which will focus on rare decays of the η and η' mesons, providing increased sensitivity in the search for violations of SM symmetries by several orders of magnitude beyond the current experimental state of the art [9].

2. $\eta^{(\prime)} \to \pi^0 l^+ l^-$ and $\eta' \to \eta l^+ l^-$ semileptonic decays

In this work, the SM calculations assume that the $\eta^{(\prime)} \to \pi^0 l^+ l^-$ and $\eta' \to \eta l^+ l^-$ processes are dominated by the exchange of vector resonances; that is, they proceed through the C-conserving virtual transition $\eta^{(\prime)} \to V\gamma^*$ (with $V = \rho^0$, ω or ϕ), followed by $V \to \pi^0 \gamma^*$ (or $V \to \eta\gamma^*$) and $2\gamma^* \to l^+ l^-$ (see Fig. 1 for details).

We start by selecting an effective vertex that contains the appropriate interacting terms. The $VP\gamma$ interaction amplitude consistent with Lorentz, P, C and electromagnetic gauge invariance can be written as [10]

$$\mathcal{M}(V \to P\gamma) = g_{VP\gamma}\, \epsilon_{\mu\nu\alpha\beta}\, \epsilon^\mu_{(V)} p^\nu_V \epsilon^{*\alpha}_{(\gamma)} q^\beta \hat{F}_{VP\gamma}(q^2)\,, \tag{1}$$

where $g_{VP\gamma}$ is the coupling constant for the $VP\gamma$ transition involving on-shell photons, $\epsilon_{\mu\nu\alpha\beta}$ is the totally antisymmetric Levi-Civita tensor, $\epsilon_{(V)}$ and p_V are the polarisation and 4-momentum vectors of the initial V, $\epsilon^*_{(\gamma)}$ and q are the corresponding ones for the final γ, and $\hat{F}_{VP\gamma}(q^2) \equiv F_{VP\gamma}(q^2)/F_{VP\gamma}(0)$ is a normalised form factor to account for off-shell photons mediating the transition[1]. In addition to this, the usual QED vertex is used to describe the subsequent $2\gamma^* \to l^+l^-$ transition. Accordingly, there are six diagrams (two per vector meson) contributing to each one of the six semileptonic decay processes and the corresponding Feynman diagrams are shown in Fig. 1.

The invariant decay amplitude in momentum space can be written as

$$\mathcal{M} = ie^2 \sum_{V=\rho^0,\omega,\phi} g_{V\eta^{(\prime)}\gamma} g_{V\pi^0(\eta)\gamma} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2+i\epsilon} \frac{1}{(k-q)^2+i\epsilon} \epsilon_{\mu\nu\alpha\beta} \left[\frac{k^\mu (P-k)^\alpha (k-q)^\rho (P-k)^\delta}{(P-k)^2 - m_V^2 + i\epsilon} \right]$$
$$\epsilon_{\rho\sigma\delta}{}^{\beta} \bar{u}(p_-) \left[\gamma^\sigma \frac{\not{k} - \not{p}_+ + m_l}{(k-p_+)^2 - m_l^2 + i\epsilon} \gamma^\nu + \gamma^\nu \frac{\not{p}_- - \not{k} + m_l}{(k-p_-)^2 - m_l^2 + i\epsilon} \gamma^\sigma \right] v(p_+) \,, \tag{2}$$

where $q = p_+ + p_-$ is the sum of lepton-antilepton pair 4-momenta, e is the electron charge, and $g_{V\eta^{(\prime)}\gamma}$ and $g_{V\pi^0(\eta)\gamma}$ are the corresponding VMD coupling constants. After calculating the loop integral, one arrives at the following expression for the unpolarised squared amplitude

$$\overline{|\mathcal{M}|^2} = 4\Big\{2(P\cdot p_+)(P\cdot p_-) - m^2_{\eta^{(\prime)}}\left[(p_+\cdot p_-) + m_l^2\right]\Big\} \times \mathrm{Abs}(\Omega)^2$$
$$+ 8m_l^2\left[(P\cdot p_+) - (P\cdot p_-)\right]\mathrm{Re}(\Omega\Sigma^*) + 4m_l^2\left[(p_+\cdot p_-) - m_l^2\right]\mathrm{Abs}(\Sigma)^2 \,, \tag{3}$$

