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The three-nucleon interaction in pionful and pionless EFT

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After a brief historical perspective on the three-nucleon ($3N$) interaction, we will focus on the chiral $3N$ forces and discuss some of the main present discrepancies with experimental data on low-energy $p-d$ scattering. By including the subleading $3N$ contact interactions, expected to arise at the fifth order of the chiral expansion (N4LO), these discrepancies can be solved. However, part of the needed contributions can be regarded as of fourth order (N3LO), because they are induced by a unitary transformation devised to absorb redundant NN contact interactions at N3LO. As a result, a satisfactory description in NN and $3N$ systems can be achieved already at N3LO. Finally, the pionless case will be described, with particular emphasis on the different treatment of the $3N$ force, and on possible consequences for the chiral series.

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1. Introduction

The three-nucleon force is acknowledged since many decades as an important component of the nuclear interaction. This has been firmly established at the beginning of the current century, when very accurate models of the two-nucleon component of the nuclear interaction, describing thousands of NN scattering data with a χ^2 close to 1, became available, either in the form of phenomenological potentials, such as the AV18 [1] or the CD-Bonn [2], or derived within the chiral effective field theory (ChEFT) [3–5]. At the same time, accurate numerical methods were developed, which enabled the computation of light-nuclei binding energies from first principles, i.e. starting from the realistic NN interactions that were available [10–12].

It was then clear that extra binding should have been produced, ascribed to a three-nucleon interaction. A paradigmatic illustration of this statement is provided by the spectra of nuclei until ^{12}C: the predictions based only on two-body interaction are systematically above the experimental data, while the inclusion of $3N$ interactions brings them in spectacularly good agreement [13, 14]. But at the same time, the $3N$ force should also provide a repulsion, in order to stabilize nuclear matter, i.e. to describe correctly the saturation density [15].

These evidences were not surprising, as long as the nuclear interaction was understood as resulting from meson exchanges, after Yukawa's idea. Various mechanisms lead to non-pairwise nuclear interactions, one of the earliest being the Fujita-Miyazawa contribution, dating back to 1957 [16]. All such kinds of mechanisms are nowadays embedded in the framework of the systematic low-momentum expansion of ChEFT [17–21], as exemplified in Fig. 1. The triangular character of the "matrix of orders" gives an explanation of the dominant role of pairwise interactions. Indeed, in the Weinberg counting, the $3N$ force is a small correction that starts at the next-to-next-to-leading order (N2LO), and further suppression is obtained as more and more nucleons participate to the irreducible interaction [22–28]. Among the various interactions, the ones involving purely nuclear low-energy constants (LECs), that have to be fitted to nuclear observables, deserve a special attention. Contrary to the other vertices, which can be considered as known from processes involving more pions and less nucleons, purely contact vertices are free parameters describing the short-distance nuclear dynamics. Their number increases as one moves to higher orders seeking more accuracy. In the isospin limit there are 24 such LECs in the NN sector up to N4LO, while in the $3N$ sector there are 14 LECs at the same order [29], only a single one contributing up to N3LO. At this order, high accuracy is met in the description of NN data. However, no comparable accuracy is obtained in the $3N$ system, if one includes scattering data. In particular, a long-standing discrepancy exists between theory and experiment in $N-d$ elastic scattering, known as the A_y puzzle [30, 31]: at very low-energy the theoretical predictions underestimate the (very precise) experimental data for this polarization asymmetry by about 30%. In Ref. [31] two- and three-nucleon chiral interactions are used up to N3LO, without reaching any improvement. The disagreement diminishes as the energy is increased. Another discrepancy is found in the breakup channel of $N-d$ scattering, the so-called space-star anomaly. The effect of three-nucleon forces (3NF) up to N3LO is found to be negligible there [32].

To better understand the nature of such discrepancies we should emphasize the big difference in the number of free LECs up to N3LO in the NN and $3N$ sectors. It is true that, from the perspective of the EFT, all these LECs stand on an equal footing: in principle there is no reason to fit the

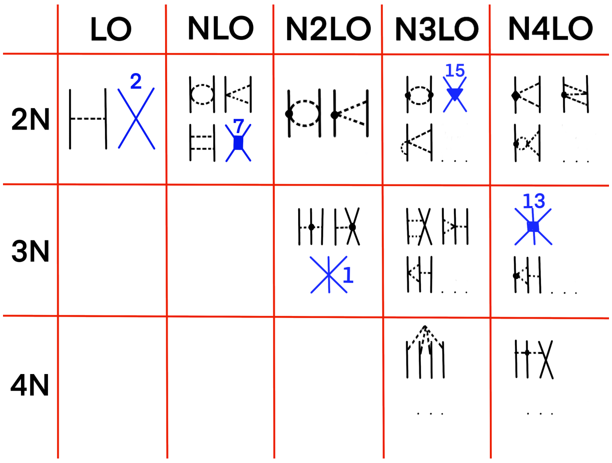

Figure 1: Representative diagrams of multi-nucleon forces at the various order of the chiral expansion.

