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Recent CKMfitter updates on global fits of the CKM matrix

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The CKMfitter group aims at performing global fits of the CKM parameters by combining efforts from both experimental and theoretical sides. This proceeding gives the most updated CKM fit results from the CKMfitter group with inputs till early 2021 (Moriond 2021).

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1. Introduction

The CKMfitter group is formed by a group of particle physicists who are interested in CP violation and flavor physics. Members either work on theory side to understand the phenomenology of flavor physics or are from BaBar, Belle, Belle II or LHCb experiments. It aims at performing a global analysis of different measurements to determine the Cabibbo-Kobayashi-Maskawa matrix parameters [1, 2] in the framework of the Standard Model (SM) and some of its extensions [3].

In the SM, Yukawa couplings are not necessarily diagonalized in interaction eigenstates, and mass eigenstates are different from interaction eigenstates. The CKM matrix is generated from the unitary matrices needed to diagonalize the mass matrix. The CKM matrix is a 3×3 unitary matrix. The unitary conditions,

$$\sum_{i \text{ or } j} V_{ij}V_{ij}^* = 1, \qquad \sum_i V_{ij}V_{ik}^* = 0, \qquad \sum_j V_{ij}V_{kj}^* = 0, \tag{1}$$

constrain the free parameters of the CKM matrix into four, A, λ, $\bar{\rho}$ and $\bar{\eta}$ in Wolfenstein parameterization [4–6], where V_{ij} is the CKM matrix element, and i is an up-type quark, u, c or t, j is a down-type quark d, s or b. The four parameters are related to the CKM matrix elements by the following relationships,

$$\lambda^2 = \frac{V_{us}V_{us}^*}{V_{ud}V_{ud}^* + V_{us}V_{us}^*}, \qquad A^2\lambda^4 = \frac{V_{cb}V_{cb}^*}{V_{ud}V_{ud}^* + V_{us}V_{us}^*}, \qquad \bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}. \tag{2}$$

The unitary conditions lead to the CKM triangles and the most frequently cited one is obtained from $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$, where the three angles, defined as

$$\alpha = \phi_2 = \arg(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}), \qquad \beta = \phi_1 = \arg(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}), \qquad \gamma = \phi_3 = \arg(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}), \tag{3}$$

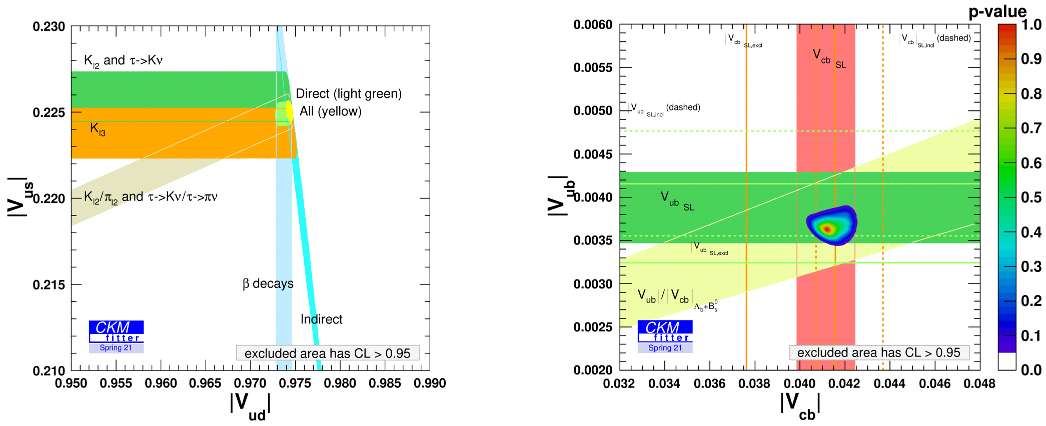
have similar sizes. This triangle is also closely related to observables in B physics.

In the CKMfitter, a frequentist approach based on a χ^2 analysis is used to combine different measurements and lattice inputs such as form factors, bag parameters. The CKM parameters are obtained by minimizing the χ^2 built from observables and the χ^2_{min} gives an indication of the goodness of the fit. The difference of the χ^2 to χ^2_{min} is used to calculate confidence level (CL) or p-value. The Range fit (Rfit) scheme is used to treat statistical and theoretical uncertainties, where different sources of uncertainties from theoretical inputs are summed linearly and the uncertainty due to theoretical assumptions is treated as a range, instead of a Gaussian distribution [7]. The statistical uncertainty is still considered as Gaussian like, where the central values are those determined by the range.

