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Some recent developments in nonleptonic B decays

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I report on three recent topics from the field of two-body nonleptonic decays of $B_{(s)}$ mesons. The computation of two-loop NNLO corrections to the leading penguin amplitudes in QCD factorisation, puzzles that have emerged in colour-allowed tree-level decays to heavy-light final states, and a combination of QCD factorisation with $SU(3)$ flavour symmetry to estimate the size of weak-annihilation amplitudes.

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1. Introduction

Nonleptonic decays of $B_{(s)}$ mesons play an essential role in the flavour-physics programmes at current and future colliders. The prospects for ever more precise measurements are therefore excellent, and that precision must be matched by theoretical predictions if we aim for improving our understanding of the mechanism of quark flavour mixing and CP violation, or even for pursuing the quest for indirect signals of new physics. On the theoretical side the bottleneck for precision is the computation of the hadronic matrix elements, where QCD effects from many different scales arise. Several approaches have been developed to get a handle on the hadronic matrix elements, each having its virtues and drawbacks. *Factorisation frameworks* such as PQCD [1, 2] or QCD factorization (QCDF) [3–5] factorise short- from long-distance physics in the heavy quark limit. However, in their present use, they don't allow for the computation of sub-leading power corrections in the heavy-quark expansion from first field-theoretical principles (see [6] for a computation of annihilation amplitudes with light-cone sum rules). *Flavour symmetries* of the light quarks [7–9] have the advantage of hardly requiring any assumption about the scales of the occurring QCD effects, and relate different decay channels to each other, thereby reducing the number of independent parameters. On the other hand, it is well-known that flavour $SU(3)$, U-spin and V-spin are severely broken by the strange-quark mass, and the lack of a rigorous implementation of flavour breaking can still be regarded as the main drawback of this approach. *Dalitz plot analyses* are mostly applied to three-body decays, and very important for phenomenology. They are mostly data-driven, but also QCD-based predictions have been worked out in recent years [10–12].

One obvious idea to get a better handle on nonleptonic decays is to combine the different approaches, with the goal of benefitting from their advantages, while at the same time minimizing the sensitivities to their individual drawbacks. One example is the so-called factorization-assisted topological amplitude approach [13–17], while combinations of factorization and flavour symmetries were studied e.g. in [18–21] and very recently in [22]. Also numerous studies of flavour symmetries in multibody final states exist (e.g. [23–25]).

In this proceedings contribution, we report on three recent studies in the field of two-body nonleptonic $B_{(s)}$ decays. The two-loop $O(\alpha_s^2)$ correction to the leading penguin amplitudes in QCD factorisation, puzzles that have emerged in colour-allowed tree-level decays to heavy-light final states, and a combination of QCD factorisation with $SU(3)$ flavour symmetry to estimate the size of weak annihilation amplitudes.

2. Penguin amplitudes and direct CP asymmetries to $O(\alpha_s^2)$

The QCD factorization formula [3, 5, 26] for charmless two-body nonleptonic B decays into two pseudoscalar mesons M_1 and M_2,

$$\begin{aligned}\langle M_1 M_2 | Q_i | \bar{B} \rangle &= i\,\frac{m_B^2}{4}\left\{ F^{BM_1}(0) \int_0^1 du\, T_i^I(u)\, f_{M_2}\phi_{M_2}(u) + (M_1 \leftrightarrow M_2) \right. \\ &\left. + \int_0^\infty d\omega \int_0^1 du dv\, T_i^{II}(\omega, v, u)\, f_B\phi_B(\omega)\; f_{M_1}\phi_{M_1}(v)\; f_{M_2}\phi_{M_2}(u) \right\} + O\left(\frac{\Lambda_{\rm QCD}}{m_b}\right), \quad (1)\end{aligned}$$

expresses the matrix element of an operator Q_i from the effective weak Hamiltonian in terms of non-perturbative $B \to M$ form factors $F^{BM}(0)$, decay constants f_M, light-cone distribution amplitudes