NN LECs to 2-body observables only. However, doing a global fit to 2- and 3-body observables would be rather demanding from a practical point of view, in view of the complexities of the 3-body problem. Moreover, in a hypothetical global fit, the statistical weight of NN data would be overwhelming, leading to little practical difference as compared to the customary procedure. The only way to let the 3-body observables having an impact could be to add a theoretical truncation error [33], which would amount at the end to a drastic reduction of the nice accuracy of chiral NN potentials at N3LO. It is also worth mentioning that, despite many attempts, there are no reasonable modifications of the NN potential which could account for the A_y puzzle [34].

2. Accuracy in the $3N$ sector

Let us now focus on the new LECs which appear at the next order, N4LO, since these may improve the description of scattering observables. In Ref. [29] we have written down all possible subleading $3N$ contact operators, and used Fierz identities to reduce their number to 13 in the $3N$ center-of-mass frame. By choosing an appropriate form for the cutoff, the resulting coordinate-space $3N$ potential can be given in a local form depending on 13 LECs $E_{i=1,\dots,13}$,

$$
\begin{aligned}
V = \sum_{i\neq j\neq k} \quad & (E_1 + E_2 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\left[Z_0''(r_{ij}) + 2\frac{Z_0'(r_{ij})}{r_{ij}}\right] Z_0(r_{ik}) \\
& + (E_5 + E_6 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j) S_{ij}\left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}}\right] Z_0(r_{ik}) \\
& + (E_7 + E_8 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k)(\mathbf{L}\cdot\mathbf{S})_{ij}\frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\
& + \left[(E_9 + E_{10}\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k)\boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}}_{ij}\boldsymbol{\sigma}_k\cdot\hat{\mathbf{r}}_{ik}\right. \\
& \left. + (E_{11} + E_{12}\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k + E_{13}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)\boldsymbol{\sigma}_k\cdot\hat{\mathbf{r}}_{ij}\boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}}_{ik}\right] Z_0'(r_{ij}) Z_0'(r_{ik}),
\end{aligned} \tag{1}
$$

where the profile functions are the Fourier transforms of the cutoff function

$$
Z_0(r;\Lambda) = \int \frac{d\mathbf{p}}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}} F(\mathbf{p}^2;\Lambda), \tag{2}
$$