2. Observables

The observables for global fits of the CKM matrix are mainly magnitudes of CKM matrix elements and phases of them. The magnitudes of CKM matrix elements are either obtained from semi-leptonic (or leptonic) decays or indirectly constrained from B^0 and B_s^0 mixing. The precision on $|V_{ud}|$ is led by measurements from super-allowed nuclear β decays. The latest survey, published

Figure 1: 2D plots of $|V_{us}|$ vs $|V_{ud}|$ (left) and $|V_{ub}|$ vs $|V_{cb}|$ (right). Details are described in the text.

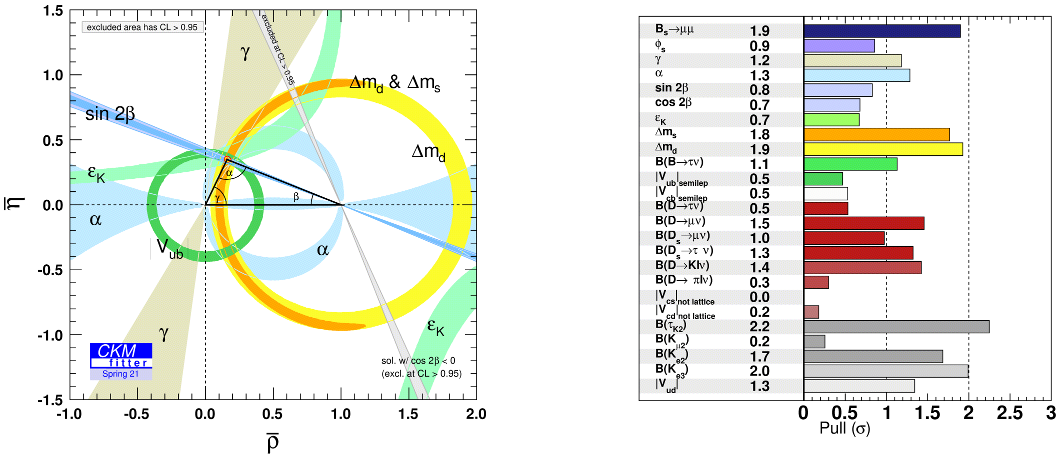


in 2020, includes improved measurements and recent calculations for radiative corrections and gives $|V_{ud}| = 0.97373 \pm 0.00031$ [8]. Comparing to our previous update in 2019, the central value is smaller by 0.00045, while the uncertainty is larger by 50%. The uncertainty includes both statistical and theoretical contributions. According to the Rfit framework, the two should be separated and the statistical contribution is found to be 0.00009. It is worth noting that the $|V_{ud}|$ from our fits without the direct measurement is 0.97440 ± 0.00006, and thus the statistical contribution should be treated properly in the Rfit scheme. In addition, different theoretical contributions are also considered carefully, and are added up linearly based on the Rfit frame, which leads to the $|V_{ud}|$ used in this update, $0.97373 \pm 0.00009 \pm 0.00053$.

The $|V_{us}|$, $|V_{cs}|$, $|V_{cd}|$, $|V_{ub}|$ and $|V_{cb}|$ are obtained from semi-leptonic (or leptonic) decays of kaon, charmed and beauty hadrons, with inputs from lattice calculations on form factors and decay constants. The $|V_{us}|$ is also constrained from τ decays into final states containing a kaon. Little changes on $|V_{us}|$ are made compared to our previous updates. The 2D plots of $|V_{ud}|$ vs $|V_{us}|$ are shown in Fig. 1. The indirect constraints from b decays are related to $|V_{ud}|$ and $|V_{us}|$ through unitarity (light blue). The yellow region of the global combination corresponds to 68% CL. The same CL is also applied for other filled regions. The direct constraint on $|V_{ud}|$ and $|V_{us}|$ is displayed as light green. The $|V_{ud}|$ from supper-allowed nuclear β decay is shown as dark blue, and $|V_{us}|$ from different kaon and τ decays are separately plotted. A deviation from unitarity of the first row is tested to be $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.00230^{+0.00218}_{-0.00023}(1\sigma)$, $-0.00230^{+0.00237}_{-0.00044}(2\sigma)$ and $-0.00230^{+0.00242}_{-0.00065}(3\sigma)$.

The new measurements of partial branching fractions of inclusive $B \to X_u l \nu$ decays with hadronic tagging [9] is included in the $|V_{ub}|$ determined from inclusive measurements. In the $|V_{cb}|$ determined from exclusive measurements, a new result is obtained based on 2020 BGL refit with preliminary non-zero recoil form factor ratio from JLQCD and the new $D \to K\pi$ branching fraction [10]. With these changes, the $|V_{ub}|$ values used in this update are $(4.16 \pm 0.12 \pm 0.31) \times 10^{-3}$, $(3.70 \pm 0.10 \pm 0.21) \times 10^{-3}$ and $(3.88 \pm 0.08 \pm 0.21) \times 10^{-3}$ for inclusive, exclusive results and the average between the two, respectively. The $|V_{cb}|$ values are $(42.2 \pm 0.4 \pm 0.5) \times 10^{-3}$, $(39.6 \pm 0.6 \pm 0.5) \times 10^{-3}$ and $(41.15 \pm 0.34 \pm 0.45) \times 10^{-3}$ for the three categories, respectively. The