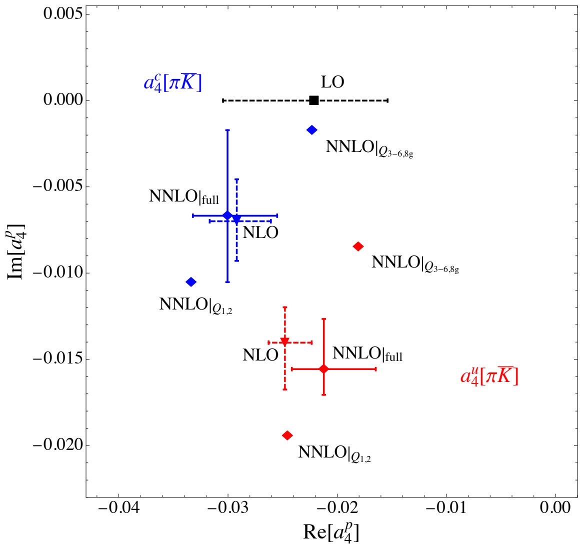

Figure 1: Anatomy of QCD corrections to $a_4^{u,c}(\pi\bar{K})$ [27]. The leading-order (LO) point is equal for both.

(LCDA) $\phi_M(u)$, and perturbatively calculable hard-scattering kernels $T_i^I(u)$, $T_i^{II}(\omega,v,u)$. Since the latter have the structure $T_i^I = 1 + \mathcal{O}(\alpha_s)$ and $T_i^{II} = \mathcal{O}(\alpha_s)$, it is clear that the (strong) rescattering phases and hence the direct CP asymmetries are of $\mathcal{O}(\alpha_s)$, or of next-to-leading power $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$. While the calculation of power corrections in QCDF remains challenging, the short-distance part at leading power has recently been completed at next-to-next-to-leading order (NNLO) [27], corresponding to $\mathcal{O}(\alpha_s^2)$. The calculation in the latter paper focused on the two-loop correction to the penguin amplitudes $a_4^u(M_1M_2)$ and $a_4^c(M_1M_2)$, which required the computation of more than a hundred two-loop diagrams involving two different scales (u and m_c^2/m_b^2), and could only be achieved by applying sophisticated multi-loop techniques, in particular for the master integrals [28]. In figure 1 we provide for $a_4^u(\pi\bar{K})$ and $a_4^c(\pi\bar{K})$ the anatomy of the QCD corrections at various orders [27]. Recently, also QED corrections became available [29, 30].

The results for the amplitudes can be used to obtain the direct CP asymmetries at $\mathcal{O}(\alpha_s^2)$. In table 1 we collect the numerical values of direct CP asymmetries for a sample of penguin-dominated charmless $B \to PP$ channels at various perturbative orders[1]. The columns labelled "NLO" and "NNLO" give the respective results with all power-suppressed terms but the short-distance dominated scalar penguin amplitude set to zero. The column "NNLO+LD" adds the previously neglected long-distance (LD) terms back, whose main effect is from weak annihilation. The numbers in table 1 reveal that the perturbatvie NNLO corrections are in general not sizeable, as can be anticipated from figure 1. The addition of the LD effects, which is done according to the scenario S_4' in [26], has a large impact on both, the central values and the uncertainties, and in many cases spoils the precision achieved at leading power. There are, however, suitably chosen combinations which are robust against power corrections, for instance the CP asymmetry difference

$$\delta(\pi\bar{K}) = A_{\rm CP}(\pi^0K^-) - A_{\rm CP}(\pi^+K^-)\,, \tag{2}$$

[1]Note that the numbers in table 1 are from [31], where at NNLO only the contribution from current-current operators to $a_4^{u,c}$ was available. A comprehensive update using also more recent results is still pending.