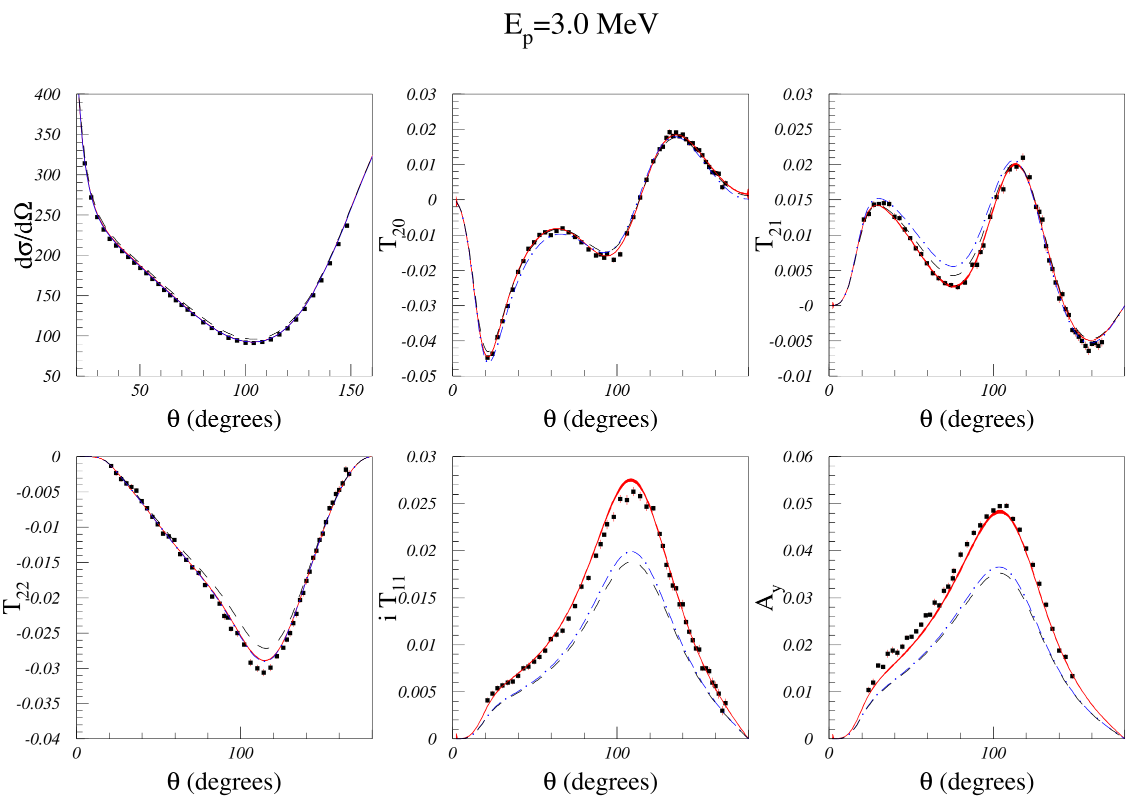

Figure 2: Two-parameter fits of the subleading $3N$ LECs to p-d data from Ref. [37] at 3 MeV proton energy. The red bands include the best fit curves for cutoff Λ between 200 and 500 MeV. Black dashed lines are the prediction with the 2-body AV18 interaction, while the blue dashed-dotted lines also include the 3-body UIX interaction.

chosen in the following as,

$$F(\mathbf{p}^2;\Lambda) = \exp\left[-\left(\frac{\mathbf{p}^2}{\Lambda^2}\right)^2\right]. \tag{3}$$

The rationale for this choice of operator basis is to involve ordinary spin-isospin operators of the NN interaction, like S_{ij} and $(\mathbf{L}\cdot\mathbf{S})_{ij}$ representing respectively the tensor and spin-orbit operators for particle pair ij, with the additional dependence on the coordinate of the third particle in the profile functions. The latter type of operators have a special relevance, since they have been proposed in the literature to address the A_y problem [35]. Indeed, by including just 2 of the 13 extra LECs, E_5 and E_7, we can fit very precise polarization data [37] on elastic $p-d$ scattering at 3 MeV laboratory energy, with huge improvement as compared to conventional $3N$ forces, as shown in Fig. 2, for a whole range of cutoffs $\Lambda = 200-500$ MeV. Also included in the fits are the triton binding energy and the doublet and quarted $N-d$ scattering lengths. The reported fits [36] have been done in a hybrid approach, using the phenomenological AV18 NN interaction and adding the subleading contact term as a residual $3N$ interaction, on the top of the conventional Urbana model (UIX) [38]. For all the explored values of the cutoff Λ the χ^2/d.o.f. can be brought down to around 2. Having fitted the data at a single energy, we can provide predictions at lower energies and compare to available

