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Chiral effective theory in the Higgs sector

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In these proceedings we give a brief overview of the chiral symmetry structure of the electroweak effective theory that extends the Standard Model. The basic ideas that lead the construction of the electroweak effective theory are presented and discussed. We show how the global electroweak chiral symmetry appears in the Standard Model and how one can exploit it to study possible deviations and new physics.

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1. Introduction

In these proceedings we discuss the effective field theory (EFT) description of the electroweak (EW) gauge and Higgs sectors and its phenomenological application put forward in Ref. [1, 2]. We adopt the non-linear realization of the EW symmetry provided by the electroweak effective theory (EWET) [3], also known as Higgs effective field theory (HEFT) or electroweak chiral Lagrangian (EWχL) [1, 4–6]. In this approach the Higgs boson is treated as an independent degree of freedom (dof), separate from the triplet of EW Nambu-Goldstone bosons ω^a (NGB), which transform non-linearly under the $\mathcal{G} = SU(2)_L \times SU(2)_R$ global chiral group. The remaining SM matter fields can be added in a rather straight-forward procedure [1, 3, 4, 7–10]. HEFT extends the most commonly considered EFT framework for the study of new physics, the Standard Model Effective Theory (SMEFT). This EFT is based on the SM symmetries but incorporates both the Higgs and the EW Goldstones all together in a complex doublet Φ. The similarities and differences of both HEFT and SMEFT approaches have been long discussed in the bibliography [11–16].

Nonetheless, since this conference is devoted to chiral dynamics, we will spend an important part of these proceedings explaining in some detail the origin of the EW chiral symmetry that provides the foundations of the HEFT approach. This symmetry, though related, is many times confused with the EW Standard Model gauge symmetry. We hope this little summary may clarify the issue for non-practitioners in the topic.

These proceedings will be organized as follows: in Sec. 2, we will present in detail the EW chiral symmetry and its spontaneous symmetry breaking in the SM. In Sec. 3, based on this chiral symmetry, we proceed and elaborate the most general chiral invariant effective Lagrangian that extends the SM and incorporate possible new physics corrections at low energies. In Sec. 4, we will have a glimpse on some related phenomenology. We will provide a summary and some final conclusions in Sec. 5.

2. Electroweak chiral symmetry and the Standard Model

2.1 A second $SU(2)$ symmetry hidden in the scalar sector

When discussing the chiral symmetry in the EW sector of the SM, one of the first questions that often pops up is where this extended $\mathcal{G} = SU(2)\times SU(2)$ chiral symmetry is. This question is biased by our learning of how the **full** SM has been built, which is based on an $G_{\rm gauge} = SU(2)_L \times U(1)_Y$ gauge symmetry. In what follows, we will ignore the colour group $SU(3)$ and just focus on the EW interactions.

Chiral symmetry appear in the limit when the gauge and fermion sectors are decoupled from the scalar sector ($g = g' = \lambda_\psi = 0$). We will see that, under this limit, the Higgs doublet sector shows an extended global symmetry $\mathcal{G} = SU(2) \times SU(2)$ beyond the EW one.

The scalar sector Lagrangian is given by

$$\mathcal{L}_{\rm SM}^{\rm sca} = \partial_\mu \Phi^\dagger \partial^\mu \Phi + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 \,. \qquad (1)$$

This Lagrangian is invariant under global $SU(2)_L$ transformations:

$$\begin{aligned} \Phi &\overset{SU(2)_L}{\longrightarrow} g_L\,\Phi\,, \\ \mathcal{L}_{\rm SM}^{\rm sca} &\overset{SU(2)_L}{\longrightarrow} \mathcal{L}_{\rm SM}^{\rm sca}\,, \end{aligned} \tag{2}$$

with the Higgs doublet,

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_2 + i\varphi_1 \\ \varphi_0 - i\varphi_3 \end{pmatrix}. \tag{3}$$

A peculiarity of the $SU(2)$ groups is their real representations, so one finds that the conjugate,

$$\Phi^c = i\sigma_2\Phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_0 + i\varphi_3 \\ -\varphi_2 + i\varphi_1 \end{pmatrix}, \tag{4}$$

also transforms as a doublet **<u>under the same</u>** $SU(2)_L$ transformation, $\Phi^c \to g_L\Phi^c$.

However, there is another $SU(2)$ invariant group hidden in this Lagrangian. This can be observed through a little rearrangement of the real scalar components in the Lagrangian:

$$\begin{aligned} \mathcal{L}_{\rm SM}^{\rm sca} &= \partial_\mu\Phi^\dagger\partial^\mu\Phi + \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 \\ &= \frac{1}{2}\sum_{k=0}^{3}(\partial_\mu\varphi^k)^2 + \frac{\mu^2}{2}\sum_{k=0}^{3}(\partial_\mu\varphi^k)^2 - \frac{\lambda}{4}\left(\sum_{k=0}^{3}(\partial_\mu\varphi^k)^2\right)^2 \\ &= \partial_\mu\chi^\dagger\partial^\mu\chi + \mu^2\chi^\dagger\chi - \lambda(\chi^\dagger\chi)^2\,, \end{aligned} \tag{5}$$

with

$$\chi = \begin{pmatrix} -\phi^+ \\ \phi^{0*} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\varphi_2 - i\varphi_1 \\ \varphi_0 + i\varphi_3 \end{pmatrix}. \tag{6}$$

Correspondingly, one has the conjugate doublet,

$$\chi^c = i\sigma_2\chi^* = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_0 - i\varphi_3 \\ \varphi_2 - i\varphi_1 \end{pmatrix}. \tag{7}$$

Note that the newly defined χ is not a linear combination of ϕ and Φ^c, and does not transform as a doublet under $SU(2)_L$. However, written in this form, it is easy to realize that there is an additional $SU(2)' \equiv SU(2)_R$ symmetry in this scalar sector:

$$\begin{aligned} \chi &\overset{SU(2)_R}{\longrightarrow} g_R\,\chi\,, \\ \mathcal{L}_{\rm SM}^{\rm sca} &\overset{SU(2)_R}{\longrightarrow} \mathcal{L}_{\rm SM}^{\rm sca}\,. \end{aligned} \tag{8}$$

Thus, one can see that the SM Higgs sector has a global chiral symmetry group $\mathcal{G} = SU(2)_L \times SU(2)_R$, wider than what could be naively expected.