f	NLO	NNLO	NNLO + LD	Exp
$\pi^-\bar{K}^0$	$0.71^{+0.13\,+0.21}_{-0.14\,-0.19}$	$0.77^{+0.14\,+0.23}_{-0.15\,-0.22}$	$0.10^{+0.02\,+1.24}_{-0.02\,-0.27}$	-1.7 ± 1.6
π^0K^-	$9.42^{+1.77\,+1.87}_{-1.76\,-1.88}$	$10.18^{+1.91\,+2.03}_{-1.90\,-2.62}$	$-1.17^{+0.22\,+20.00}_{-0.22\,-\;6.62}$	4.0 ± 2.1
π^+K^-	$7.25^{+1.36\,+2.13}_{-1.36\,-2.58}$	$8.08^{+1.52\,+2.52}_{-1.51\,-2.65}$	$-3.23^{+0.61\,+19.17}_{-0.61\,-\;3.36}$	-8.2 ± 0.6
$\pi^0\bar{K}^0$	$-4.27^{+0.83\,+1.48}_{-0.77\,-2.23}$	$-4.33^{+0.84\,+3.29}_{-0.78\,-2.32}$	$-1.41^{+0.27\,+5.54}_{-0.25\,-6.10}$	1 ± 10
$\delta(\pi\bar{K})$	$2.17^{+0.40\,+1.39}_{-0.40\,-0.74}$	$2.10^{+0.39\,+1.40}_{-0.39\,-2.86}$	$2.07^{+0.39\,+2.76}_{-0.39\,-4.55}$	12.2 ± 2.2
$\Delta(\pi\bar{K})$	$-1.15^{+0.21\,+0.55}_{-0.22\,-0.84}$	$-0.88^{+0.16\,+1.31}_{-0.17\,-0.91}$	$-0.48^{+0.09\,+1.09}_{-0.09\,-1.15}$	-14 ± 11

Table 1: Direct CP asymmetries in percent [31]. Theoretical uncertainties are CKM and hadronic, respectively.

which is still at the heart of the $B \to K\pi$ puzzle, or the asymmetry sum rule

$$\Delta(\pi\bar{K}) = A_{\rm CP}(\pi^+K^-) + \frac{\Gamma_{\pi^-\bar{K}^0}}{\Gamma_{\pi^+K^-}}\,A_{\rm CP}(\pi^-\bar{K}^0) - \frac{2\Gamma_{\pi^0K^-}}{\Gamma_{\pi^+K^-}}\,A_{\rm CP}(\pi^0K^-) - \frac{2\Gamma_{\pi^0\bar{K}^0}}{\Gamma_{\pi^+K^-}}\,A_{\rm CP}(\pi^0\bar{K}^0)\,, \quad (3)$$

which is expected to be small [32]. The numbers in table 1 confirm that power corrections are numerically much better under control for these quantities than for most of the direct CP asymmetries themselves. An updated value for $\Delta(\pi\bar{K})$ including QED corrections can be found in [29]. On the experimental side one of the urgently missing pieces is the CP asymmetry in the $\pi^0\bar{K}^0$ channel, which constitutes a very important measurement at Belle II. Recent progress in this direction was reported in [33].

3. Puzzles in tree-level color-allowed decays

For two-body colour-allowed nonleptonic tree-level decays such as $\bar{B}^0_{(s)} \to D^{(*)+}_{(s)}L^-$ (L being a light pseudoscalar meson) QCDF is expected to work very well: Both the colour-suppressed tree and the penguin amplitudes are absent, and effects from spectator scattering and weak annihilation are power suppressed [4]. Moreover, weak annihilation is absent if the decay is flavour-specific, i.e. if all final-state flavours are distinct as in $\bar{B}^0_s \to D^{(*)+}_s\pi^-$ and $\bar{B}^0 \to D^{(*)+}K^-$ (but not in $\bar{B}^0 \to D^{(*)+}\pi^-$). Besides the branching fractions themselves we will consider the following ratios,

$$\mathcal{R}^{P(V)}_{s/d} = \frac{\mathcal{B}(\bar{B}^0_s \to D^{(*)+}_s\pi^-)}{\mathcal{B}(\bar{B}^0 \to D^{(*)+}K^-)}\,, \quad \mathcal{R}^{V/P}_s = \frac{\mathcal{B}(\bar{B}^0_s \to D^{*+}_s\pi^-)}{\mathcal{B}(\bar{B}^0_s \to D^+_s\pi^-)}\,, \quad \mathcal{R}^{V/P}_d = \frac{\mathcal{B}(\bar{B}^0 \to D^{*+}K^-)}{\mathcal{B}(\bar{B}^0 \to D^+K^-)}\,. \quad (4)$$