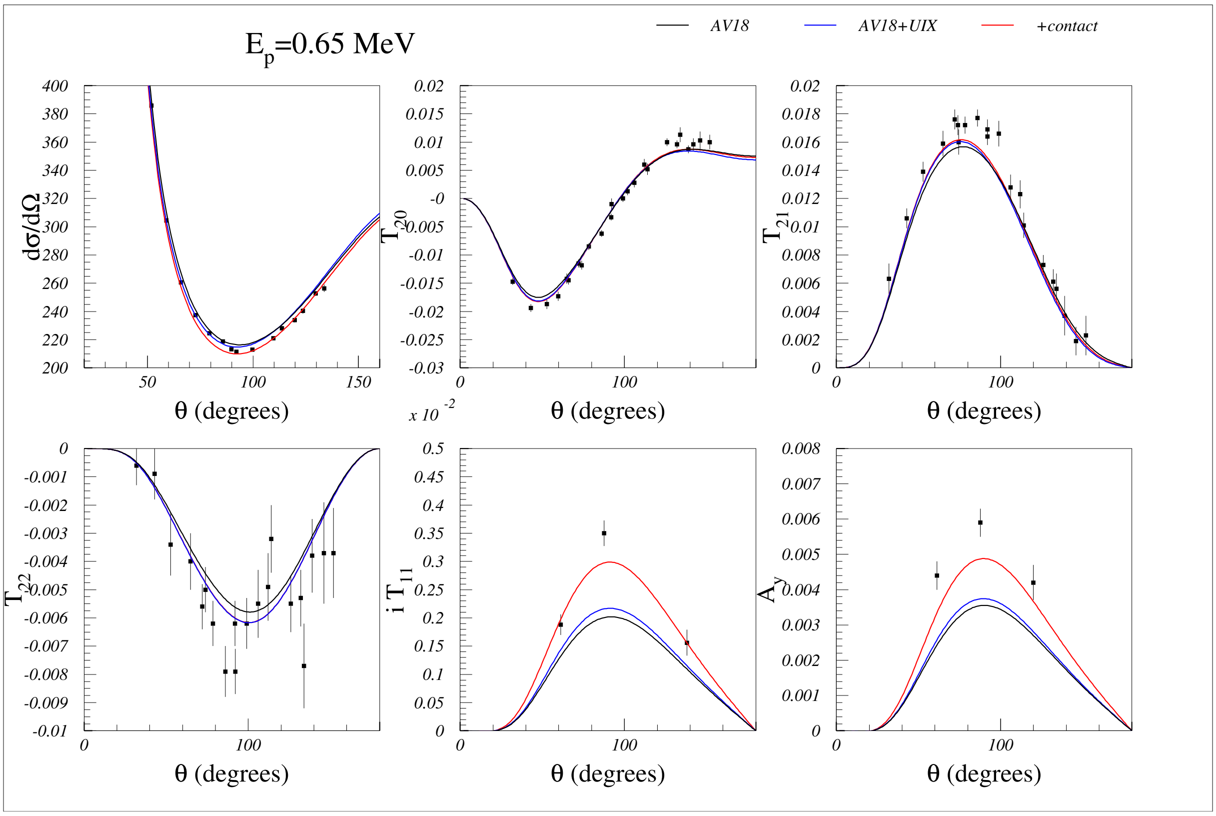

Figure 3: Predictions of the fitted models (red bands) to the same observables at 0.65 MeV proton energy, as compared to experimental data from Ref. [39]. The black and blue lines correspond respectively to the predictions with AV18 and AV18+UIX interactions.

experimental data. We show in Fig. 3 the predictions at 0.65 MeV proton energy, the lowest energy where actual data exist [39]. We can conclude that also the energy dependence is captured by the fitted contact interactions. The message to retain from this exploration is that decisive improvement can be expected in the description of $3N$ scattering data at N4LO, once the considered interactions are included (see also Ref. [40]). However, in the following we will argue that this improvement can actually be expected already at N3LO, due to a freedom to reshuffle contributions between N3LO NN interaction and N4LO $3N$ interaction encoded in a unitary transformation [41].

3. A unitary ambiguity

It was already shown in Ref. [9] that 3 of the 15 contact NN operators contributing at N3LO are in fact redundant, because they can be absorbed by a suitable unitary transformation of the kinetic energy operator H_0: defining the unitary transformation in terms of its generators,

$$U = \exp \sum_{n=1}^{5} \alpha_n T_n, \tag{4}$$

with α_n real parameters and T_n antihermitian operators respecting all the symmetries of the theory, we have

$$U^\dagger H_0 U = \sum_{n=1}^{5} \alpha_i \sum_{i=1}^{15} c_{ni} O_i^{(4)}, \tag{5}$$

where $O_i^{(4)}$ $(i = 1, ..., 15)$ denote the 15 NN contact operators appearing at the N3LO and c_{ni} are just numerical coefficients. The generators T_n can be found, in the EFT perspective, among the most general ones expressible as local products of field operators, ordered according to the increasing number of derivatives. If we ignore pion fields, the simplest such generators have a two-body character and involve two derivatives,

$$\begin{aligned}
T_1 &= \int d^3\mathbf{x} N^\dagger \overleftrightarrow{\nabla}^i N \nabla^i (N^\dagger N), & (6)\\
T_2 &= \int d^3\mathbf{x} N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^i (N^\dagger \sigma^j N), & (7)\\
T_3 &= \int d^3\mathbf{x} \left[N^\dagger \overleftrightarrow{\nabla}^i \sigma^i N \nabla^j (N^\dagger \sigma^j N) + N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^j (N^\dagger \sigma^i N) \right], & (8)\\
T_4 &= i\epsilon^{ijk} \int d^3\mathbf{x} N^\dagger \overleftrightarrow{\nabla}^i N N^\dagger \overleftrightarrow{\nabla}^j \sigma^k N, & (9)\\
T_5 &= \int d^3\mathbf{x} \left[N^\dagger \overleftrightarrow{\nabla}^i \sigma^i N \nabla^j (N^\dagger \sigma^j N) - N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N \nabla^j (N^\dagger \sigma^i N) \right], & (10)
\end{aligned}$$