with Ω and Σ defined as

$$\Omega = \sum_{V=\rho^0,\omega,\phi} \alpha_V + \sigma_V \,, \quad \Sigma = \sum_{V=\rho^0,\omega,\phi} \beta_V + \tau_V \,, \tag{4}$$

where the parameters α_V, β_V, σ_V and τ_V are (see Ref. [11] for details)

$$\alpha_V = e^2 \frac{g_{V\eta^{(\prime)}\gamma} g_{V\pi^0(\eta)\gamma}}{16\pi^2} \int dxdydz \left[\frac{2A_1}{\Delta_{1V} - i\epsilon} - \frac{B_1}{(\Delta_{1V} - i\epsilon)^2} \right] , \tag{5}$$

$$\beta_V = e^2 \frac{g_{V\eta^{(\prime)}\gamma} g_{V\pi^0(\eta)\gamma}}{16\pi^2} \int dxdydz \left[\frac{2C_1}{\Delta_{1V} - i\epsilon} - \frac{D_1}{(\Delta_{1V} - i\epsilon)^2} \right] , \tag{6}$$

$$\sigma_V = e^2 \frac{g_{V\eta^{(\prime)}\gamma} g_{V\pi^0(\eta)\gamma}}{16\pi^2} \int dxdydz \left[\frac{2A_2}{\Delta_{2V} - i\epsilon} - \frac{B_2}{(\Delta_{2V} - i\epsilon)^2} \right] , \tag{7}$$

$$\tau_V = e^2 \frac{g_{V\eta^{(\prime)}\gamma} g_{V\pi^0(\eta)\gamma}}{16\pi^2} \int dxdydz \left[\frac{2C_2}{\Delta_{2V} - i\epsilon} - \frac{D_2}{(\Delta_{2V} - i\epsilon)^2} \right] . \tag{8}$$

In addition, the denominators in Eqs. (5–8), i.e. Δ_{1V} and Δ_{2V}, can be expressed as

$$\Delta_{1V} = 2yz(P\cdot q) + 2xy(p_+\cdot q) + (y-1)yq^2 + 2xz(P\cdot p_+) + x^2 m_l^2$$
$$+ z\left[(z-1)m^2_{\eta^{(\prime)}} + m_V(m_V - i\Gamma_V)\right] ,$$
$$\Delta_{2V} \equiv \Delta_{1V} \;\; \text{with} \;\; p_+ \leftrightarrow p_- \,, \tag{9}$$

Decay	$\Gamma_{\rm th}$	$\mathrm{BR}_{\rm th}$	$\mathrm{BR}_{\rm exp}$
$\eta \to \pi^0 e^+ e^-$	$2.7(1)(1)(2) \times 10^{-6}$ eV	$2.0(1)(1)(1) \times 10^{-9}$	$< 7.5 \times 10^{-6}$ (CL=90%) [8]
$\eta \to \pi^0 \mu^+ \mu^-$	$1.4(1)(1)(1) \times 10^{-6}$ eV	$1.1(1)(1)(1) \times 10^{-9}$	$< 5 \times 10^{-6}$ (CL=90%) [12]
$\eta' \to \pi^0 e^+ e^-$	$8.7(5)(6)(6) \times 10^{-4}$ eV	$4.5(3)(4)(4) \times 10^{-9}$	$< 1.4 \times 10^{-3}$ (CL=90%) [12]
$\eta' \to \pi^0 \mu^+ \mu^-$	$3.3(2)(4)(3) \times 10^{-4}$ eV	$1.7(1)(2)(2) \times 10^{-9}$	$< 6.0 \times 10^{-5}$ (CL=90%) [12]
$\eta' \to \eta e^+ e^-$	$8.3(0.5)(0.1)(3.5) \times 10^{-5}$ eV	$4.3(0.3)(0.2)(1.8) \times 10^{-10}$	$< 2.4 \times 10^{-3}$ (CL=90%) [12]
$\eta' \to \eta \mu^+ \mu^-$	$3.0(0.2)(0.1)(1.1) \times 10^{-5}$ eV	$1.5(1)(1)(5) \times 10^{-10}$	$< 1.5 \times 10^{-5}$ (CL=90%) [12]