In the factorisation formula [4]

$$\langle D^{(*)+}_{(s)}L^-|\,Q_i\,|\bar{B}^0_{(s)}\rangle = \sum_j F_j^{\bar{B}_{(s)}\to D^{(*)}_{(s)}}(M_L^2)\int_0^1 du\,T_{ij}(u)\phi_L(u) + O\left(\frac{\Lambda_{\rm QCD}}{m_b}\right) \quad (5)$$

the hard functions $T_{ij}(u)$ are known to two loops [34], and the form factors have been examined in a recent precision study [35]. As expected, the bottleneck to a precision prediction are the power

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	PDG	QCDF prediction	discrepancy
$\mathcal{B}(\bar{B}_s^0 \to D_s^+\pi^-)$	3.00 ± 0.23	4.42 ± 0.21	$\sim 4\sigma$
$\mathcal{B}(\bar{B}^0 \to D^+K^-)$	0.186 ± 0.020	0.326 ± 0.015	$\sim 5\sigma$
$\mathcal{B}(\bar{B}_s^0 \to D_s^{*+}\pi^-)$	2.0 ± 0.5	$4.3^{+0.9}_{-0.8}$	$\sim 2\sigma$
$\mathcal{B}(\bar{B}^0 \to D^{*+}K^-)$	0.212 ± 0.015	$0.327^{+0.039}_{-0.034}$	$\sim 3\sigma$
$\mathcal{R}^P_{s/d}$	16.1 ± 2.1	$13.5^{+0.6}_{-0.5}$	$< 1\sigma$
$\mathcal{R}^V_{s/d}$	9.4 ± 2.5	$13.1^{+2.3}_{-2.0}$	$< 1\sigma$
$\mathcal{R}^{V/P}_s$	0.66 ± 0.16	$0.97^{+0.20}_{-0.17}$	$< 1\sigma$
$\mathcal{R}^{V/P}_d$	1.14 ± 0.15	1.01 ± 0.11	$< 1\sigma$

Table 2: Theory vs. experiment for flavour-specific, colour-allowed tree-level decays. Branching ratios are given in units of 10^{-3}, their ratios are defined in eq. (4).

corrections. In [36], power corrections from several effects were identified and their size estimated: higher twist effects to the light-meson LCDA, hard-collinear gluon emission from the spectator quark q and from the heavy quarks b and c, and soft-gluon exchange between the $B \to D$ and the light-meson system. The total size of the next-to-leading power compared to the leading-power contributions were conservatively estimated to be below the percent level [36], which supports the picture of these decays being very clean.

However, when comparing to the experimental measurements a puzzling pattern arises, which we summarize in table 2. The experimental values for the branching ratios are consistenly below the theoretical predictions, between $\sim 2\sigma$ and $\sim 5\sigma$ depending on the channel. The ratios of branching fractions are, on the other hand, in agreement within uncertainties. The numbers could be brought into agreement with a universal, non-factorizable contributions of $O(-15-20\%)$ on the amplitude level. At the moment, it is unclear where this contribution could arise. QED corrections were studied in [30], with the outcome that they ease the tension but are too small to explain the discrepancy. Rescattering effects were considered in [37] and also found to be too small. Also the input parameters can be considered reliable. Issues on the experimental side are also unlikely since the decays have a large branching fraction and only charged particles in the final state. Moreover, a recent measurement from Belle [38] confirms earlier findings. Effects of physics from beyond the Standard Model (BSM) have also been investigated in a couple of recent papers. In [39] it was found that the tension can be partially explained by a left-handed W'-model, while still being compatible with other flavour and collider bounds. New tensor structures were analysed in [40], some of them can explain the data at the 1σ-level. In the same reference also a model-dependent analysis, e.g. with a colourless charged scalar, was carried out. In [41] a combination with dijet searches was performed, where mediators with various $SU(3) \times SU(2) \times U(1)$ quantum numbers were considered. For a good portion of the scenarios considered there, it was pointed out that the parameter space to explain the nonleptonic tree-level puzzle is to a large extent already ruled out by dijet searches. Other recent studies that combine tree-level nonleptonic decays with lifetimes [42], with $B_s^0 \to D_s^{\mp}K^{\pm}$ decays [43, 44], and with CP violation in the mixing and decay of $B^0_{(s)}$ mesons [45] are also available.