where $N^\dagger \overleftrightarrow{\nabla}^i N = N^\dagger (\nabla^i N) - (\nabla^i N^\dagger) N$, and $N(x)$ denotes the non-relativistic nucleon field operator. As usual, Fierz identities allow to express the operators involving the isospin Pauli matrices in terms of the above ones. The last two operators were not considered in Ref. [9], because they depend on the total momentum $\mathbf{P}$ of the nucleon-nucleon pair. Specifically, the associated unitary transformation of the kinetic energy generates a $\mathbf{P}$-dependent NN interaction, which we denote by $V_{NN}(\mathbf{P})$

$$U^\dagger H_0 U = H_0 + [H_0, \alpha_4 T_4 + \alpha_5 T_5] = H_0 + V_{NN}(\mathbf{P}), \tag{11}$$

having the following momentum space expression in the two-nucleon system

$$V_{NN}(\mathbf{P}) = 8i\frac{\alpha_4}{m} \mathbf{k} \cdot \mathbf{Q} \mathbf{Q} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) + 4\frac{\alpha_5}{m} \mathbf{k} \cdot \mathbf{Q} \mathbf{k} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2), \tag{12}$$

where $\mathbf{k} = \mathbf{p}' - \mathbf{p}$ and $\mathbf{Q} = (\mathbf{p} + \mathbf{p}')/2$ depend on the initial and final relative momenta $\mathbf{p}$ and $\mathbf{p}'$ respectively, and m is the nucleon mass. In principle, a $\mathbf{P}$-dependent NN interaction is conceivable, provided it satisfies all constraints from relativity [42–45]. These are encoded in the Poincaré commutation relations among the group generators, which can be imposed order by order in powers of soft momenta [46]. As a result of such an analysis, one finds a class of $\mathbf{P}$-dependent interactions with fixed coefficients, which represent $1/m$ boost corrections to lower order interactions. In addition, further interactions are possible with unspecified magnitude, in the form of two independent operators each multiplied by a corresponding LEC. They correspond exactly to the interactions entering in $V_{NN}(\mathbf{P})$ [41]. In other words, the NN contact interaction at N3LO depends on 17 free LECs, $D_{1,...,17}$, with the additional LECs parametrizing the $\mathbf{P}$-dependent interaction,

$$V_{NN}(\mathbf{P}) = iD_{16} \mathbf{k} \cdot \mathbf{Q} \mathbf{Q} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) + D_{17} \mathbf{k} \cdot \mathbf{Q} \mathbf{k} \times \mathbf{P} \cdot (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2). \tag{13}$$

The two additional structures were ignored in the literature, since the NN interaction was constructed in the center-of-mass frame, in which they vanish identically. Therefore there is no way to determine the associated LECs from NN data: they are redundant, in the same way as the three combinations of LECs $D_{1,...,15}$ identified in Ref. [9]. The redundancy is clearly reflected in the existence of a suitable unitary transformation to cancel their contribution. However, they are only redundant in the NN system: indeed the NN pairs are not constrained to have zero total momentum in the overall center-of-mass frame of a larger system. Therefore the interaction parametrized by the two new LECs D_{16} and D_{17} should not be considered as a two-nucleon force, but rather a contribution to multi-nucleon forces. A similar conclusion can be drawn about the interactions generated by T_n with $n = 1, 2, 3$.

