

Summary of Working Group 5 of the “11th International Workshop on the CKM Unitarity Triangle (CKM 2021)”

Joachim Brod,^a Resmi P. K.^b and Wenbin Qian^c

^a*Department of Physics, University of Cincinnati,
345 Clifton Ct, Cincinnati, OH 45221, U.S.A.*

^b*Aix Marseille Univ, CNRS/IN2P3, CPPM
163 avenue de Luminy, Case 902 13288 Marseille cedex 09, France*

^c*School of Physics, University of Chinese Academy of Sciences
19(A) Yuquan Road, Beijing, China*

E-mail: brodjm@ucmail.uc.edu, resmi.pk@cern.ch, wenbin.qian@cern.ch

We summarize recent developments presented in working group 5: direct CP violation, including ϕ_3/γ determination from $B \rightarrow D^{(*)}K^{(*)}$ decays, charmless B decays (including $K\pi$ puzzle, extraction of angle ϕ_2/α), as well as polarization and branching fraction measurements.

*11th International Workshop on the CKM Unitarity Triangle (CKM2021)
22-26 November 2021
The University of Melbourne, Australia*

1. Introduction

The working group 5 of the international workshop on the CKM Unitarity Triangle focuses on experimental and phenomenology progresses on physics related to direct CP violation, including ϕ_3/γ determination from $B \rightarrow D^{(*)}K^{(*)}$ decays, charmless B decays ($K\pi$ puzzle, extraction of angle ϕ_2/α etc.). Polarization and branching fraction measurements are also topics discussed in the working group. In this proceeding contribution, we give a brief summary of recent progresses presented in CKM2021.

2. ϕ_3/γ measurements

The determination of the standard CKM unitarity triangle angle $\phi_3 \equiv \gamma \equiv \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$ from $B \rightarrow DK$ and $B \rightarrow \bar{D}K$ decays is theoretically extremely clean. The reason is that the $B \rightarrow DK$ transitions receive contributions only from tree operators, and none from penguin operators; furthermore, all the relevant matrix elements can be obtained from data if enough D -decay channels are measured. The sensitivity to ϕ_3/γ arises from the interference of $b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$ decay amplitudes, which have a relative weak phase ϕ_3/γ . These quark-level transitions mediate the $B \rightarrow DK$ decays, with the D^0 and \bar{D}^0 subsequently decaying into a common final state, which allows the two decay channels to interfere. Several variants of the method have been proposed: decays into CP eigenstates [1, 2], decays into flavor states [3], decays into multibody states [4–6]. Other possibilities include the decays of neutral B mesons [7, 8], multibody B decays [9–12], and D^* or D^{**} decays [13, 14].

The above set of methods has several sources of theoretical errors. Most of them can be reduced once more statistics becomes available. For instance, in the past the $D \rightarrow K_S\pi^+\pi^-$ Dalitz plot needed to be modeled using a sum of Breit-Wigner resonances or using the K-matrix formalism. Utilizing the data from entangled $\psi(3770) \rightarrow D\bar{D}$ decays measured at charm factories, this uncertainty can in principle be completely avoided [5]. The related error is now statistics-dominated. Novel sources for correlated $D - \bar{D}$ pairs have recently been proposed [15].