However, one finds that the potential has non-trivial minima with $\langle\, |\Phi|^2 \,\rangle = v^2/2 \neq$, with $v = \mu/\sqrt{\lambda}$. Hence, this chiral symmetry $\mathcal{G}$ becomes spontaneously broken by the choice of the vacuum, traditionally taken to be $\langle\, \Phi \,\rangle^T = \left(0, v/\sqrt{2}\right)$ (and, correspondingly, $\langle\, \chi \,\rangle^T = \left(0, v/\sqrt{2}\right)$). This vacuum remains nevertheless invariant under the diagonal subgroup $\mathcal{H} = SU(2)_{L+R} \in \mathcal{G}$, with $g_L = g_R$. This subgroup is usually denoted a custodial or isospin symmetry. This spontaneous symmetry breaking (SSB) gives place to an $SU(2)_L \times SU(2)_R/SU(2)_{L+R}$ coset pattern for the corresponding SSB chiral Nambu-Goldstone bosons (NGB). This structure has been exploited along the years for the construction of EW effective theories [1] following the principles of the CCWZ formalism [17, 18].

2.2 A complex doublet Φ as a real 4–plet $\vec{\varphi}$

This extended symmetry can be also understood, maybe in a clearer way if one expresses the scalar sector Lagrangian in terms of its real components, gather in the real 4–vector: $\vec{\varphi}^T = (\varphi_1, \varphi_2, \varphi_3, \varphi_0,)$:

$$\mathcal{L}_{\rm SM}^{\rm sca} \;=\; \partial_\mu \Phi^\dagger \partial^\mu \Phi + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 \;=\; \frac{1}{2}(\partial_\mu \vec{\varphi})^2 + \frac{\mu^2}{2}\vec{\varphi}^2 - \frac{\lambda}{4}\left(\vec{\varphi}^2\right)^2 \,. \tag{9}$$

This Lagrangian is invariant under rotations $g \in SO(4)$,

$$\begin{aligned} \vec{\varphi} \quad &\stackrel{SO(4)}{\longrightarrow} \quad g\,\vec{\varphi}\,, \\ \mathcal{L}_{\rm SM}^{\rm sca} \quad &\stackrel{SO(4)}{\longrightarrow} \quad \mathcal{L}_{\rm SM}^{\rm sca}\,. \end{aligned} \tag{10}$$

However, one finds non-trivial minima in the potential with $\vec{\varphi} \neq \vec{0}$. Traditionally one takes $\langle \vec{\varphi} \rangle^T = (0, 0, 0, v)$ (although any other choice with $\langle \vec{\varphi}^2 \rangle = v^2$ is just as good), which is no longer invariant under the whole $\mathcal{G} = SO(4)$ group but still remains unchanged under $\mathcal{H} = SO(3)$ transformations of its first three components. This leads to an $SO(4)/SO(3)$ coset pattern for the NGB arising from the chiral SSB. This symmetry pattern has been employed in the past to described, e.g., pion interactions at low energies through the CCWZ formalism [17–19], as one has the local isomorphisms $SO(4) \sim SU(2) \times SU(2)$ and $SO(3) \sim SU(2)$.

2.3 A complex doublet Φ as a bi-fundamental representation Σ

We now discuss one final alternative approach that illustrates how the Higgs sector has a larger symmetry than the $SU(2) \times U(1)_Y$ invariance of the SM. First, let us construct the bi-fundamental 2×2 representation Σ from the Φ doublet:

$$\Sigma \;=\; (\Phi^c,\, \Phi) \;=\; \begin{pmatrix} \overline{\chi^c} \\ \overline{\chi} \end{pmatrix} = \frac{1}{\sqrt{2}}\left(\varphi_0 + i\varphi_a \sigma^a\right)\,. \tag{11}$$

By means of this matrix one can easily rewrite the Higgs Lagrangian in the form,

$$\begin{aligned} \mathcal{L}_{\rm SM}^{\rm sca} \;&=\; \partial_\mu \Phi^\dagger \partial^\mu \Phi + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 \\ &= \frac{1}{2}{\rm Tr}\{\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\} + \frac{\mu^2}{2}{\rm Tr}\{\Sigma\Sigma^\dagger\} - \frac{\lambda}{4}\left({\rm Tr}\{\Sigma\Sigma^\dagger\}\right)^2\,, \end{aligned} \tag{12}$$

where we made use of ${\rm Tr}\{\Sigma\Sigma^\dagger\} = {\rm Tr}\{\Phi^c\overline{\Phi^c} + \Phi\overline{\Phi}\} = \overline{\Phi^c}\Phi^c + \overline{\Phi}\Phi = 2\overline{\Phi}\Phi$, and analogous trace relations.

This Lagrangian is invariant under the global group $\mathcal{G} = SU(2)_L \times SU(2)_R$, this is, under unitary transformations of Σ acting either from the left ($g_L \in SU(2)_L$) or from the right ($g_R^\dagger \in SU(2)_R$) in the way,

$$\begin{aligned} \Sigma \quad &\overset{SU(2)_L\times SU(2)_R}{\longrightarrow} \quad g_L \Sigma g_R^\dagger \,, \\ \mathcal{L}_{\rm SM}^{\rm sca} \quad &\overset{SU(2)_L\times SU(2)_R}{\longrightarrow} \quad \mathcal{L}_{\rm SM}^{\rm sca} \,. \end{aligned} \tag{13}$$

One may notice that the left transformations g_L are related to the transformation $\Phi \to g_L \Phi$ whereas the right transformations g_R are given by $\chi \to g_R \chi$.