4. Induced $3N$ forces at N3LO

If we start from the complete set of NN contact interactions, including the **P**-dependent ones parametrized by the LECs D_{16} and D_{17}, the appropriate choice of unitary transformation so as to cancel the latter contributions corresponds to the parameters,

$$\alpha_4 = -\frac{m}{8} D_{16}, \quad \alpha_5 = -\frac{m}{4} D_{17}, \tag{14}$$

as it is clear from Eqs. (12) and (13). Similar choices for the parameters α_1, α_2 ,α_3 allow to further reduce the number of NN contact LECs to 12. However, the same unitary transformation, when applied to the LO NN contact interaction, generates a genuine $3N$ force, of the same kind as the subleading contact interaction discussed in Sec. 2. More explicitly, this transformation induces a shift in the LECs E_i as follows:

$$\begin{aligned}
\delta E_1 &= \alpha_1 (C_S + C_T) + \alpha_2 (C_S - 2C_T), && (15)\\
\delta E_2 &= 3\alpha_2 C_T + 2\alpha_3 C_T - 4\alpha_4 C_T + 2\alpha_5 C_T, && (16)\\
\delta E_3 &= 2\alpha_1 C_T + \alpha_2 (2C_S - C_T) + \frac{2}{3}\alpha_3 (2C_S - C_T) + 4\alpha_4 C_T - 2\alpha_5 C_T, && (17)\\
\delta E_4 &= \frac{2}{3}\alpha_1 C_T + \frac{1}{3}\alpha_2 (2C_S - 7C_T) - \frac{2}{3}\alpha_3 C_T + \frac{4}{3}\alpha_4 C_T - \frac{2}{3}\alpha_5 C_T, && (18)\\
\delta E_5 &= 2\alpha_1 C_T + 2\alpha_2 (C_S - 2C_T) + \frac{2}{3}\alpha_3 (2C_S - C_T) + 4\alpha_4 C_T - 2\alpha_5 C_T, && (19)\\
\delta E_6 &= \frac{2}{3}\alpha_1 C_T + \frac{2}{3}\alpha_2 (C_S - 2C_T) - \frac{2}{3}\alpha_3 C_T + \frac{4}{3}\alpha_4 C_T - \frac{2}{3}\alpha_5 C_T, && (20)\\
\delta E_7 &= 8\alpha_4 C_T, && (21)\\
\delta E_8 &= \frac{1}{3}\delta E_7, && (22)\\
\delta E_9 &= 3\alpha_1 C_T + 3\alpha_2 (C_S - 2C_T) + 2\alpha_3 (C_S - 2C_T) - \alpha_4 (C_S - 5C_T) - 4\alpha_5 C_T, && (23)\\
\delta E_{10} &= \alpha_1 C_T + \alpha_2 (C_S - 2C_T) - \frac{1}{3}\alpha_4 (3C_S - 7C_T), && (24)\\
\delta E_{11} &= 3\alpha_1 C_T + 3\alpha_2 (C_S - 2C_T) + 2\alpha_3 (C_S - 2C_T) + \alpha_4 (C_S - 5C_T) + 4\alpha_5 C_T, && (25)\\
\delta E_{12} &= \alpha_1 C_T + \alpha_2 (C_S - 2C_T) + \frac{1}{3}\alpha_4 (3C_S - 7C_T), && (26)\\
\delta E_{13} &= -8\alpha_4 C_T + 4\alpha_5 C_T. && (27)
\end{aligned}$$

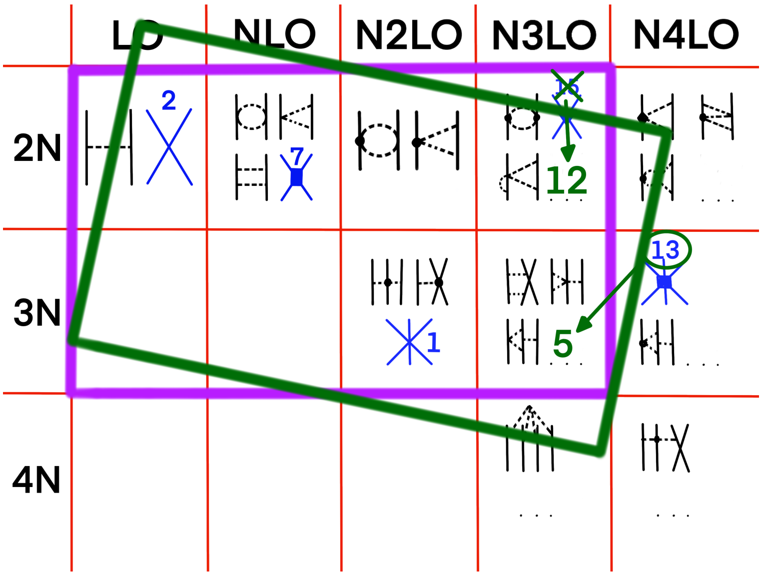

Figure 4: The green rectangle represents the unitary-transformed set of NN and $3N$ interactions up to N3LO. Apart from a reshuffling of LECs, there are two more LECs as compared to the purple rectangle, corresponding to the **P**-dependent NN interaction of Eq. (13), which were ignored in the literature so far.