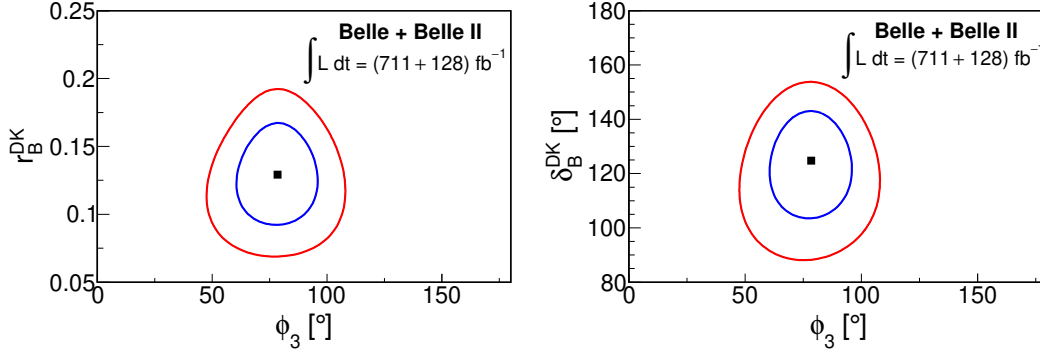
Other sources of reducible uncertainties are neutral D and K mixing (for final states with K_S). Both of these effects can be included by modifying the expressions for the decay amplitudes, taking meson mixing into account, and then using experimentally measured mixing parameters [16–20]. Similarly, the inclusion of $\Delta\Gamma_S$ can be important if ϕ_3/γ is extracted from untagged $B_s \rightarrow D\phi$ decays and can be achieved once $\Delta\Gamma_S$ is well measured [21].

Likewise, CP violation in the D system can be taken into account by appropriately modifying the expressions for the decay amplitudes (and using the fact that in Cabibbo-allowed D decays there is no direct CP violation) [22–26].

Yet another source of reducible theory error are QED radiative corrections to the decay widths. The uncertainties from this source are expected to be below present experimental sensitivity on γ so that not much work has been done on them. Since the corrections are CP conserving they can be reabsorbed in the CP-even measured hadronic quantities and would not affect γ , as long as in the measurements the radiative corrections are treated consistently between different decay modes.

The first irreducible theory error on γ thus arises from higher-order electroweak corrections and cannot be eliminated using just experimental information. The resulting uncertainty was calculated

Figure 1: Two dimensional confidence regions obtained for $\phi_3 - r_B^{DK}$ (left) and $\phi_3 - \delta_B^{DK}$ (right) using $B^+ \rightarrow D^0(K_S^0 h^+ h^-)K^-$ decays from a combined dataset at Belle and Belle II



for the $B \rightarrow DK$ modes in Ref. [27] and for $B \rightarrow D\pi$ in Ref. [28]. The resulting uncertainty is $\delta\gamma^{DK}/\gamma \lesssim \mathcal{O}(10^{-7})$ for the $B \rightarrow DK$. It can be somewhat larger for $B \rightarrow D\pi$ due to a possible accidental cancellation of matrix elements; however, an effect larger than $\delta\gamma^{D\pi}/\gamma \lesssim \mathcal{O}(10^{-4})$ is very unlikely.

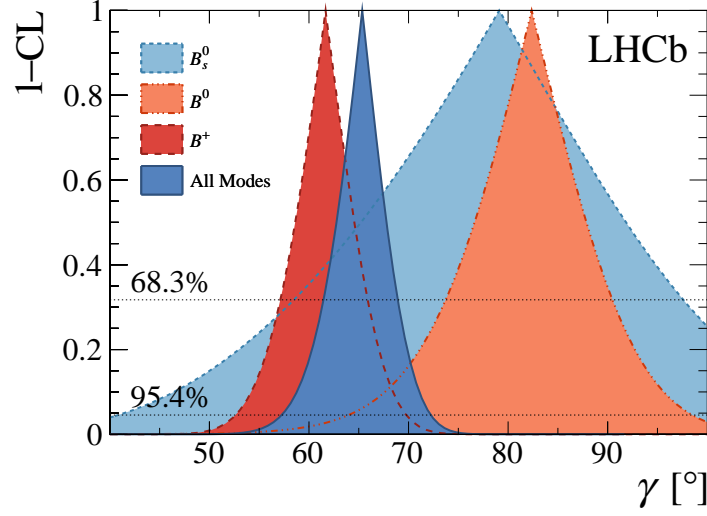
In summary, tree-level $B \rightarrow DK$ and related decays continue to provide a unique opportunity for clean measurements of ϕ_3/γ where new-physics contributions are expected to be small (see Refs. [29, 30] for an analysis of possible new-physics effects).

