PROCEEDINGS OF SCIENCE

Exclusive $|V_{ub}|$ determinations using Padé Approximants

Sergi Gonzàlez-Solís,[a,*] **Pere Masjuan**[b,c] **and Camilo Rojas**[b,c]

[a] *Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA*

[b] *Grup de Física Teòrica, Departament de Física, Universitat Autònoma de Barcelona, 08193 Bellaterra (Barcelona), Spain*

[c] *Institut de Física d'Altes Energies (IFAE) and The Barcelona Institute of Science and Technology, Campus UAB, 08193 Bellaterra (Barcelona), Spain*

E-mail: sergig@lanl.gov

We determine the CKM parameter $|V_{ub}|$ from the exclusive semileptonic decays $B \to \pi\ell\nu_\ell$ and $B_s \to K\ell\nu_\ell$, employing Lattice QCD theoretical information on the form factors in combination with experimental measurements of the differential branching ratio distributions. Using Padé approximants to the participating form factors we obtain $|V_{ub}| = 3.86(11) \times 10^{-3}$ and $|V_{ub}| = 3.58(9) \times 10^{-3}$ from the analyses of the individual decay channels, respectively, and $|V_{ub}| = 3.68(5) \times 10^{-3}$ from a simultaneous analysis of both decays. Our results highlight the importance of the decay $B_s \to K\mu\nu_\mu$ in complementing the traditional $B \to \pi\ell\nu_\ell$ one in the exclusive determination of $|V_{ub}|$, and strengthens the case for precise measurements of the differential $B_s \to K\ell\nu_\ell$ decay rate with a finer resolution of the q^2 bins, as it would definitely allow achieving more conclusive results for $|V_{ub}|$.

Preprint number: LA-UR-22-22737

11th International Workshop on the CKM Unitarity Triangle (CKM2021)
22-26 November 2021
The University of Melbourne, Australia

[*] Speaker

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1. Introduction

The Cabibbo-Kobayashi-Maskawa (CKM) matrix describes how quarks mix under the weak interaction. Its matrix elements, denoted by V_{ij} for a $j \to i$ quark transition, are fundamental parameters of the Standard Model (SM), and knowledge of their magnitude with high accuracy is absolutely required for precise SM test. In this contribution we focus on $|V_{ub}|$, one of the least-known CKM elements which governs the strength of $b \to u$ transitions, and we are going to consider exclusive processes only. The most competitive exclusive determination of $|V_{ub}|$ is obtained from the decay channel $B \to \pi\ell\nu_\ell$, which has generally exhibited a tension with inclusive determinations [1]. The semileptonic $B_s \to K\ell\nu_\ell$ also depends on the CKM element $|V_{ub}|$, and in fact can play an important role in reassessing the result and addressing the exclusive versus inclusive $|V_{ub}|$ puzzle. Recently, the first experimental data on $B_s \to K\ell\nu_\ell$ became available by the LHCb Collaboration, which measured the partial branching ratio distribution in two regions of q^2 [2]. In our work, we use these data to determine $|V_{ub}|$ and illustrate the potential of a combined analysis of the decays $B \to \pi\ell\nu_\ell$ and $B_s \to K\ell\nu_\ell$.

2. Decay amplitude and form factors

The differential decay rate for the decay $B \to \pi\ell\nu_\ell$ can be written as:

$$\begin{aligned}\frac{d\Gamma(B\to\pi\ell\nu_\ell)}{dq^2} &= \frac{G_F^2|V_{ub}|^2\lambda^{1/2}(m_B^2,m_\pi^2,q^2)}{128 m_B^3\pi^3 q^2}\left(1-\frac{m_\ell^2}{q^2}\right)^2 \\ &\times \left\{m_\ell^2(m_B^2-m_\pi^2)^2|f_0(q^2)|^2+\frac{2q^2}{3}\lambda(m_B^2,m_\pi^2,q^2)\left(1+\frac{m_\ell^2}{2q^2}\right)|f_+(q^2)|^2\right\}, \quad (1)\end{aligned}$$

where $q_\mu = (p_B - p_\pi)_\mu = (p_\ell + p_{\nu_\ell})_\mu$ is the transferred momentum to the dilepton pair, and $\lambda(x,y,z) = (x+y-z)^2 - 4xy$ is the Kallen function. The hadronic physics that we require is contained in $f_+(q^2)$ and $f_0(q^2)$ in Eq. (1), which are, respectively, the vector and scalar form factors corresponding to the exchange of $J^P = 1^-$ and 0^+ particles. For the decay $B_s \to K\ell\nu_\ell$, the distribution is that of Eq. (1) but replacing $m_B \to m_{B_s}, m_\pi \to m_K$ and the $B \to \pi$ form factors by the $B_s \to K$ ones.