Nonetheless, the potential minima lead to a non-trivial vev $\langle \Sigma \rangle \neq 0$ given by the condition $\mathrm{Tr}\{\Sigma\Sigma^\dagger\} = v^2$. The usual vacuum choice is given in this representation by $\langle \Sigma \rangle = \frac{v}{\sqrt{2}} \mathbf{1_{2\times 2}}$. One can observe that this vacuum is no longer invariant under the whole chiral group $\mathcal{G} = SU(2)_L \times SU(2)_R$. Still, it remains invariant under the diagonal custodial subgroup $\mathcal{H} = SU(2)_{L+R}$ with $g_L = g_R \equiv g_C$, this is, $\langle \Sigma \rangle \to g_C \langle \Sigma \rangle g_C^\dagger = \langle \Sigma \rangle$.

Later on, in the EFT construction, we will use this bi-fundamental representation to describe the Higgs interactions. In addition, we will employ the CCWZ formalism [17, 18], where the NGB transform non-linearly under chiral transformations that do not leave the vacuum invariant. For the sake of this, it is useful to express the SM Higgs doublet, or the related matrix Σ, in a modulus-phase form,

$$\Sigma = \frac{1}{\sqrt{2}}(v+h)\, U(\omega^i) \,, \tag{14}$$

where we have taken the usual vev orientation $\langle \Sigma \rangle = \frac{v}{\sqrt{2}} \mathbf{1_{2\times 2}}$, h parametrizes the modulus of Φ and the 2×2 unitary matrix $U(\omega^i)$ depends non-linearly on three real fields $(\omega_1, \omega_2, \omega_3)$, the Goldstone fields, that parametrize the orientation of Φ with respect to its vaccuum direction. Under this decomposition, the scalar Lagrangian becomes,

$$\begin{aligned} \mathcal{L}_{\rm SM}^{\rm sca} \quad &= \; = \frac{1}{2}\mathrm{Tr}\{\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\} + \frac{\mu^2}{2}\mathrm{Tr}\{\Sigma\Sigma^\dagger\} - \frac{\lambda}{4}\left(\mathrm{Tr}\{\Sigma\Sigma^\dagger\}\right)^2 \\ &= \frac{(v+h)^2}{4}\mathrm{Tr}\{\partial_\mu U \partial^\mu U^\dagger\} + \frac{1}{2}(\partial_\mu h)^2 - V(h) \,, \end{aligned} \tag{15}$$

with $V(h) = -\frac{\mu^2}{2}(h+v)^2 + \frac{\lambda}{4}(h+v)^4$. In this non-linear Higgs representation, the previous variations of Σ and the SM Lagrangian under chiral transformations have now the form:

$$\begin{aligned} h \quad &\overset{SU(2)_L\times SU(2)_R}{\longrightarrow} \quad h \,, \\ U \quad &\overset{SU(2)_L\times SU(2)_R}{\longrightarrow} \quad g_L U g_R^\dagger \,, \\ \mathcal{L}_{\rm SM}^{\rm sca} \quad &\overset{SU(2)_L\times SU(2)_R}{\longrightarrow} \quad \mathcal{L}_{\rm SM}^{\rm sca} \,. \end{aligned} \tag{16}$$

One then finally realize that the complex doublet Φ, or 4–plet $\vec{\varphi}$ accepts a decomposition in term of a singlet component (h) and a triplet transforming non-linearly $(\omega_1, \omega_2, \omega_3)$. It is important to remark that symmetry alone does not force you to have the singlet and triplet components forming a 4–plet (or a complex doublet). Thus, if we give up renormalizability and intend to construct a general EW low-energy effective theory one should not take this *a priori* assumption for granted and h and U should be taken as independent multiplets of $\mathcal{G}$ if one desires to perform really model-independent searches of new physics.

Additionally, in order to complete the SM one needs to reconnect the scalar sector with the gauge bosons and SM fermions. The SM has a gauge $SU(2)_L \times U(1)_Y \in \mathcal{G}$ symmetry which gets spontaneously broken down to the EM subgroup, $U(1)_{EM}$. Only the gauge fields associeted with these generators are physical. The remaining ones are nevertheless handle through auxiliary spurionic fields ℓ_μ and r_μ that allow keeping the covariance under $\mathcal{G}$, and are set to the physical EW gauge fields $W_\mu^{1,2,3}$ and B_μ at the end of the day. The partial gauging of $\mathcal{G}$ introduces and explicit chiral breaking. In these proceedings we are ignoring the QCD $SU(3)_C$ gauge group, which does not play any role in the EW effective theory beyond some radiative corrections.

In addition, one needs to add the SM fermions. For that, one introduces spurionic auxiliary Yukawa fields that allow keeping the chiral invariance. When these auxiliary Yukawa fields are set to the SM Yukawa couplings, chiral symmetry is explicit broken, but in the way it is broken in the SM.

3. Building the EW effective theory

3.1 Particle content, symmetry and counting

In order to construct your EFT, one needs to answer three question:

1. What is your particle content? This is, what are fields of the the soft modes or light degrees of freedom in the effective Lagrangian $\mathcal{L}_{\text{EFT}}$?

2. What is your symmetry? This is, what symmetry invariance do you use to construct the EFT operators in $\mathcal{L}_{\text{EFT}}$? Notice that this is not the same as the previous point. HEFT, SMEFT and the SM based on the same symmetry.

3. What is the power counting? This is, according to what principle so you classify the infinite series of effective operators in $\mathcal{L}_{\text{EFT}}$? What are the most important operators –leading order (LO)–? What are the second most important operators –next-to-leading order (NLO)–? Etc.