New Belle results for $\bar{B}^0 \rightarrow D^+\pi^-/K^-$ [31] and Belle 2 for $B \rightarrow D^{(*)}h$ [32] have been presented. (Here and in the following, h stands for either π or K .) The ratio of branching fractions of the cabibbo favoured and cabibbo suppressed modes of this type facilitates tests of theoretical predictions, particularly those of factorization and SU(3) symmetry breaking in QCD. The combined Belle and Belle II [33] measurement with $\bar{B}^+ \rightarrow D^0(K_S^0 h^+ h^-)K^-$ gives $\phi_3 = (78.4 \pm 11.4 \pm 0.5 \pm 1.0)^\circ$ (statistical, systematic, D -mixing-related errors) has the total uncertainty reduced from 15° to 11° compared to the previous combination, with an uncertainty of 3° expected with 10 ab^{-1} of data at Belle II. Two dimensional confidence regions for the observables of interest are shown in Fig. 1. A measurement of ϕ_3 with the four-body D decay of $K_S^0 \pi^+ \pi^- \pi^0$ is demonstrated with dataset from Belle and the uncertainty from this single mode is expected to be 4.4° with the full data set of 50 ab^{-1} anticipated at Belle II [34].

Results presented by the LHCb collaboration include $B^\pm \rightarrow D^{(*)}h^\pm$ [35], and the current single most precise measurement from $B^+ \rightarrow Dh^+$ with $D \rightarrow K_S h^+ h^-$, giving $68.7_{-5.1}^{+5.2}$ [36]. Preliminary results for $B^\pm \rightarrow Dh^\pm$, with $D \rightarrow h^\pm h^\mp \pi^0$, as well as for the first observation of $B_{(s)} \rightarrow \bar{D}^*(2007)K^\pm \pi^\mp$, were also shown. The first LHCb combination of both γ and charm mixing measurements was presented and yields $\gamma = 65.4_{-4.2}^{+3.8}$ [37]. The one dimensional 1 - CL profiles from a combination of various B decays is shown in Fig. 2.

The inputs on D meson decays from the charm factories are crucial for improving the ϕ_3/γ measurements. The strong-phase difference parameters for $D^0 \rightarrow K_S^0 h^+ h^-$ using the current dataset of 2.93 fb^{-1} at BESIII are reported [38–40]. These serve as important inputs for the Belle (II) and LHCb measurements described above. The strong-phase parameters for $D^0 \rightarrow K^- \pi^+ \pi^0$ and $D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$ from BESIII are also presented [41]. It is demonstrated that the sensitivity of γ can be improved by a binned measurement in $D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$ phase space.

Figure 2: One dimensional 1 - CL profiles for γ from a combination of different decays of B meson species at LHCb



Several new developments that are just started to be implemented have been discussed. On possibility, leading to a BESIII + LHCb intercollaboration effort are $D^0 \rightarrow K_S \pi^+ \pi^-$ decays with an unbinned model-independent analysis [42]. Further, a double Dalitz-plot analysis of the modes $B^0 \rightarrow D^0 K^+ \pi^-$, $D^0 \rightarrow K_S \pi^+ \pi^-$ [43] is expected to lead to a sensitivity of 10° with LHC run I & II data, and 2.5° with 50/fb of data.

Other ideas involve b -flavored baryons (these decays are unique for LHCb) [44], e.g, $\Lambda_b^0 \rightarrow D\Lambda(\rightarrow p\pi^-)K^-$. Λ decay are difficult to reconstruct, and a preliminary study of $\Lambda^{*0} \rightarrow pK^-$ has been performed instead [45]. Even if CPV measured, the complexity of baryons vs. mesons makes it non-trivial to extract ϕ_3/γ .

By combining all modes, the hope is to exceed the precision quoted in the LHCb and Belle 2 performance papers.