The present best knowledge of the vector and scalar $B \to \pi$ and $B_s \to K$ form factors are obtained from Lattice-QCD calculations in the large-q^2 region, which are then extrapolated to the full kinematic range, *i.e.* $0 < q^2 < (m_B - m_\pi)^2$, using parametrizations based on resonance-exchange ideas [3] or the z-expansion [4]. These parametrizations are in a form or another a certain kind of Padé approximant [5], which we will use in this work. Here, we only briefly review them, referring to Refs. [5–7] for further details.

In short, Padé approximants (PA in what follows) to a given function are ratios of two polynomials (with degree M and N, respectively)

$$P_N^M(q^2) = \frac{\sum_{j=0}^M a_j (q^2)^j}{\sum_{k=0}^N b_k (q^2)^k} = \frac{a_0 + a_1 q^2 + \cdots + a_M (q^2)^M}{1 + b_1 q^2 + \cdots + b_N (q^2)^N}\,, \quad (2)$$

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with coefficients determined after imposing a set of a accuracy-through-order conditions with the function $f(q^2)$ one wants to approximate:

$$f(q^2) - P_N^M(q^2) = O(q^2)^{M+N+1}\,. \tag{3}$$

In this contribution we will employ PA to parametrize both $B \to \pi$ and $B_s \to K$ vector and scalar form-factors in order to extrapolate the large-q^2 region's calculations obtained from Lattice-QCD to the full kinematic range.

3. Determination of $|V_{ub}|$

3.1 Fits to $B \to \pi\ell\nu_\ell$

In this section we determine $|V_{ub}|$ performing fits to the $B \to \pi\ell\nu_\ell$ differential branching ratio distribution experimental measurements combined with the $B \to \pi$ form factor Lattice-QCD simulated data. To this end, we minimize the following χ^2-like function,

$$\chi^2_{B\pi} \;=\; N\left(\frac{\chi^2_{\rm data}}{N_{\rm data}} + \frac{\chi^2_{\rm Lattice}}{N_{\rm Lattice}}\right)\,, \tag{4}$$

where $N_{\rm data}$ is the number of experimental points, $N_{\rm Lattice}$ the number of the Lattice form factor q^2-points, and $N = N_{\rm data} + N_{\rm Lattice}$. The above definition ensures the χ^2 function with a smaller number of points is well represented in $\chi^2_{B\pi}$, and is not overridden by that with a larger number of points. The expressions for the individual χ^2 functions in Eq. (4), $\chi^2_{\rm data}$ and $\chi^2_{\rm Lattice}$, can be found in [6]. For the fit, we use the data in 13 bins of q^2 ($N_{\rm data} = 13$) from the HFLAV group [8], which results from the average of the four most precise measurements of the differential $B \to \pi\ell\nu_\ell$ decay rate from BaBar [9, 10] and Belle [11, 12], and the Lattice QCD information on the vector and scalar form factors from the FLAG group [13]. For our analysis, we have generated, respectively, 3 and 2 data points for the vector and scalar form factors at three representative values of q^2 from their z-fits, which we have gathered in Table I in [6].

For the dominant vector form factor, we have performed fits with Padé sequences of the type $P_1^M(q^2)$ and $P_2^M(q^2)$, where the poles are left free to be fitted, and of the type $T_1^M(q^2)$ and $P_{1,1}^M(q^2)$, where the $B^*(1^-)$ pole is fixed to the PDG mass, $m_{B^*(1^-)} = 5.325$ GeV [1]. In both type of sequences we reach $M = 3$ and $M = 2$ as the best approximants with the current data. Our best is obtained with a $P_{1,1}^2$ approximant, which yields:

$$|V_{ub}| = 3.86(11) \times 10^{-3}\,, \tag{5}$$

Our $|V_{ub}|$ value in Eq. (5) is larger, and slightly more precise than, the FNAL/MILC result, $|V_{ub}| = 3.72(16) \times 10^{-3}$ [14], and the FLAG reported value, $|V_{ub}| = 3.73(14) \times 10^{-3}$ [13]. The reason for that is (in part) due to the adopted χ^2 fit function in Eq. (4), which we consider as more democratic. In addition, this procedure has an impact on the comparison with respect to $|V_{ub}|$ determinations from inclusive decays $B \to X_u\ell\nu_\ell$, $|V_{ub}| = 4.25(12)^{+15}_{-14}(23) \times 10^{-3}$ [1], with which our values differ by only 1.35σ. In Fig. 1, we show the differential branching ratio distribution (left plot) and the outputs for the vector and scalar form factors (right plot) resulting from our preferred fit $P_{1,1}^2$.