Table 1: Decay widths and branching ratios for the six C-conserving decays $\eta^{(\prime)} \to \pi^0 l^+ l^-$ and $\eta' \to \eta l^+ l^-$ ($l = e$ or μ). First error is experimental, second is down to numerical integration and third is due to model dependency.

where x, y and z are the Feynman integration parameters. Finally, the VMD coupling constants are parametrised using the phenomenological model described in Ref. [13]. The associated numerical values for the parameters of the model are obtained from an optimisation fit to $V \to P\gamma$ and $P \to V\gamma$ radiative decays, where $V = \rho^0$, ω, ϕ and $P = \pi^0$, η, η' (see Ref. [11] for further details).

We are now in a position to perform the computations required to obtain quantitative results for the decay widths and branching ratios of the six C-conserving semileptonic decays. Our theoretical predictions are summarised in Table 1, where the first error quoted is experimental, second is down to numerical integration and third is due to model dependency. We also show in this table the most up-to-date experimental upper bounds. Last but not least, theoretical results for the dilepton energy spectra of the six semileptonic decays are presented in Fig. 2. The decay widths and dilepton energy spectra for the two $\eta \to \pi^0 l^+ l^-$ processes obtained using our approach appear to be in good agreement with other results available in the published literature. To the best of our knowledge, the theoretical predictions for the four $\eta' \to \pi^0 l^+ l^-$ and $\eta' \to \eta l^+ l^-$ decays that we have presented in this work are the first predictions from theory that have been published.

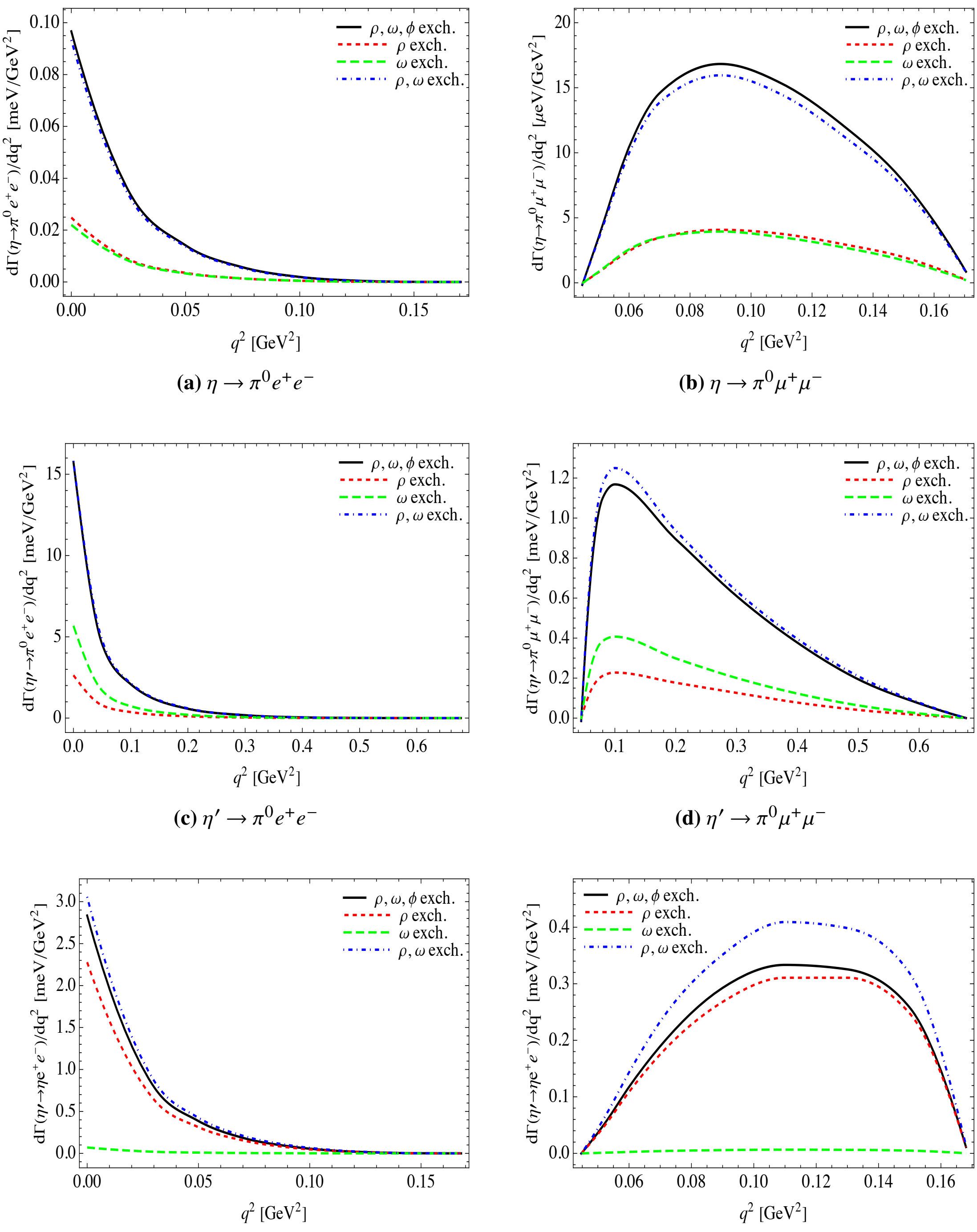
3. CP violation in $\eta^{(\prime)} \to \pi^0 \mu^+ \mu^-$ and $\eta' \to \eta \mu^+ \mu^-$