Correspondingly, in the case of HEFT one has:

1. HEFT has the SM content, understanding this as the EW Goldstone triplet ω^a, the singlet Higgs h, the gauge bosons and the SM fermions.

2. The whole HEFT has a gauge symmetry $G_{SM} = SU(2)_L \times U(1)_Y$ (and QCD) that becomes spontenaoulsy broken in the form $G_{SM} \to H_{SM} = U(1)_{EM}$, giveng place to the masses $W^\pm$ and Z gauge bosons. HEFT, as it happens in the SM scalar sector, has an extended global chiral symmetry $\mathcal{G} = SU(2)_L \times SU(2)_R \times U(1)_{B-L} \supset G_{SM}$, which gets spontaneously broken down to the global custodial group $\mathcal{H} = SU(2)_{L+R} \times U(1)_{B-L} \supset H_{SM}$. The Baryon-minus-Lepton-number group is required in the presence of fermions to embed the SM groups G_{SM} and H_{SM}.

   This exact symmetry of the scalar sector is softly broken in the full Standard Model when the fermion and EW gauge-boson interactions are plugged in. Therefore, we need to explicitly break the chiral symmetry in exactly the same way it is done in the SM: there is a $L \Longleftrightarrow R$ asymmetry for $g = g'$, as there is a different gauging of the $\mathcal{G}$ generators (at a practical level, this would be solve in the limit $g' = 0$); there is a $t \leftrightarrow b$ splitting in the fermion doublets, as the top and bottom components have different masses for $\lambda_t \neq \lambda_b$.

3. In the case of the HEFT one use the chiral counting, which sorts out the contribution to the scattering amplitudes according to a power expansion $T \sim \sum_k c_k p^k$, in the external momenta (energy) or masses of the light particle, $p \sim E \sim m$, with the typical chiral expansion combination $\sim \frac{p^2}{16\pi^2 v^2}$. Indeed, the SM counting is a particular case of this chiral counting, as $\frac{g^{(\prime)\,2}}{16\pi^2} \sim \frac{m^2_{W,Z}}{16\pi^2 v^2}$, $\frac{\lambda^2_{\rm Fer}}{16\pi^2} \sim \frac{m^2_{\rm Fer}}{16\pi^2 v^2}$, $\frac{\lambda}{16\pi^2} \sim \frac{m_h^2}{16\pi^2 v^2}$.

We would like to note the similarities and differences between HEFT and SMEFT [11]: both effective EFT's are based on the gauge symmetry G; however, SMEFT includes the Higgs h and EW Goldstones ω^a as a complex doublet Φ, not as separate degrees of freedom. In addition, HEFT is organized according to the chiral counting, while SMEFT is organized according to an expansion in the canonical dimension of the operators.

The effective Lagrangian is organized in growing powers of p, standing p for any soft scale of the EFT (external momenta, masses m_h, m_W, etc.) [7]:

$$\mathcal{L}_{\rm EWET} = \sum_{\hat{d}\geq 2} \mathcal{L}_{\rm EWET}^{(\hat{d})} . \tag{17}$$

So far the Large Hadron Collider (LHC) has not found any trace of beyond the Standard Model (BSM) states with masses below 1 TeV. Likewise, no significant deviation has been observed in the low-energy interactions between Standard Model (SM) particles. Effective field theories are then the natural approach. In [3, 7, 20] we discuss the possibility of strongly-coupled BSM scenarios with the approximate custodial symmetry invariance of the SM, exact in the SM scalar sector. We develop an invariant Lagrangian under $\mathcal{G} = SU(2)_L \times SU(2)_R$, which spontaneously breaks down to the custodial subgroup $\mathcal{H} = SU(2)_{L+R}$ and generates the electroweak (EW) would-be Goldstone bosons ω^a, described a unitary 2×2 matrix $U(\omega)$. In these (non-linear) EW chiral Lagrangian with a light Higgs (ECLh), the low-energy amplitude $\mathcal{M}$ has an expansion in powers of infrared scales p (external momenta and SM masses) of the form (e.g., for $2 \to 2$ processes) [3, 7, 20–24],

$$\mathcal{M} \sim \underbrace{\frac{p^2}{v^2}}_{\text{LO (tree)}} + \Bigg(\underbrace{a_k^r}_{\text{NLO (tree)}} - \underbrace{\frac{\Gamma_k}{16\pi^2}\ln\frac{p}{\mu} + \ldots}_{\text{NLO (1-loop)}} \Bigg) \frac{p^4}{v^4} + \mathcal{O}(p^6) . \tag{18}$$

The EW effective theory (EWET) Lagrangian operators can be sorted out based on their chiral dimension:

$$\mathcal{L}_{\rm EWET} = \mathcal{L}_2 + \mathcal{L}_4 + \ldots \tag{19}$$

where the operators in $\mathcal{L}_{\hat{d}}$ are of $\mathcal{O}(p^{\hat{d}})$ [3, 7, 20–23]. Covariant derivatives and masses are $\mathcal{O}(p)$ [19, 25] and each fermion field scales like $\mathcal{O}(p^{1/2})$ in naive dimensional analysis (NDA) [3, 7, 10, 20, 22, 23]. The $\mathcal{G}$–invariant operators in $\mathcal{L}_{\rm EWET}$ are built with the Goldstone tensors