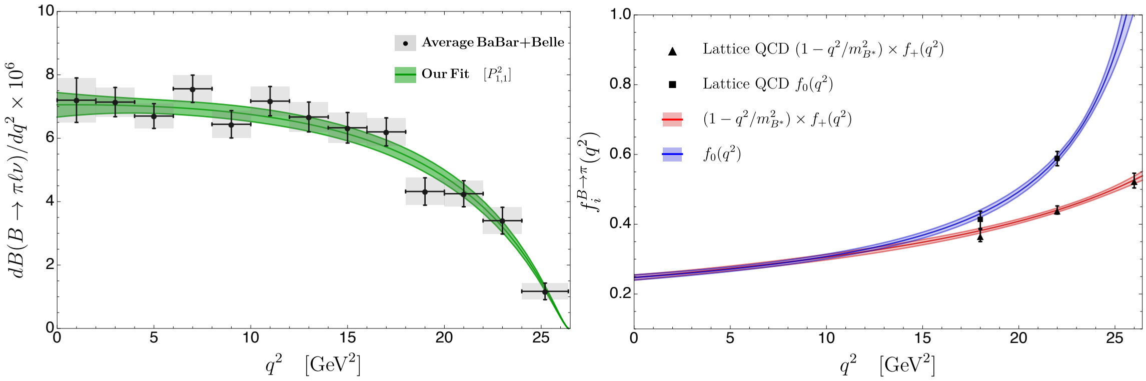

Figure 1: *Left*: Averaged BaBar and Belle $B \to \pi\ell\nu$ differential branching ratio distribution (gray) [8] as compared to our $P^2_{1,1}$ result (green) obtained in combined fits as presented in [6]. *Right*: Output for the $B \to \pi$ vector (red) and scalar (blue) form factors.

3.2 Fits to $B_s \to K\ell\nu_\ell$

For the determination of $|V_{ub}|$ from the decay $B_s \to K\ell\nu_\ell$, we follow a strategy similar to that of the previous section for $B \to \pi\ell\nu$, using the form factors shape information from theory given by the RBC/UKQCD [15] and FNAL/MILC [16] Lattice-QCD Collaborations, along with the recent experimental information on the decay spectrum released by the LHCb Collaboration, $BR(B_s \to K^-\mu^+\nu_\mu) = 0.36(2)(3)\times10^{-4}$ for $q^2 < 7$ GeV^2 and $BR(B_s \to K^-\mu^+\nu_\mu) = 0.70(5)(6)\times10^{-4}$ for $q^2 > 7$ GeV^2 [2]. The best fit is obtained with a $P^3_{1,1}$ approximant for the vector form factor, and taking a P^0_1 approximant for the scalar one. We obtain $|V_{ub}| = 3.58(9)\times10^{-3}$, which is a 2.5% error and represent a shift of about $(1.8-2)\sigma$ downwards with respect to the value $|V_{ub}| = 3.86(11)\times10^{-3}$ determined from the decay $B \to \pi\ell\nu_\ell$ (cf. Eq. (5)). Despite the differing results, we note that an important aspect to improve the compatibility results for $|V_{ub}|$ is the binned measurement of the $B_s \to K\ell\nu_\ell$ differential branching ratio distribution, and most importantly its low-energy region, which fixes the q^2-dependence of the form factors at low-energies. In this sense, the experimental information is presently limited to the two LHCb experimental points, which are rather thick for an accurate extraction of the functional behavior of the form factors, specially at low-energies. Therefore, new and more precise measurements of the decay rate with a thinner resolution of the q^2 bins will definitely allow obtain more conclusive results from the $B_s \to K\ell\nu_\ell$ decay. A graphical account of our fit with the $P^3_{1,1}$ approximant is presented in Fig. 2 for the differential branching ratio distribution (left plot) and the output for the vector and scalar form factors (right plot).

3.3 Combined fits to

In the previous sections we have extracted $|V_{ub}|$ and the corresponding form factor parameters from individual fits to the decays $B \to \pi\ell\nu_\ell$ and $B_s \to K^-\mu^+\nu_\mu$ experimental data combined with the Lattice-QCD information on the corresponding vector and scalar form factors. In this section, we explore the potential of performing simultaneous fits to all experimental and theoretical information on both exclusive decays to determine $|V_{ub}|$. As in the preceding sections, we have tried various Padé sequences [6]. The best fit is obtained with a $P^2_{1,1}$ and $P^3_{1,1}$ approximant for the $B \to \pi$ and $B_s \to K$ vector form factors, respectively. The resulting fit parameters and the