The parameters α_1,...,α_5 should be regarded in these equations as determined in terms of the LECs D_i of the N3LO NN contact interactions, as exemplified in Eq. (14). Notice also that they are enhanced by powers of m, which, according to the Weinberg counting [23], brings an inverse power of the soft scale. Therefore, even the order mismatch between the absorbed N3LO NN interaction and the induced N4LO $3N$ interaction is compensated: the resulting $3N$ force can be legitimately regarded as N3LO. We point out in particular that the spin-orbit LECs E_7 and E_8 are induced precisely by the new LEC D_{16} parametrizing the **P**-dependent NN interaction, whose relevance is thus demonstrated, also in the light of the results in Sec. 2.

The unitary transformation of the LO πN interaction Hamiltonian,

$$H_{\pi N} = \frac{g_A}{2F_\pi} \int d\mathbf{x} N^\dagger \overrightarrow{\nabla} \pi^a \tau^a \cdot \vec{\sigma} N, \tag{28}$$

generates πNN contact interactions which give rise to a $3N$ force contribution of contact-one-pion-exchange topology,

$$\begin{aligned} V_{3NF} = -\frac{g_A}{F_\pi} \sum_{i\neq j\neq k} \frac{\mathbf{k}_k \cdot \boldsymbol{\sigma}_k\, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k}{k_k^2 + m_\pi^2} & \left\{ \alpha_1 \mathbf{k}_j \cdot \mathbf{k}_k\, \mathbf{k}_k \cdot \boldsymbol{\sigma}_i \right. \\ & + \alpha_2 \left[\mathbf{k}_j \cdot \mathbf{k}_k\, \mathbf{k}_k \cdot \boldsymbol{\sigma}_j + 2i \mathbf{k}_j \cdot (\mathbf{Q}_i - \mathbf{Q}_j)\, \mathbf{k}_k \cdot \boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j \right] \\ & + (\alpha_3 + \alpha_5) \left[k_k^2 \mathbf{k}_j \cdot \boldsymbol{\sigma}_j - 2i \mathbf{k}_j \cdot \boldsymbol{\sigma}_j\, \mathbf{k}_k \cdot \mathbf{Q}_i \times \boldsymbol{\sigma}_i + 2i \mathbf{Q}_j \cdot \boldsymbol{\sigma}_j\, \mathbf{k}_k \cdot \mathbf{k}_j \times \boldsymbol{\sigma}_i \right] \\ & + (\alpha_3 - \alpha_5) \left[\mathbf{k}_j \cdot \mathbf{k}_k\, \mathbf{k}_k \cdot \boldsymbol{\sigma}_j + 2i \mathbf{k}_j \cdot \boldsymbol{\sigma}_j\, \mathbf{k}_k \cdot \mathbf{Q}_j \times \boldsymbol{\sigma}_i - 2i \mathbf{Q}_i \cdot \boldsymbol{\sigma}_j\, \mathbf{k}_k \cdot \mathbf{k}_j \times \boldsymbol{\sigma}_i \right] \\ & \left. -2\alpha_4 \left[\mathbf{k}_k \cdot \boldsymbol{\sigma}_i\, \mathbf{k}_k \cdot \mathbf{Q}_j \times \boldsymbol{\sigma}_j - 2i \mathbf{k}_k \cdot \mathbf{Q}_i \left(\mathbf{k}_k \cdot \mathbf{Q}_i\, \mathbf{Q}_j \cdot \boldsymbol{\sigma}_i - \mathbf{k}_k \cdot \mathbf{Q}_j\, \mathbf{Q}_i \cdot \boldsymbol{\sigma}_i \right) \right] \right\}. \end{aligned} \tag{29}$$

Being enhanced by a factor of m, this kind of $3N$ force, which naively is twice suppressed as compared to the so-called c_D term arising at N2LO, should be regarded instead as N3LO, similarly