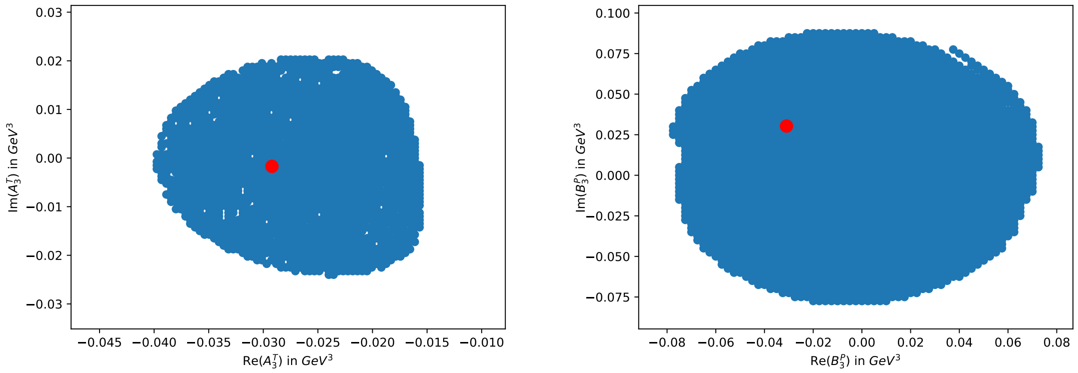

Figure 2: Two sample amplitudes determined from the $SU(3)$-irreducible fit. The red point denotes the best-fit point, the blue area the 1σ confidence region.

4. Combining QCD factorisation and flavour symmetries

In a recent work [22] we estimate the potential size of the weak-annihilation amplitudes in QCDF in a data-driven approach combined with the $SU(3)$ flavour symmetry of the light quarks. To this end we use the linear relations between the so-called topological and $SU(3)$-invariant description of the decay amplitude [46], and determine the $SU(3)$-invariant amplitudes through a χ^2-fit, for which we use experimental input for branching fractions (23 measurements and 6 upper bounds) and CP asymmetries (17 measurements and one upper bound) from $B \to PP$ decays. The fit parameters are made up of 20 complex amplitudes (10 for tree and 10 for penguins), of which two complex amplitudes and one overall phase can be absorbed. Together with the $\eta - \eta'$ mixing angle, we therefore fit for 36 real parameters. The best-fit point is determined from 10^9 randomly generated points in our 36-dimensional space, with some refinements for which we refer to [22]. The uncertainties are determined through a likelihood ratio test, and the p value is determined from Wilk's theorem with two degrees of freedom. In figure 2 we show the fit result for two sample $SU(3)$-irreducible amplitudes. To assess the physical consequences of the fit, the results for the amplitudes are translated back to branching fractions and direct CP asymmetries for more than 30 $B \to PP$ channels. Quantitatively, the goodness of the fit is reflected by $\chi^2/d.o.f = 0.851$. Hence, the vast majority of the numbers between theory and experiment are in agreement, though with still sizeable uncertainties in certain cases. Moreover, some of our numbers represent predictions for yet unmeasured channels, in particular those from B_s^0 decays and those with $\eta^{(\prime)}$ in the final state.

We then investigate the connection between the $SU(3)$-irreducible representation and QCDF. We establish the transformation rules between the topological description and QCDF, and show that the number of independent complex amplitudes equals 18 in both approaches (see also [47]), and that the relation is again linear. Together with the relations in [46] this establishes the transformation between the $SU(3)$-irreducible and the QCDF amplitudes, and hence allows for the translation of the $SU(3)$-fit results into constraints on QCDF amplitudes. In particular, the fit gives a quantitative estimate of the size of the annihilation amplitudes as dictated by data (see also [48]). Our main finding is that the most constrained weak annihilation amplitudes are below 0.04, see left panel of figure 3. However, values up to ~ 0.3 are allowed in certain cases (see right panel of figure 3),