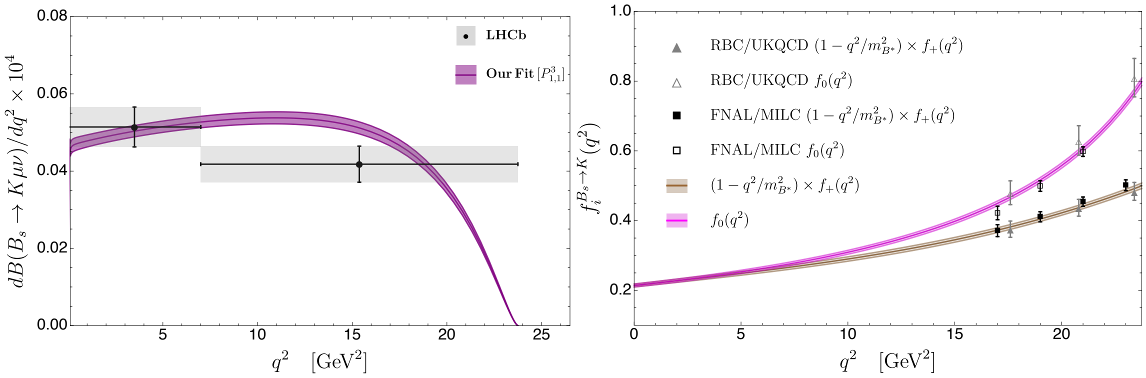

Figure 2: *Left*: LHCb $B_s \to K^- \mu^+ \nu_\mu$ differential branching ratio distribution (gray) [2] as compared to our best fit result (purple) obtained in combined fits as presented in [6]; the two LHCb data points are placed in the middle of each bin and have been divided by the bin width. *Right*: Output for the $B_s \to K$ vector (brown) and scalar (magenta) form factors compared to the Lattice-QCD data of Ref. [15].

correlation matrix are presented in [6]; our results corresponds, to the best of our knowledge, to the first correlated results between the $B \to \pi$ and $B_s \to K$ form factors, which can serve as guidance for those Lattice Collaborations that are planning making available the full theoretical correlation between form factors for different process in their final results [16]. The resulting value for $|V_{ub}|$ from the combined analysis is found to be:

$$|V_{ub}| = 3.68(5) \times 10^{-3}\,, \tag{6}$$

which is only a 1.4% error, and have been obtained taking into account the restrictions $f_+^{B\to\pi}(0) = f_0^{B\to\pi}(0)$ and $f_+^{B_s\to K}(0) = f_0^{B_s\to K}(0)$ simultaneously. Concerning the latter, we obtain:

$$f_{+,0}^{B\pi}(0) = 0.255(5)\,, \qquad f_{+,0}^{B_sK}(0) = 0.211(3)\,. \tag{7}$$

We would like to note that the result in Eq. (6) corresponds to the most precise determination of $|V_{ub}|$ to date, and that this value is shifted about 1.4σ downwards with respect to $|V_{ub}| = 3.86(11) \times 10^{-3}$ extracted from $B \to \pi\ell\nu_\ell$ alone, and about 1σ upwards with respect to $|V_{ub}| = 3.58(9) \times 10^{-3}$ obtained from the individual analysis of the $B_s \to K\ell\nu_\ell$ channel. A graphical account of the results of our fits as compared to the experimental and Lattice form factor data is given in [6].

4. Summary

In this contribution, based on [6], we have explored the role of the decay $B_s \to K\ell\nu_\ell$ in complementing the traditional channel $B \to \pi\ell\nu_\ell$ in the determination of the CKM element $|V_{ub}|$. The motivation of this study is the first reported measurement of the branching ratio of the decay $B_s \to K^-\mu^+\nu_\mu$ by the LHCb Collaboration [2], making this analysis of timely interest. Our analysis has been based on the use of Padé approximants to the corresponding form factors, and proceeded in three steps, obtaining $|V_{ub}| = 3.86(11) \times 10^{-3}$ and $|V_{ub}| = 3.58(9) \times 10^{-3}$ from the individual analyses of the decays $B \to \pi\ell\nu_\ell$ and $B_s \to K\ell\nu_\ell$, respectively, and $|V_{ub}| = 3.68(5) \times 10^{-3}$ from a simultaneous analysis of both decays including an error from the PA convergence. The process of

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performing a combined fit to both decays also tests for their compatibility, and the result is a $|V_{ub}|$ that stays $\sim 1\sigma$ away from the $|V_{ub}|$ results extracted from the individual decay modes. In this sense, more precise experimental measurements of the differential $B_s \to K\ell\nu_\ell$ decay distribution with a finer resolution of the q^2 bins will help achieve more conclusive results. As a benefit of our analysis, in [6] we have provided calculations for different phenomenological observables such as total decay rates, ratio of τ-to-μ differential decay rates or the forward-backward asymmetry.

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