3. Charmless B decays

Charmless B decays are ideal places to search for physics beyond the Standard Model (SM) as amplitudes from tree-level contributions (with the $b \rightarrow u$ transition) and loop-level contributions (with the $b \rightarrow s$ or $b \rightarrow d$ transition) are at a similar level and both are suppressed in the SM. New physics entering into loop diagrams can be revealed by comparing measured quantities with predictions from the SM. Interesting observables are branching fractions, time-dependent and time-independent CP violation. These observables can also be used to extract CKM parameters, such as the angle α . Comparing CKM parameters obtained from different processes offers alternative methods for new physics hunting.

3.1 Theory of non-leptonic two-body B decays

A generic SM amplitude for a B decay is a product of CKM factors, Wilson coefficients that are known perturbatively up to next-to-next-to-leading-logarithmic accuracy in the SM [46–50],

and hadronic matrix elements. Nonperturbative QCD dynamics makes the calculation of the latter a challenge. While the matrix elements for leptonic and semi-leptonic decays can be parameterized in terms of decay constants and form factors, respectively, the theory for non-leptonic decays relies on some form of factorization. QCD factorization [51–53] allows for systematic calculations to arbitrary order in α_s and leading power Λ/m_b , while pQCD [54–56] is based on k_T factorization and avoids end-point singularities. Both approaches have countless phenomenological applications. A different approach relies on flavor symmetries (e.g. $SU(3)$ or isospin) [57]; however, inclusion of symmetry breaking can be difficult. The combination of factorization and symmetry leads to the factorization-assisted topological-amplitude approach [58–60].

In WG5, several (recent) developments in QCD factorization were further discussed. While tree amplitudes are known at next-to-next-to-leading order (NNLO) for quite some time [61, 62], the prediction of direct CP asymmetries at next-to-leading order (NLO) requires also the penguin amplitudes at NNLO [63] (also QED corrections are now available [64]). There is still some tension between theory and experiment in some modes; both theoretical and experimental uncertainties are still non-negligible. For a discussion of the “ $K\pi$ ” puzzle, see Sec. 3.2. Recent theoretical effort [65] on the discrepancy between SM theory and experiment for the $\bar{B}_{(s)}^0 \rightarrow D_{(s)}^+ \{\pi^-, K^-\}$ has also been discussed. QCD factorization for these modes is particularly clean, and the size of power corrections has been estimated to be tiny [65]. Nevertheless, some of the measured branching ratios are in significant tension with the SM prediction. The origin of these discrepancies is still under discussion [66, 67], but physics beyond the SM could be an alternative explanation [68, 69]. Lastly, the results of a fit to data of flavor- $SU(3)$ invariant amplitudes, based on a topological decomposition of the subamplitudes for $B \rightarrow PP$ decays [70], have been presented [71]. Among the results is a set of “transformation rules”, used to obtain constraints on amplitudes in QCD factorization. In the future, both further theoretical progress and new experimental data will help to address some of the above-mentioned discrepancies.

3.2 $K\pi$ puzzle

The so-called “ $K\pi$ ” puzzle is a long-standing issue in charmless B decays. Naive Isospin relationship indicates that the direct CP violation of $B^+ \rightarrow K^+\pi^0$, $A_{CP}(B^+ \rightarrow K^+\pi^0)$ is equal to $A_{CP}(B^0 \rightarrow K^+\pi^-)$. However, the world averages of the experimental results [72] give $A_{CP}(B^+ \rightarrow K^+\pi^0) = (+4.0 \pm 2.1)\%$ and $A_{CP}(B^0 \rightarrow K^+\pi^-) = (-8.4 \pm 0.4)\%$. There are around 6σ difference between the two, whether it is due to strong dynamic effects or due to new physics is still unknown. To reduce the impacts from strong dynamics, a more precise test based on QCD sum rule [73] is suggested where