In Ref. [14], the possibility of observing new physics signatures via CP-violating effects at REDTOP was assessed using muon polarisation observables in η leptonic decays. In particular, the purely leptonic channels $\mu^+\mu^-$, $\mu^+\mu^-\gamma$, and $\mu^+\mu^-\ell^+\ell^-$ were studied, finding that CP violation in the $\mu^+\mu^-$ final state could be observed at REDTOP, whilst evading neutron electric dipole moment (nEDM) constraints.

We investigate here the semileptonic $\eta^{(\prime)} \to \pi^0 \mu^+ \mu^-$ and $\eta' \to \eta \mu^+ \mu^-$ decays using the SMEFT as the general theoretical framework to capture new physics. Defining the following momenta $q = p_{\mu^+} + p_{\mu^-} = p_{\eta^{(\prime)}} + p_{\pi(\eta)}$, $\bar{q} = p_{\mu^+} - p_{\mu^-}$, and $k = p_{\eta^{(\prime)}} - p_{\pi(\eta)}$, the most general form factor decomposition for $\langle \mu^+ \mu^- | iT | \eta^{(\prime)} \pi^0(\eta) \rangle = i(2\pi)^4 \delta(p_{\mu^+} + p_{\mu^-} - p_{\eta^{(\prime)}} - p_{\pi(\eta)}) \mathcal{M}$ is

$$\mathcal{M} = m_\mu (\bar{u} v) F_1 + (\bar{u} i \gamma^5 v) F_2 + (\bar{u} \not{k} v) F_3 + i (\bar{u} \not{k} \gamma^5 v) F_4 \,, \tag{10}$$

[1]For simplicity of the calculation, we neglect the q^2 dependence of the transition form factor in Eq. (1).



(a) $\eta \to \pi^0 e^+ e^-$ **(b)** $\eta \to \pi^0 \mu^+ \mu^-$

(c) $\eta' \to \pi^0 e^+ e^-$ **(d)** $\eta' \to \pi^0 \mu^+ \mu^-$

(e) $\eta' \to \eta e^+ e^-$ **(f)** $\eta' \to \eta \mu^+ \mu^-$

Figure 2: Dilepton energy spectra corresponding to the six C-conserving semileptonic decay processes $\eta^{(\prime)} \to \pi^0 l^+ l^-$ and $\eta' \to \eta l^+ l^-$ ($l = e$ or μ) as a function of the dilepton invariant mass q^2.