$U(\omega)$, functions $\mathcal{F}_k$ of the Higgs singlet h, its derivatives $\partial_{\mu_1}...\partial_{\mu_n}h$, the gauge fields and the SM fermions ψ [1, 10, 35, 36]. From the chiral counting point of view $\mathcal{L}^{\rm SM}$ would be $O(p^2)$ but its underlying renormalizable structure makes all $\Gamma_k = 0$ and ensures the absences of higher-dimension divergences [24, 26]. The most important contributions to a given process are given by the operators of lowest chiral dimension. The leading order (LO) contribution is $O(p^2)$ and is given by tree-level diagrams with only $\mathcal{L}_2$ vertices. Likewise, the one-loop contribution with only $\mathcal{L}_2$ vertices is $O(p^4)$; it is suppressed in (18) with respect to the LO by a factor $p^2/\Lambda_{\rm NL}^2$, with $\Lambda_{\rm NL}^2 \sim 16\pi^2 v^2 \Gamma_k^{-1} \gtrsim 3$ TeV (with $v = (\sqrt{2}G_F)^{-1/2} \approx 246$ GeV). This suppression factor is related to the non-linearity of the ECLh and $\Lambda_{\rm NL} \to \infty$ when the Higgs can be embedded in a complex doublet Φ [24]. [1]

In these proceedings [3, 7, 20] we focus our attention on the tree-level next-to-leading order (NLO) contributions. They are $O(p^4)$ and are provided by tree-level diagrams with one $\mathcal{L}_4$ vertex with low-energy coupling a_k (LEC) and an arbitrary number of $\mathcal{L}_2$ vertices. They get contributions from tree-level heavy resonance exchanges. At low energies, these $O(p^4)$ terms in (18) are typically suppressed with respect to the LO amplitude, $O(p^2)$, by a factor $a_k p^2/v^2 \sim p^2/M_R^2$ [3, 7, 20, 46, 47].

3.2 LO Lagrangian, $O(p^2)$

The leading order (LO) term is of $O(p^2)$. In addition to all the Standard Model (SM) operators, $\mathcal{L}_{\rm EWET}^{(2)}$ contains some new physics interactions [1, 5, 6],

$$\begin{aligned}\mathcal{L}_{\rm EWET}^{(2)} &= \sum_\xi \left[i\,\bar\xi \gamma^\mu d_\mu \xi - v\left(\,\bar\xi_L\, \mathcal{Y}\, \xi_R \,+\, \text{h.c.}\right)\right] \\ &-\frac{1}{2g^2}\langle \hat W_{\mu\nu}\hat W^{\mu\nu}\rangle_2 - \frac{1}{2g'^2}\langle \hat B_{\mu\nu}\hat B^{\mu\nu}\rangle_2 - \frac{1}{2g_s^2}\langle \hat G_{\mu\nu}\hat G^{\mu\nu}\rangle_3 \\ &+\frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{2}m_h^2 h^2 - V(h/v) + \frac{v^2}{4}\mathcal{F}_u(h/v)\,\langle u_\mu u^\mu\rangle_2\,,\end{aligned} \tag{20}$$

where the EW Goldstones ω^a are non-linearly realized through the usual exponential parametrization $u(\omega) = \exp(i\vec\sigma\vec\omega/(2v))$, with the Pauli matrices σ^a and the EW scale $v = \left(\sqrt{2}G_F\right)^{-1/2} = 246$ GeV. The Higgs field h is a singlet and can enter through undetermined functions $\mathcal{F}(h/v)$ that may appear in front of the chiral operators. In particular, the Goldstone kinetic term introduces the factor

$$\mathcal{F}_u(h/v) \;=\; 1 + \frac{2\kappa_W h}{v} + \frac{c_{2V} h^2}{v^2} + O(h^3)\,. \tag{21}$$

The couplings κ_W and c_{2V} parametrize the hWW and $hhWW$ interactions. These low-energy constants (LEC) are normalized such that $\kappa_W = c_{2V} = 1$ in the SM. An alternative notation $a = \kappa_W$ and $b = c_{2V}$ is used in some works.

It is important to note that the SM Lagrangian is a particular case of $\mathcal{L}_{\rm EWET}^{(2)}$: regardless of the considered coordinates for the Higgs and the EW Goldstones (complex doublet or modulus-phase), the theory is renormalizable, this is, non additional operator is ever needed to cancel out the ultraviolet divergences.

[1] Ref. [26] provides a geometrical interpretation in terms of the curvature of metric of the internal weak space of the Higgs. In the flat-space limit one has $\Lambda \to \infty$, with Λ the validity cut-off of the EFT. Further studies have shown that the presence of curvature in the scalar manifold is not, however, the key difference between HEFT and SMEFT-type theories. See, e.g., Refs. [11, 15, 16, 26].

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3.3 NLO Lagrangian, $\mathcal{O}(p^4)$

At next-to-leading order (NLO), $\mathcal{O}(p^4)$, the effective Lagrangian has the structure [3, 7],

$$\begin{aligned}\mathcal{L}^{(4)}_{\rm EWET} &= \sum_{i=1}^{12} \mathcal{F}_i(h/v)\, \mathcal{O}_i + \sum_{i=1}^{3} \widetilde{\mathcal{F}}_i(h/v)\, \widetilde{\mathcal{O}}_i \\ &+ \sum_{i=1}^{8} \mathcal{F}_i^{\psi^2}(h/v)\, \mathcal{O}_i^{\psi^2} + \sum_{i=1}^{3} \widetilde{\mathcal{F}}_i^{\psi^2}(h/v)\, \widetilde{\mathcal{O}}_i^{\psi^2} \\ &+ \sum_{i=1}^{10} \mathcal{F}_i^{\psi^4}(h/v)\, \mathcal{O}_i^{\psi^4} + \sum_{i=1}^{2} \widetilde{\mathcal{F}}_i^{\psi^4}(h/v)\, \widetilde{\mathcal{O}}_i^{\psi^4}\,. \end{aligned} \tag{22}$$

The precise form of these NLO operators can be found in Refs. [2, 3, 5, 7]. Any non-zero value of the $\mathcal{O}(p^4)$ LECs would signal the presence of new physics (NP). Some examples of NLO computations within the EWET can be found in Refs. [4, 27, 28].