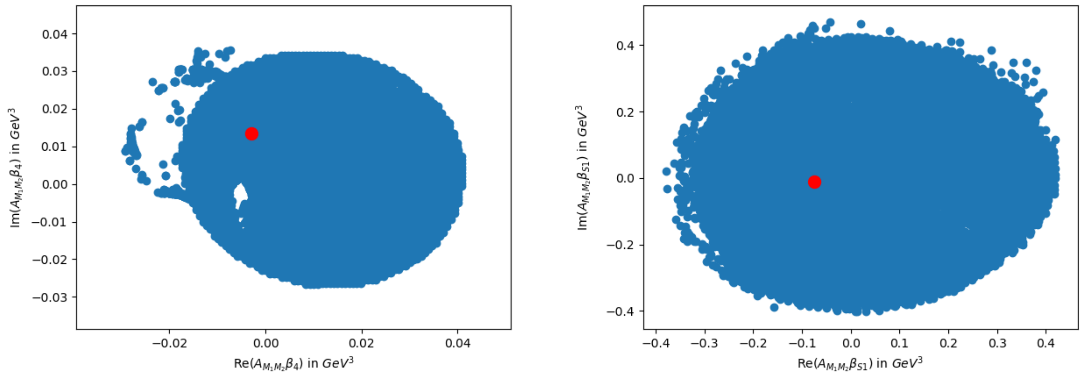

Figure 3: Fit results for two sample annihilation amplitudes. The colour coding is the same as in figure 2.

which can to a large part be attributed to sizeable uncertainties in several of the experimental input parameters. Also effects of $SU(3)$ breaking are briefly addressed in [22]. However, this important topic certainly deserves further dedicated studies in the future.

5. Conclusion

Despite all the progress in nonleptonic $B_{(s)}$ decays, the field is eagerly awaiting further improvements. On the experimental side a key input to the asymmetry sum rule is the measurement of the direct CP asymmetry in $\bar{B}^0 \to \pi^0 \bar{K}^0$ (for a first step in this direction see [33]). Moreover, measurements of branching ratios and direct CP asymmetries in $B_{(s)}$ decays to $\eta^{(\prime)}$ will yield a more comprehensive picture in the comparison between theory and experiment. On the theoretical side it is indispensable to get power corrections and flavour symmetry breaking under better control. For a very recent work see [49].

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References

[1] Y.-Y. Keum, H.-n. Li and A.I. Sanda, *Fat penguins and imaginary penguins in perturbative QCD*, *Phys. Lett. B* **504** (2001) 6 [hep-ph/0004004].

[2] C.-D. Lu, K. Ukai and M.-Z. Yang, *Branching ratio and CP violation of $B \to \pi\pi$ decays in perturbative QCD approach*, *Phys. Rev. D* **63** (2001) 074009 [hep-ph/0004213].

[3] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, *QCD factorization for $B \to \pi\pi$ decays: Strong phases and CP violation in the heavy quark limit*, Phys. Rev. Lett. **83** (1999) 1914 [hep-ph/9905312].

[4] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, *QCD factorization for exclusive, nonleptonic B meson decays: General arguments and the case of heavy light final states*, *Nucl. Phys. B* **591** (2000) 313 [hep-ph/0006124].

[5] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, *QCD factorization in $B \to \pi K, \pi\pi$ decays and extraction of Wolfenstein parameters*, *Nucl. Phys. B* **606** (2001) 245 [hep-ph/0104110].

[6] A. Khodjamirian, T. Mannel, M. Melcher and B. Melic, *Annihilation effects in $B \to \pi\pi$ from QCD light-cone sum rules*, *Phys. Rev. D* **72** (2005) 094012 [hep-ph/0509049].

[7] D. Zeppenfeld, *SU(3) Relations for B Meson Decays*, *Z. Phys. C* **8** (1981) 77.

[8] M.J. Savage and M.B. Wise, *SU(3) Predictions for Nonleptonic B Meson Decays*, *Phys. Rev. D* **39** (1989) 3346.

[9] M. Gronau and D. London, *Isospin analysis of CP asymmetries in B decays*, *Phys. Rev. Lett.* **65** (1990) 3381.

[10] S. Kränkl, T. Mannel and J. Virto, *Three-body non-leptonic B decays and QCD factorization*, *Nucl. Phys. B* **899** (2015) 247 [1505.04111].

[11] R. Klein, T. Mannel, J. Virto and K.K. Vos, *CP Violation in Multibody B Decays from QCD Factorization*, *JHEP* **10** (2017) 117 [1708.02047].

[12] T. Huber, J. Virto and K.K. Vos, *Three-Body Non-Leptonic Heavy-to-heavy B Decays at NNLO in QCD*, *JHEP* **11** (2020) 103 [2007.08881].