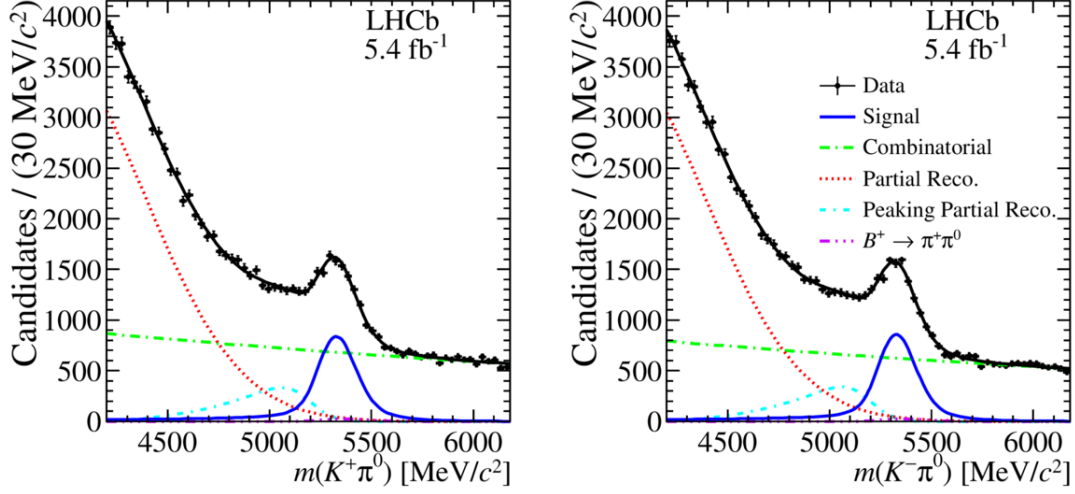
$$A_{CP}(B^0 \rightarrow K^+\pi^+) + A_{CP}(B^+ \rightarrow K^0\pi^+) \frac{B(B^+ \rightarrow K^0\pi^+) \tau_{B^0}}{B(B^0 \rightarrow K^+\pi^-) \tau_{B^+}} = \quad (1)$$

$$A_{CP}(B^+ \rightarrow K^+\pi^0) \frac{2B(B^+ \rightarrow K^+\pi^0) \tau_{B^0}}{B(B^0 \rightarrow K^+\pi^-) \tau_{B^+}} + A_{CP}(B^0 \rightarrow K^0\pi^0) \frac{2B(B^0 \rightarrow K^0\pi^0) \tau_{B^0}}{B(B^0 \rightarrow K^+\pi^-) \tau_{B^+}}.$$

One can find that in addition to the observables in $B^0 \rightarrow K^+\pi^+$ and $B^+ \rightarrow K^+\pi^0$ decays, CP violation and branching fraction measurements are also needed for $B^+ \rightarrow K^0\pi^+$ and $B^0 \rightarrow K^0\pi^0$. The last one is especially challenge and needs more efforts from experimental side.

The LHCb experiment, supposed to perform less well on neutral final states, is trying to make its contribution in understanding the puzzle by further improving the CP violation measurement on

Figure 3: Invariant-mass distribution of the selected candidates with fits overlaid for $B^+ \rightarrow K^+\pi^0$ (left) and $B^- \rightarrow K^-\pi^0$ (right).