3.4 1 loop corrections: $\mathcal{O}(p^4)$ renormalization

The HEFT Lagrangian is sorted out according to the chiral dimension of its operators, based on the scaling D_μ, $m_{\rm Bos}$, $m_{\rm Fer}$, $y_{\rm Fer}$, $g^{(\prime)} \sim \mathcal{O}(p)$ [3, 4, 7], such that the observables have an expansion $\mathcal{M} \sim \sum_n \mathcal{M}_n (p^2)^n$, where p stands for the appropriate combination of soft scales of the EFT (k_i^μ, m_j). For instance,

$$\mathcal{M}_{2\to 2} = \underbrace{\frac{a_{\mathcal{M}} p^2}{v^2}}_{\rm LO\ (tree)} + \underbrace{\frac{\mathcal{F}_{\mathcal{M}}\, p^4}{v^4}}_{\rm NLO\ (tree)} + \underbrace{\left(\frac{p^4 \Gamma_{\mathcal{M}}}{16\pi^2 v^4} \ln\frac{\mu}{p} + \dots\right)}_{\rm NLO\ (1-loop)} + \underbrace{\mathcal{O}(p^6)}_{\rm NNLO+\dots}\,, \tag{23}$$

where $a_{\mathcal{M}}$ ($\mathcal{O}(p^2)$) and $\mathcal{F}_{\mathcal{M}}$ ($\mathcal{O}(p^4)$) are the corresponding combination of renormalized couplings, with $\Gamma_{\mathcal{M}}$ providing their running [24, 26, 31–33]. Actually, in a rather baroque way, the SM is also organized in a chiral expansion, with $\frac{p^2}{16\pi^2 v^2} = \frac{g^{(\prime)2}}{16\pi^2}, \frac{\lambda^2_{\rm Fer}}{16\pi^2}, \frac{\lambda}{16\pi^2}$. [2] Since the latter p's are not external momenta but coupling constants, the SM amplitudes (arranged like Eq. (23)) do not diverge like a power of the energy. Obviously, both in SM and BSM scenarios, particular kinematic regimes may require further refinements of the EFT, as there may be further hierarchies between EFT scales.

Path integral functional methods allow one to extract the ultraviolet (UV) renormalization of the effective Lagrangian. Considering fluctuation η_i of the HEFT fields around their equation of motion (EoM) solutions, and expanding $\mathcal{L}_{\mathcal{O}(p^2)}$ in powers of η_i, the $\mathcal{O}(\eta^2)$ terms yield the one-loop contributions [24, 26]. The boson loop contributions to the effective action are given by

$$\int {\rm d^d x}\; \mathcal{L}^{\rm (NL),1\ell}_{NLO} = \frac{i}{2} {\rm tr}\log\left(-\frac{\delta^2 \mathcal{L}_{\mathcal{O}(p^2)}}{\delta\eta_i\,\delta\eta_j}\right) = -\frac{\mu^{d-4}}{16\pi^2 (d-4)} \int {\rm d^d x} \sum_k \Gamma_k \mathcal{O}_k + {\rm finite}\,, \tag{24}$$

The explicit form of the remaining gauge boson and fermion loop UV divergences can be found in [31–33] and further discussions on gauge-fixing are provided in Ref. [34]. These scalar one-loop contributions in (24) are $\mathcal{O}(p^4)$ [24, 26] and the BSM corrections are suppressed by $\Delta\mathcal{K}^2 =$

[2] At LO $g^2 = 4m_W^2/v^2$, $g'^2 = 4(m_Z^2 - m_W^2)/v^2$, $\lambda = m_H^2/(2v^2)$ and $y^2_{\rm Fer} = m^2_{Fer}/(2v^2)$.

$\big((\mathcal{F}_u')^2/\mathcal{F}_u - 4\big)$ and $\Omega = \big(2\mathcal{F}_u''/\mathcal{F}_u - (\mathcal{F}_u'/\mathcal{F}_u)^2\big)$ [24], vanishing in the SM limit. It is interesting to remark the geometrical analysis in [26], where the manifold of scalar fields $(h, \vec{\omega})$ has an associated metric g_{ij} =diag$\{\mathcal{F}_u(h) g_{ab}(\omega), 1\}$ in the HEFT. [3] Interestingly, the corresponding curvature [4] $\mathbb{R}$ is proportional to $\Delta\mathcal{K}^2$ and Ω. Thus, the scale suppressing the one-loop corrections with respect to the LO is $\Lambda^{-2} = \mathbb{R}/(4\pi)^2$. Barring non-derivative operators, the theory becomes non-interacting in the flat limit (the SM) –both at the tree and loop level–. This points out that the true expansion scale of the EFT in general is not the one that explicitly appears suppressing the EFT operators and individual diagrams (e.g., $4\pi v$ in the HEFT), which depends on the particular choice of coordinates for the scalar fields; the low-energy amplitudes derived from $\mathcal{L}_{O(p^2)}$ rather seem to be organized via an expansion in a more intrinsic and coordinate-independent scale, $\mathbb{R}$:

$$\textbf{Geometric-EWET}\Big|_{\mathcal{L}=\mathcal{L}_{O(p^2)}}: \qquad \mathcal{M} \;=\; \mathbb{R}p^2 \;+\; \frac{\mathbb{R}^2 p^4}{(4\pi)^2} \;+\; O(p^6)\,. \tag{25}$$

When testing for new physics, loop corrections must be taken into account to explain experiments with the necessary precision. One of the typical studies within the EFT framework is vector bosons scattering. At high energies often only boson loop corrections are kept because of its dependence with powers of the energy while fermion corrections are neglected.

Some final words referring fermion loop corrections: the expression for the fermion contributions are proportional to the mass of the fermion inside the loop and couplings of the Lagrangian instead of proportional to derivatives [29, 30]. Some of this couplings still allow an $O(10\%)$ deviation from the SM [4]. Nevertheless, the top Yukawa is so large that for moderate energies this type of contributions may numerically compete with truly derivarivative interactions. This is the reason why it is relevant to test the importance of these fermion contributions when considering the whole range of possible values within the HEFT framework [29].