where the $F_i \equiv F_i(q^2, \bar{q} \cdot k)$ form factors have been introduced. The connection to the $\eta^{(\prime)} \to \pi^0 \mu^+ \mu^-$ and $\eta' \to \eta \mu^+ \mu^-$ decays is obtained via crossing symmetry with $k \to p_{\eta^{(\prime)}} + p_{\pi(\eta)}$. General considerations on discrete symmetries can be used to show that electromagnetic interactions can only contribute to $F_1(q^2, [\bar{q}\cdot k]^{2n})$ and $F_3(q^2, [\bar{q}\cdot k]^{2n+1})$, with $n = 0, 1, 2\ldots$, and that they can, in turn, be expressed in terms of the Σ and Ω parameters from Eq. (4) as $F_1 = \Sigma$ and $F_3 = \frac{1}{2}\Omega$. Turning to the BSM *CP*-violating contribution, one starts by assuming that the SMEFT provides a correct description of Nature. Accordingly, new physics degrees of freedom are expected to lie above the electroweak scale and, therefore, only the SM particle spectra are considered. New-physics effects come from higher dimension operators starting with $D = 6$. The contribution from the different operators is outlined in Ref. [14] where it is found that Fermi operators involving quarks and leptons provide the most significant contribution and, therefore, the ones we focus on

$$O^{prst}_{\ell edq} = (\bar{\ell}^i_p e_r)(\bar{d}_s q^i_t)\,, \qquad O^{(1)prst}_{\ell equ} = (\bar{\ell}^i_p e_r)(\bar{q}^j_s u_t)\epsilon_{ij}\,, \tag{11}$$

where *prst* are family indices (i.e. $p, r, s, t = 1$ or 2) [15]. These operators produce a non-vanishing *CP*-odd F_2 form factor [16]

$$F_2 = \left[\mathrm{Im}\, c^{2211}_{\ell edq}\, \langle 0|\, \bar{d}d\, |\eta^{(\prime)}\pi^0(\eta)\rangle + \mathrm{Im}\, c^{2222}_{\ell edq}\, \langle 0|\, \bar{s}s\, |\eta^{(\prime)}\pi^0(\eta)\rangle - \mathrm{Im}\, c^{(1)2211}_{\ell equ}\, \langle 0|\, \bar{u}u\, |\eta^{(\prime)}\pi^0(\eta)\rangle\right]/v^2\,, \tag{12}$$

where $v^{-2} = \sqrt{2}G_F$ and the corresponding hadronic matrix elements are obtained using LN_c χPT at NLO, which, after renormalising the fields and diagonalising the mass matrix (cf. Ref. [16]), can be expressed as

$$\langle 0|\, \bar{u}u/\bar{d}d\, |\eta\pi^0\rangle = \pm B_0 \left[\left(1 - \frac{m^2_\eta - m^2_\pi}{M^2_S}\right)(c\phi_{23} \pm \epsilon_{13} s\phi_{23}) - \left(c\phi_{23} - \frac{s\phi_{23}}{\sqrt{2}}\right)\frac{\tilde{\Lambda}}{3}\right]\left(\frac{M^2_S}{M^2_S - s}\right)\,, \tag{13}$$

$$\langle 0|\, \bar{s}s\, |\eta\pi^0\rangle = -2B_0\epsilon_{13}\left[\left(1 - \frac{m^2_\eta + 3m^2_\pi - 4m^2_K}{M^2_S}\right)s\phi_{23} + \frac{\tilde{\Lambda}}{3}\left(\frac{c\phi_{23}}{\sqrt{2}} - s\phi_{23} - \frac{\epsilon_{12} s\phi_{23}}{\sqrt{2}\epsilon_{13}}\right)\right]\left(\frac{M^2_S}{M^2_S - s}\right)\,, \tag{14}$$

where we have introduced the scale invariant parameter $\tilde{\Lambda} = \Lambda_1 - 2\Lambda_2$, ϕ_{23} is the η-η' mixing angle in the quark-flavour basis, ϵ_{12} and ϵ_{13} are first order approximations to the corresponding ϕ_{12} and ϕ_{13} isospin-breaking mixing angles in the π^0-η and π^0-η' sectors, respectively (cf. Ref. [13] for further details), and M_S is the mass of a generic octet scalar resonance. Note in particular that Eq. (14) vanishes in the absence of isospin breaking, largely suppressing the amplitude ($\epsilon_{13} \sim O(1\%)$). The corresponding expressions for $\eta \to \eta'$ can be obtained by substituting $c\phi_{23} \to s\phi_{23}$, $s\phi_{23} \to -c\phi_{23}$ and $m_\eta \to m_{\eta'}$. For the $\eta' \to \eta\mu^+\mu^-$ decay, the matrix elements read