[13] H.-n. Li, C.-D. Lu and F.-S. Yu, *Branching ratios and direct CP asymmetries in $D \to PP$ decays*, *Phys. Rev. D* **86** (2012) 036012 [1203.3120].

[14] Q. Qin, H.-n. Li, C.-D. Lü and F.-S. Yu, *Branching ratios and direct CP asymmetries in $D \to PV$ decays*, *Phys. Rev. D* **89** (2014) 054006 [1305.7021].

[15] S.-H. Zhou, Y.-B. Wei, Q. Qin, Y. Li, F.-S. Yu and C.-D. Lu, *Analysis of Two-body Charmed B Meson Decays in Factorization-Assisted Topological-Amplitude Approach*, *Phys. Rev. D* **92** (2015) 094016 [1509.04060].

[16] C. Wang, Q.-A. Zhang, Y. Li and C.-D. Lu, *Charmless $B_{(s)} \to VV$ Decays in Factorization-Assisted Topological-Amplitude Approach*, *Eur. Phys. J. C* **77** (2017) 333 [1701.01300].

[17] H.-Y. Jiang, F.-S. Yu, Q. Qin, H.-n. Li and C.-D. Lü, *D^0-$\overline{D}^0$ mixing parameter y in the factorization-assisted topological-amplitude approach*, *Chin. Phys. C* **42** (2018) 063101 [1705.07335].

[18] M. Gronau, O.F. Hernandez, D. London and J.L. Rosner, *Electroweak penguins and two-body B decays*, Phys. Rev. D **52** (1995) 6374 [hep-ph/9504327].

[19] S. Descotes-Genon, J. Matias and J. Virto, *Exploring $B_{d,s} \to KK$ decays through flavour symmetries and QCD-factorisation*, Phys. Rev. Lett. **97** (2006) 061801 [hep-ph/0603239].

[20] H.-Y. Cheng and S. Oh, *Flavor SU(3) symmetry and QCD factorization in $B \to PP$ and PV decays*, JHEP **09** (2011) 024 [1104.4144].

[21] Y.-K. Hsiao, C.-F. Chang and X.-G. He, *A global $SU(3)/U(3)$ flavor symmetry analysis for $B \to PP$ with $\eta - \eta'$ Mixing*, Phys. Rev. D **93** (2016) 114002 [1512.09223].

[22] T. Huber and G. Tetlalmatzi-Xolocotzi, *Estimating QCD-factorization amplitudes through SU(3) symmetry in $B \to PP$ decays*, Eur. Phys. J. C **82** (2022) 210 [2111.06418].

[23] B. Bhattacharya, M. Gronau, M. Imbeault, D. London and J.L. Rosner, *Charmless $B \to PPP$ decays: The fully-symmetric final state*, Phys. Rev. D **89** (2014) 074043 [1402.2909].

[24] B. Bhattacharya and D. London, *Using U spin to extract γ from charmless $B \to PPP$ decays*, JHEP **04** (2015) 154 [1503.00737].

[25] I. Bediaga, T. Frederico, P.C. Magalhaes and D.T. Machado, *Global CP asymmetries in charmless three-body B decays with final state interactions*, Phys. Lett. B **824** (2022) 136824 [2109.01625].

[26] M. Beneke and M. Neubert, *QCD factorization for $B \to PP$ and $B \to PV$ decays*, Nucl. Phys. B **675** (2003) 333 [hep-ph/0308039].

[27] G. Bell, M. Beneke, T. Huber and X.-Q. Li, *Two-loop non-leptonic penguin amplitude in QCD factorization*, JHEP **04** (2020) 055 [2002.03262].

[28] G. Bell and T. Huber, *Master integrals for the two-loop penguin contribution in non-leptonic B-decays*, JHEP **12** (2014) 129 [1410.2804].

[29] M. Beneke, P. Böer, J.-N. Toelstede and K.K. Vos, *QED factorization of non-leptonic B decays*, JHEP **11** (2020) 081 [2008.10615].

[30] M. Beneke, P. Böer, G. Finauri and K.K. Vos, *QED factorization of two-body non-leptonic and semi-leptonic B to charm decays*, JHEP **10** (2021) 223 [2107.03819].