Figure 2: Global fit results of the CKM parameters (left) and the pull distributions of the input observables (right).



new ratio of $|V_{ub}/V_{cb}|$ from $B_s^0 \to K^+\mu^-\nu_\mu$ and $B_s^0 \to D_s^+\mu^-\nu_\mu$, $0.0946 \pm 0.0041 \pm 0.0068$ [11] is also included in the global fit, however, only high q^2 region is considered, which uses LQCD inputs. One should note that there are clear tension between low and high q^2 regions, where different theoretical inputs are used. The new $|V_{cb}|$ measurement from LHCb [12] is also not considered as knowledge of $B \to D^* l\nu$ is required in its measurement, which leads to large correlations between measurements and efforts are needed to understand it better.

In Fig. 1, the $|V_{ub}|$ and $|V_{cb}|$ determined from exclusive and inclusive semi-leptonic B decays are shown with solid and dashed borders, while the averages of the two are shown as filled areas with green and red colors, respectively. The diagonal colored band corresponds to the determination of $|V_{ub}|/|V_{cb}|$ from Λ_b and B_s^0 decays. The rainbow oval region indicates the indirect determination of $|V_{ub}|$ and $|V_{cb}|$ from the global fit, without any information from semi-leptonic or leptonic b decays. Uncertainties from direct measurements are still large and efforts are needed to improve the sensitivities.

In addition to the measurements of the magnitudes of the CKM matrix elements, phase measurements also add important constraints to the determination of the CKM parameters. The interesting observables are α (ϕ_2), β (ϕ_1), γ (ϕ_3), ϕ_s. The angle α is determined using isospin analyses of the $B \to \pi\pi$, $B \to \rho\pi$ and $B \to \rho\rho$ systems [13]. The sensitivity of the angle β is driven by the $B \to (c\bar{c})K$ decays and the angle γ is obtained from a global fit of $B \to D^{(*)}K^{(*)}$ using the ADS, GLW or GGSZ methods [14–18]. The ϕ_s is measured from the $B_s^0 \to (c\bar{c})(KK,\pi\pi)$ decays with a time-dependent approach. The ϵ_K also offers important constraints over the $\bar{\rho}$-$\bar{\eta}$ plane.

3. Results

The global fit results in this update are shown in Fig. 2. The $\chi^2_{\rm min}$, corresponding to a p-value of 29%, is increased slightly compared to the 2019 results. The Wolfenstein parameters are determined to be

$$\begin{aligned} A &= 0.8132^{+0.0119}_{-0.0060}, \qquad \lambda = 0.22500^{+0.00024}_{-0.00022}, \\ \bar{\rho} &= 0.1566^{+0.0085}_{-0.0048}, \qquad \bar{\eta} = 0.3475^{+0.0118}_{-0.0054}, \end{aligned} \tag{4}$$

and the Jarlskog invariant to be $J = (3.044^{+0.068}_{-0.084}) \times 10^{-5}$. For a comparison, the Wolfenstein parameters obtained in the 2019 update are

$$\begin{aligned} A &= 0.8235^{+0.0056}_{-0.0145}, \qquad \lambda = 0.22484^{+0.00025}_{-0.00006}, \\ \bar{\rho} &= 0.1569^{+0.0102}_{-0.0061}, \qquad \bar{\eta} = 0.3499^{+0.0079}_{-0.0065}. \end{aligned} \tag{5}$$

The differences are mainly driven by the change on $|V_{ud}|$. The global fits are also performed using different sets of selected observables, such as CP violation only or CP conserving only observables, observables determined only from tree-level processes or with loop-level processes involved etc., all show consistent pictures. More results can be obtained from the CKMfitter webpage. The pull distributions for different observables are also shown in Fig. 2, where pull is defined by the square root of the differences between the $\chi^2_{\rm min}$ with and without the observable under consideration. No large pull value is found. Some of the observables have pull values equal to 0, which is a character of the Rfit scheme.

4. CKMlive

The CKMfitter is making effort to share the software with the community. A framework CKMlive has been developed where researchers outside the CKMfitter group can run their dedicated analyse to extract CKM matrix elements with the CKMfitter software. With user-defined inputs, plots and fit results can be produced. The CKMlive is well supported and questions can be sent to ckmlive@clermont.in2p3.fr.

5. Summary

The precision on the CKM parameters determined from global fits is improved dramatically in the past two decades. An overall consistency still remains. The CKMfitter group will continue to update global fit results in order to identify any discrepancies between different observables.

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