$$\langle 0|\, \bar{u}u/\bar{d}d\, |\eta'\eta\rangle = B_0\left[\left(1 - \frac{m^2_{\eta'} + m^2_\eta - 2m^2_\pi}{M^2_S}\right)\left(\frac{s2\phi_{23}}{2} \mp \epsilon_{13} c2\phi_{23}\right) - \left(\frac{c2\phi_{23}}{\sqrt{2}} + s2\phi_{23}\right)\frac{\tilde{\Lambda}}{3}\right]\left(\frac{M^2_S}{M^2_S - s}\right)\,, \tag{15}$$

$$\langle 0|\, \bar{s}s\, |\eta'\eta\rangle = -B_0\left[\left(1 - \frac{m^2_{\eta'} + m^2_\eta + 2m^2_\pi - 4m^2_K}{M^2_S}\right)s2\phi_{23} + \left(\sqrt{2}c2\phi_{23} - s2\phi_{23}\right)\frac{\tilde{\Lambda}}{3}\right]\left(\frac{M^2_S}{M^2_S - s}\right)\,. \tag{16}$$

In order to quantify the *CP*-violating effects, one constructs asymmetries that arise as a result of the interference of the SM *CP*-even and the SMEFT *CP*-odd amplitudes. Accordingly, we define the longitudinal and transverse asymmetries as follows [16]

$$A_L = \frac{N(c\theta_{e^+} > 0) - N(c\theta_{e^+} < 0)}{N(c\theta_{e^+} > 0) + N(c\theta_{e^+} < 0)} = -\frac{2}{3} \frac{\int ds dc\theta \lambda_K^{1/2} \beta_\mu m_\mu \left[\beta_\mu s \,\mathrm{Im}\, F_1 F_2^* + 2\lambda_K^{1/2} c\theta \,\mathrm{Im}\, F_3 F_2^*\right]}{64(2\pi)^3 m_\eta^3 \int d\Gamma}, \quad (17)$$

$$A_T = \frac{N[s(\bar{\phi}-\phi) > 0] - N[s(\bar{\phi}-\phi) < 0]}{N[s(\bar{\phi}-\phi) > 0] + N[s(\bar{\phi}-\phi) < 0]} = \frac{\pi}{18} \frac{\int ds dc\theta \lambda_K^{1/2} \beta_\mu m_\mu \left[\beta_\mu s \,\mathrm{Re}\, F_1 F_2^* + 2\lambda_K^{1/2} c\theta \,\mathrm{Re}\, F_3 F_2^*\right]}{64(2\pi)^3 m_\eta^3 \int d\Gamma}, \quad (18)$$

where the polar angles $\theta_{e^\pm}$ refer to those of the $e^\pm$ in the $\mu^\pm$ rest frames, $\phi(\bar{\phi})$ correspond to the azimuthal $e^\pm$ angles in the $\mu^\pm$ rest frames, and N is the number of η decays.

Making use of Eqs. (13–16) allows one to compute F_2, which can then be plugged into Eqs. (17) and (18), together with the theoretical expressions for F_1 and F_3 from Eq. (4), to find [16]

$$A_L^{\eta\to\pi^0\mu^+\mu^-} = -0.19(6)\,\mathrm{Im}\, c_{\ell equ}^{(1)2211} - 0.19(6)\,\mathrm{Im}\, c_{\ell edq}^{2211} - 0.020(9)\,\mathrm{Im}\, c_{\ell edq}^{2222}, \quad (19)$$

$$A_T^{\eta\to\pi^0\mu^+\mu^-} = 0.07(2)\,\mathrm{Im}\, c_{\ell equ}^{(1)2211} + 0.07(2)\,\mathrm{Im}\, c_{\ell edq}^{2211} + 7(3)\times 10^{-3}\,\mathrm{Im}\, c_{\ell edq}^{2222}, \quad (20)$$

$$A_L^{\eta'\to\pi^0\mu^+\mu^-} = -0.04(8)\,\mathrm{Im}\, c_{\ell equ}^{(1)2211} - 0.04(8)\,\mathrm{Im}\, c_{\ell edq}^{2211} + 10(3)\times 10^{-3}\,\mathrm{Im}\, c_{\ell edq}^{2222}, \quad (21)$$