3.5 HEFT vs SMEFT

In the case with a large mass gap with the lightest NP state R, as experiments seem to be indicating, the EFT approach appears as the most convenient one for future data analysis and interpretation. The aim of this subsection is to discuss two questions that are often posed in relation with the so-called non-linear HEFT:

- *"Since, experimentally, we seem to be so close to the SM, is it not enough to consider the SMEFT framework to parametrize NP effects at low energies?*
- *"Are not BSM loops essentially negligible?"*

The answer in both cases is *no*: the SMEFT is appropriate and loops are sub-subdominant only in particular cases (although this includes a broad set of NP scenarios, such as supersymmetry), as we will discuss. The two possible representations of the EFT based on the SM symmetries for any BSM extension are:

[3] The indices i, j run through the four scalars while a, b are restricted to the $\vec{\omega}$ coordinates. The $SO(4)$ non-linear sigma model metric $g_{ab}(\vec{\omega})$ is provided, e.g., in [26]. Different coordinate choices give different metrics: e.g., in the SMEFT, g_{ij} =diag$\big\{(1 + H/v)^2 g_{ab}(\vec{\omega}), 1 + P(H)\big\}$.

[4] All the discussion in these proceedings refers to the associated Riemann $\mathbb{R}_{ijk\ell}$, Ricci $\mathbb{R}_{ij}$ and scalar curvature $\mathbb{R}$ tensors [26], which will loosely denote as "curvature" $\mathbb{R}$.

- **SMEFT:** the EFT is build in terms of the complex doublet with the general form $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = (1 + H/v)U(\omega^a)\langle\, 0|\Phi|0 \,\rangle$ (with the vacuum expectation value $v = 0.246$ TeV and the unitary matrix U containing the electroweak (EW) Goldstones ω^a), leading to an EFT Lagrangian organized according to the canonical dimension D of the operators [4, 37]

$$\begin{aligned} \mathcal{L}^{(\mathrm{SMEFT})} &= \mathcal{L}_{SM\,(D\le 4)} + \sum_{D>4} \mathcal{L}_D = |D_\mu \Phi|^2 + \frac{c_H}{\Lambda^2}\left(\partial_\mu(\Phi^\dagger \Phi)\right)^2 + \dots \\ &= \frac{(v+H)^2}{4}\langle\, D_\mu U^\dagger D_\mu U \,\rangle + \frac{1}{2}(1 + P(H))\,(\partial_\mu h)^2 + \dots \end{aligned} \tag{26}$$

- **HEFT:** the EFT is built in terms of 1 singlet h and 3 EW Goldstones ω^a encoded in the NL representation provided by the unitary matrix $U(\omega^a)$, leading to an EFT organized according the chiral dimension d of the operators ($\mathcal{L}_{SM} \subset \mathcal{L}_{O(p^2)}$) [1, 4, 38]

$$\mathcal{L}^{(\mathrm{HEFT})} = \mathcal{L}_{O(p^2)} + \mathcal{L}_{O(p^{\ge 4})}\,, \tag{27}$$

$$\mathcal{L}_{O(p^2)} = \frac{v^2}{4}\mathcal{F}_u(h)\langle\, D_\mu U^\dagger D^\mu U \,\rangle + \frac{1}{2}(\partial_\mu h)^2 + \dots\,, \qquad \mathcal{L}_{O(p^4)} = \sum_k \mathcal{F}_k(h) O_k\,,$$

with $\mathcal{F}_u(h) = 1 + 2ah/v + bh^2/v^2 + O(h^3)$ having an analytical expansion around $h = 0$. [5]

An extensive analyses on the SMEFT↔HEFT relation can be found in [11]. Here, we provide a simplified discussion. It is possible to rewrite the SMEFT (26) in the HEFT form (27):

$$[\textbf{SMEFT} \to \textbf{HEFT}]: \qquad \frac{dh}{dH} = \sqrt{1 + P(H)} \quad \Longrightarrow \quad h = \int_0^H \sqrt{1 + P(H)}\, dH\,. \tag{28}$$

Likewise, provided there exists an $SU(2)_L \times SU(2)_R$ fixed point h^* with $\mathcal{F}_u(h^*) = 0$ [26], it is always possible to revert the complete HEFT Lagrangian (27) into the SMEFT form in terms of Φ in (26):

$$[\textbf{HEFT} \to \textbf{SMEFT}]: \qquad \Phi^\dagger \Phi = (v + H)^2/2 = \frac{v^2}{2}\mathcal{F}_u(h)\,. \tag{29}$$

The issue is not whether we write our all-order EFT Lagrangian in the form (26) or (27), the problem is to make sense of the EFT perturbative expansion in that realization [37]. Thus, if one assumes the SMEFT (26) and rewrites it in the $\mathcal{L}^{(\mathrm{NL})}$ form, the deviations from the SM in the hWW and $hhWW$ vertices, respectively $\Delta(a^2) = a^2 - 1$ and $\Delta b = b - 1$, have the precise form [11, 39] [6]

$$[\textbf{SMEFT} \to \textbf{HEFT}]: \; \Delta(a^2) = -2c_H v^2/\Lambda^2 + \dots,\; \Delta b = -4 c_H v^2/\Lambda^2 + \dots \Longrightarrow 2\Delta(a^2) = \Delta b + \dots \tag{30}$$

where the dots stand for the $O\left((v/\Lambda)^{n\ge 4}\right)$ contributions from SMEFT operators with $D \ge 8$. The next couple of examples shows the potential issues of the SMEFT representation:

[5] The a and b terms provide the hWW and $hhWW$ interactions, respectively, with normalization $a^{SM} = b^{SM} = 1$.