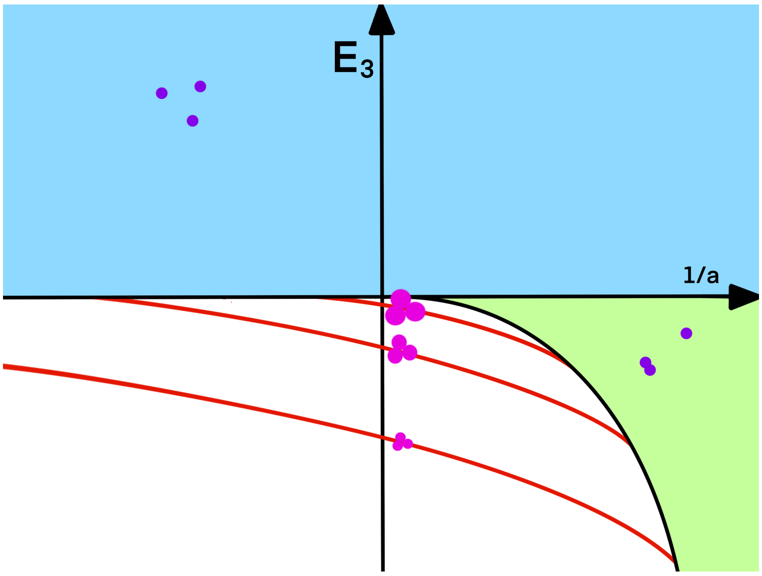

Figure 5: Qualitative Efimov plot representing the spectrum of 3-body states for the different values of the 2-body scattering length. The colored regions represent the continuum states, either 2+1 or 1+1+1. The red lines represent the binding energies of the self-similar 3-body bound states.

to the induced contact interaction. Work to investigate the phenomenological relevance of this kind of induced $3N$ force is under way. An updated matrix of orders for nuclear interactions in ChEFT should be drawn, as a result of the reshuffling of LECs between two- and three-nucleon LECs realized by the unitary transformation: at N3LO we have only 12 LECs parametrizing the NN contact interactions and 5 LECs parametrizing $3N$ interactions of mixed topology, contrary to the generally accepted wisdom [47, 48]. This is illustrated pictorially in Fig. 4.

5. The pionless case

In the pionless theory ($\not\pi$EFT) the pion mass is a heavy scale and all the interactions are short-ranged [49–52]. What makes the effective theory interesting and non-trivial is the emergence of another soft scale, the inverse of the scattering lengths a, reflected also in the shallow nature of the deuteron, extending in space well beyond the interaction range r. Similarly to what happens in ChEFT, where all powers of p/M_π, p denoting the generic soft momentum scale, are resummed e.g. in chiral logarithms, in $\not\pi$EFT the non-trivial dependence on pa is addressed while $pr \ll 1$. The small expansion parameter is r/a in the same sense that it is M_π/Λ for ChEFT. Analogously to low-energy theorems in ChEFT, valid in the chiral limit, in $\not\pi$EFT we have universal relations among observables, reflecting their scaling in a, for very large a. The two EFTs are expansion around different points in some parameter space, the chiral limit and the unitary limit [53]. Their domain of validity might even not overlap, the real question being which critical point the real world is closer to.

One of the most amazing correlations controlled by the unitary limit is the Efimov effect [54, 56, 57], which consists in the emergence of an infinite tower of three-bosons bound states for zero-range two-body interactions in the unitary limit, cfr. Fig. 5. These bound states are self-similar, related to each other by a discrete scale invariance which reflects in turn a limit cycle of the renormalization group [58, 59]. This means that the short distance dynamics does not decouple at large distances in the 3-body system. As a result a 3-body parameter has to be introduced to specify the size of the 3-body states described by the effective theory. This scale defines the theory, and it has to be present already at LO. The extent to which these kinds of universal correlations are fulfilled in nuclear physics is an open question, depending on the proximity of the physical world to the unitary limit. In Ref. [60] a number of these correlations was explored using a Gaussian characterization of nuclear systems in the Efimov window, i.e. choosing Gaussian potentials to guide the systems towards the unitary limit. Examples of such universal features, amenable to the specific deviation from the unitary limit, include the lack of excited states in the 3- and 4-nucleon systems, as well as the very small value of the neutron-deuteron doublet scattering length. It is important to stress that these universal features depend solely on the proximity of the unitary critical point and would be present irrespective of the value of the pion mass. Their imprints in nuclear physics clearly challenge ChEFT, since they are difficult to reproduce without strong fine tunings in the low-energy expansion, which might destabilize the chiral series. In particular, ChEFT could inherit the promotion of the contact $3N$ force to leading order from the $\not\pi$EFT [61], thus improving its convergence properties with possibly less orders necessary to achieve accuracy.

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