$$A_T^{\eta'\to\pi^0\mu^+\mu^-} = 3(6)\times 10^{-3}\,\mathrm{Im}\, c_{\ell equ}^{(1)2211} + 3(6)\times 10^{-3}\,\mathrm{Im}\, c_{\ell edq}^{2211} - 7(2)\times 10^{-4}\,\mathrm{Im}\, c_{\ell edq}^{2222}, \quad (22)$$

$$A_L^{\eta'\to\eta\mu^+\mu^-} = -5(39)\times 10^{-3}\,\mathrm{Im}\, c_{\ell equ}^{(1)2211} + 5(46)\times 10^{-3}\,\mathrm{Im}\, c_{\ell edq}^{2211} - 0.08(1)\,\mathrm{Im}\, c_{\ell edq}^{2222}, \quad (23)$$

$$A_T^{\eta'\to\eta\mu^+\mu^-} = 7(50)\times 10^{-5}\,\mathrm{Im}\, c_{\ell equ}^{(1)2211} - 6(65)\times 10^{-5}\,\mathrm{Im}\, c_{\ell edq}^{2211} + 1(19)\times 10^{-3}\,\mathrm{Im}\, c_{\ell edq}^{2222}, \quad (24)$$

where the error quoted accounts for both the numerical integration and the model-dependence uncertainties, with the latter strongly dominating over the former.

To assess the sensitivity to new physics, we estimate the expected number of events at REDTOP, which can be obtained from the projected statistics[2] of 5×10^{12} η/yr and 5×10^{10} η'/yr, and the SM branching ratios for the three muonic processes from Table 1. The REDTOP sensitivity to each of the SMEFT *CP*-violating Wilson coefficients from Eq. (11) can be estimated by setting to zero two out of the three coefficients in Eqs. (19–24). The corresponding results are summarised in Table 2. We also show in this table the REDTOP sensitivity to the same coefficients from $\eta \to \mu^+\mu^-$ [14], as well as the bounds set by the nEDM experiment using the most recent measurements from Ref. [19].

If one compares the sensitivities to the *CP*-violating Wilson coefficients obtained from the $\eta^{(\prime)} \to \pi^0\mu^+\mu^-$ and $\eta' \to \eta\mu^+\mu^-$ decays with the bounds extracted from nEDM experiments, one must conclude that the projected REDTOP statistics are not competitive enough for the semileptonic processes that we have studied, which can be attributed to the isospin-breaking suppression in the hadronic matrix elements, subject to the assumption that new physics can be parametrised by the SMEFT. Consequently, the leptonic $\eta \to \mu^+\mu^-$ decay studied in Ref. [14] remains the most promising channel to be studied at REDTOP.

[2]A total production of 2.5×10^{13} η/yr and 2.5×10^{11} η'/yr is expected [17], with assumed reconstruction efficiencies of approximately 20% [18].

Table 2: Summary of REDTOP sensitivities to (the imaginary parts of) the Wilson coefficients associated to the SMEFT *CP*-violating operators in Eq. (11) for the processes studied in this work, as well as the $\eta \to \mu^+\mu^-$ decay analysed in Ref. [14]. In addition, the upper bounds from nEDM experiments are given in the last row for comparison purposes.

Process	Asymmetry	$\mathrm{Im}\, c_{\ell equ}^{(1)2211}$	$\mathrm{Im}\, c_{\ell edq}^{2211}$	$\mathrm{Im}\, c_{\ell edq}^{2222}$
$\eta \to \pi^0\mu^+\mu^-$	A_L	0.0695	0.0720	0.686
	A_T	0.194	0.203	1.93
$\eta' \to \pi^0\mu^+\mu^-$	A_L	2.36	2.56	10.96
	A_T	33.1	35.8	154
$\eta' \to \eta\mu^+\mu^-$	A_L	67.5	78.5	4.46
	A_T	5264	5549	328
$\eta \to \mu^+\mu^-$	A_L	0.007	0.007	0.005
nEDM	-	≤ 0.001	≤ 0.002	≤ 0.02

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