[6] There is another dimension-6 operator in the SMEFT, c_T [4, 37], that modifies the Goldstone kinetic term once $\mathcal{L}^{(\mathrm{SMEFT})}$ is rewritten in the $\mathcal{L}^{(\mathrm{HEFT})}$ form. However, it does not contributes to $\mathcal{F}_u(h)$ in $\mathcal{L}_{O(p^2)}$, but to the $a_0 O_0$ Longhitano operator [1], NLO due to its large experimental suppression in the T oblique parameter [4, 38, 44].

1. **Dilaton Higgs models [40]:** formulated in the NL representation (27), they obey the constraint $\Delta(a^2) = \Delta b$. Thus, there are large corrections in (30) from operators of dimension $D > 6$ if one writes $\mathcal{L}^{(\mathrm{NL})}$ in the L form (26), leading to a breakdown of the SMEFT expansion [37].

2. $SO(N)/SO(N-1)$ **composite Higgs models (CHM) [41]:** Interestingly enough, the strongly interacting CHM can always be rewritten in the SMEFT form [26]. [7] However, its SMEFT v/f expansion is poorly convergent for $v \sim f$ at any energy E (or not at all), [8] while its $\mathcal{L}^{(\mathrm{HEFT})}$ gives an EFT with loops suppressed by powers of $E^2/(16\pi^2 v^2)$ [4], at least [9] valid for $E \ll 4\pi v$.

4. A glimpse on phenomenology

In this section we have a quick look at an (incomplete) selection of BSM studies within the chiral effective framework. Some works consider a pure HEFT analysis. In others, we will find further assumptions, as the existence of a strongly interacting UV completion, short-distance constraints and sum-rules. Depending on its quantum numbers, the observables are contributed by resonances with different characteristics (lighter, heavier, narrow, broad or even absence of resonant contributions) which may enter in competition with NLO loops of a similar size, as in happens in Quantum Chromodynamics. This is specially relevant in quantities where the $O(p^2)$ is absent and start at NLO [4]:

- **One-loop resonance computation of the oblique S and T parameters [44]:** the two-Weinberg sum-rule scenario in [44] yielded an $O(p^4)$ tree-level contribution suppressed at the 95% CL by a scale $\Lambda \gtrsim 5$ TeV (given by the vector and axial-vector resonance mass contributions $-(M_V^2 + M_A^2)/(4M_V^2 M_A^2)$ to the HEFT coupling a_1 [4]) and the $O(p^4)$ loops suppressed by $\Lambda \gtrsim 30$ TeV (related to the chiral log coefficient $(a^2-1)/(192\pi^2 v^2)$ [42, 44]).

- $\gamma\gamma \to ZZ$ **[42]:** Based on the Run-1 fit [43], the $O(p^4)$ tree-level contribution is suppressed at the 95% CL by a scale $\Lambda \gtrsim 1.4$ TeV (provided by the combination $2ac_\gamma/v^2$ [42]) and the $O(p^4)$ loops suppressed by $\Lambda \gtrsim 2.6$ TeV (given by $(a^2-1)/(4\pi^2 v^2)$ [42]).

In summary: BSM extensions may, in general, contain more than one hard scale. Which one is the lightest one in the low-energy EFT depends on the particular case. The HEFT organized through a chiral expansion provides a systematic approach to tackle these issues, valid up to $E = 4\pi v$ (or higher energies [10]) and is expected to lead to promising NP collider signals in future years [45].

[7] In particular, these $SO(N)/SO(N-1)$ CHM obey the relation in Eq. (30).

[8] Note that the 95%CL determination $a > 0.8$ [43] implies a rather loose lower bound for the $SO(N)/SO(N-1)$ spontaneous symmetry breaking scale [41] $f = v/\sqrt{1-a^2} > 0.4$ TeV for the CHM scale.

[9] Explicit computations show that even if individual loop diagrams are suppressed in this way, the full one-loop amplitude shows a much stronger suppression $(1-a^2)E^2/(16\pi^2 v^2) = E^2/(16\pi^2 f^2)$ for the HEFT derived from the $SO(5)/SO(4)$ CHM [42].

[10] Due to subtle cancelations in close-to-SM scenarios the HEFT might be valid up to energies way higher than $4\pi v$ as it happens, e.g., in the SM or the $SO(5)/SO(4)$ CHM [41]. In the latter, individual 1-loop diagrams in the HEFT representation have a suppression $O(p^2/(16\pi^2 v^2))$ but a strong cancellation shows up after summing them up, recovering the $E^2/(16\pi^2 f^2)$ loop suppression one obtains in the underlying $SO(5)$ non-linear sigma model diagram by diagram [42].

5. Conclusions

In these proceedings we discuss the effective field theory (EFT) description of the electroweak (EW) gauge and Higgs sectors and its phenomenological applications. We adopt the non-linear EW effective theory realization of the EW symmetry [1, 3, 7, 20], also known as Higgs effective field theory (HEFT) or electroweak chiral Lagrangian (EWChL) [1, 4–6]. The basic ingredient of the EFT are the SM symmetries, in particular the EW symmetry-breaking pattern $SU(2)_L \times SU(2)_R / SU(2)_{L+R}$, where the chiral group gets spontaneously broken down to the custodial subgroup.[11]

Following the CCWZ formalism [17, 18] one constructs the corresponding effective Lagrangian, with its effective operators sorted out according to the chiral $O(p^d)$ counting.

The EWET couplings can be predicted in terms of resonance parameters; different resonance quantum numbers lead to different patterns for the LECs [3, 7, 20, 46]. Further assumptions about the UV structure of the underlying theory can be used to refine the predictions [3, 7, 20, 44]. In thhese proceedings we have provided a couple of examples to show that composite resonances with masses of a few TeV ($M_R \sim 4\pi v \approx 3$ TeV) are compatible with present direct and indirect searches, and can easily comply with the low-energy electroweak precision tests.

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