[31] G. Bell, M. Beneke, T. Huber and X.-Q. Li, *Two-loop current-current operator contribution to the non-leptonic QCD penguin amplitude*, Phys. Lett. **B750** (2015) 348 [1507.03700].

[32] M. Gronau, *A Precise sum rule among four $B \to K\pi$ CP asymmetries*, Phys. Lett. B **627** (2005) 82 [hep-ph/0508047].

[33] Belle-II collaboration, *First search for direct CP-violating asymmetry in $B^0 \to K^0\pi^0$ decays at Belle II*, 2104.14871.

[34] T. Huber, S. Kränkl and X.-Q. Li, *Two-body non-leptonic heavy-to-heavy decays at NNLO in QCD factorization*, *JHEP* **09** (2016) 112 [1606.02888].

[35] M. Bordone, N. Gubernari, D. van Dyk and M. Jung, *Heavy-Quark expansion for $\bar{B}_s \to D_s^{(*)}$ form factors and unitarity bounds beyond the $SU(3)_F$ limit*, *Eur. Phys. J. C* **80** (2020) 347 [1912.09335].

[36] M. Bordone, N. Gubernari, T. Huber, M. Jung and D. van Dyk, *A puzzle in $\bar{B}^0_{(s)} \to D^{(*)+}_{(s)}\{\pi^-, K^-\}$ decays and extraction of the f_s/f_d fragmentation fraction*, *Eur. Phys. J. C* **80** (2020) 951 [2007.10338].

[37] M. Endo, S. Iguro and S. Mishima, *Revisiting rescattering contributions to $\overline{B}_{(s)} \to D^{(*)}_{(s)}M$ decays*, *JHEP* **01** (2022) 147 [2109.10811].

[38] Belle collaboration, *Study of $\overline{B}^0 \to D^+h^-(h = K/\pi)$ decays at Belle*, *Phys. Rev. D* **105** (2022) 012003 [2111.04978].

[39] S. Iguro and T. Kitahara, *Implications for new physics from a novel puzzle in $\bar{B}^0_{(s)} \to D^{(*)+}_{(s)}\{\pi^-, K^-\}$ decays*, *Phys. Rev. D* **102** (2020) 071701 [2008.01086].

[40] F.-M. Cai, W.-J. Deng, X.-Q. Li and Y.-D. Yang, *Probing new physics in class-I B-meson decays into heavy-light final states*, *JHEP* **10** (2021) 235 [2103.04138].

[41] M. Bordone, A. Greljo and D. Marzocca, *Exploiting dijet resonance searches for flavor physics*, *JHEP* **08** (2021) 036 [2103.10332].

[42] A. Lenz and G. Tetlalmatzi-Xolocotzi, *Model-independent bounds on new physics effects in non-leptonic tree-level decays of B-mesons*, *JHEP* **07** (2020) 177 [1912.07621].

[43] R. Fleischer and E. Malami, *Using $B_s^0 \to D_s^{\mp}K^{\pm}$ Decays as a Portal to New Physics*, 2109.04950.

[44] R. Fleischer and E. Malami, *Revealing New Physics in $B_s^0 \to D_s^{\mp}K^{\pm}$ Decays*, 2110.04240.

[45] T. Gershon, A. Lenz, A.V. Rusov and N. Skidmore, *Testing the Standard Model with CP-asymmetries in flavour-specific non-leptonic decays*, 2111.04478.

[46] X.-G. He and W. Wang, *Flavor SU(3) Topological Diagram and Irreducible Representation Amplitudes for Heavy Meson Charmless Hadronic Decays: Mismatch and Equivalence*, *Chin. Phys. C* **42** (2018) 103108 [1803.04227].

[47] M. Beneke, "Theory of two-body non-leptonic b decays." 2019, https://indico.mitp.uni-mainz.de/event/177/.

[48] C. Bobeth, M. Gorbahn and S. Vickers, *Weak annihilation and new physics in charmless $B \to MM$ decays*, *Eur. Phys. J. C* **75** (2015) 340 [1409.3252].

[49] C.-D. Lü, Y.-L. Shen, C. Wang and Y.-M. Wang, *Enhanced Next-to-Leading-Order Corrections to Weak Annihilation B-Meson Decays*, 2202.08073.

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