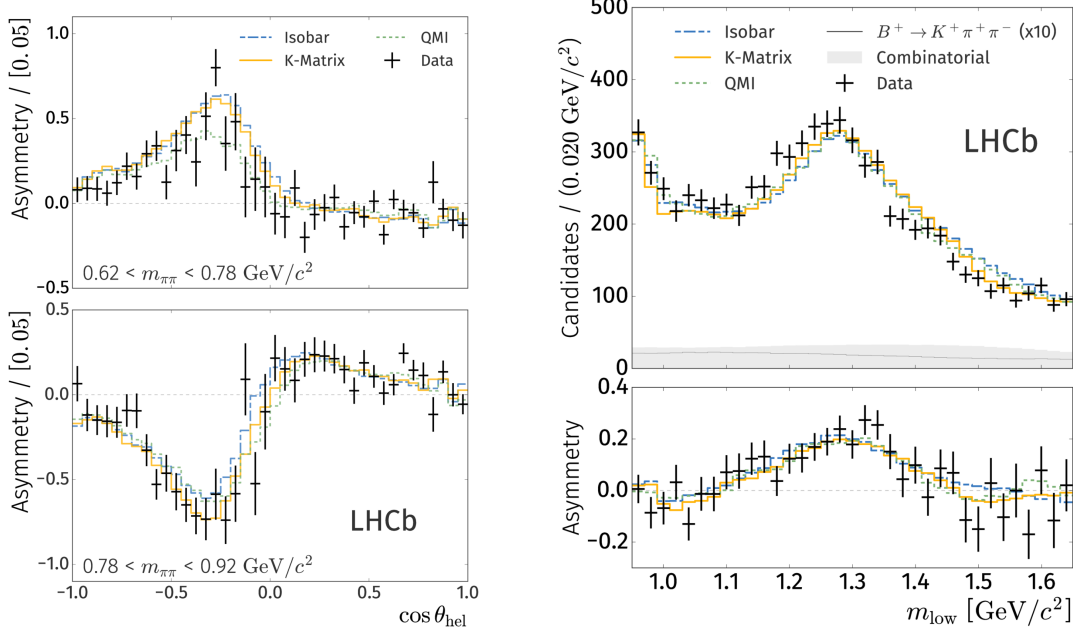
$B^+ \rightarrow K^+\pi^0$ [74]. The invariant mass distributions of the selected $B^\pm \rightarrow K^\pm\pi^0$ are shown in Fig. 3. The invariant-mass resolution is larger and background level is higher than those with charged final states [75], however, the results are still quite promising and indicate potential of the LHCb detector on neutral final states. The direct CP asymmetry of this decay has been measured to be $(2.5 \pm 1.5 \pm 0.6 \pm 0.3)\%$, where the first uncertainty is statistical, the second systematic and the last one due to external inputs. This is the most precise determination to date. By combining with the previous measurements, the new world average is obtained to be $A_{CP}(B^+ \rightarrow K^+\pi^0) = (3.1 \pm 1.7)\%$, which leads to $A_{CP}(B^+ \rightarrow K^+\pi^0) - A_{CP}(B^0 \rightarrow K^+\pi^-) = (11.5 \pm 1.4)\%$, further enhancing the discrepancy to be more than 8σ .

3.3 CP violation in three-body charmless B decays

Using data collected during year 2011-2012, LHCb has found very complicated and interesting CP violation pattern over the three-body phase space of B^+ decaying into $h^+h^-\pi^+$ final states, where h and h' are kaons or pions [76, 77]. Whether these complicated CP violation pattern is due to new physics contribution or not is still unknown. To solve this problem, understanding strong dynamics over the phase space is needed. The LHCb experiment has performed amplitude analyses of $B^+ \rightarrow \pi^+\pi^-\pi^+$ and $B^+ \rightarrow K^+K^-\pi^+$ to obtain CP violation for each of the resonant contributions and thus sheds more light on the puzzle [78–80].

In the amplitude analysis of $B^+ \rightarrow \pi^+\pi^-\pi^+$ [78, 79], three different models have been applied to describe the $\pi^+\pi^-$ S -wave, namely, the Isobar model where different resonant contributions are explicitly considered and described with dedicated line-shapes, the K-matrix model where unitary condition is preserved and a quasi-model-independent method. The K-matrix model uses a five-pole and five-channel matrix, obtained from scattering experiments and the production vector is determined from data. The quasi-model-dependent method determines amplitudes in different intervals of $m_{\pi^+\pi^-}$ directly from data, thus avoiding any model assumption. The three descriptions

Figure 4: The CP asymmetry obtained from data with fits superimposed as a function of $\cos\theta_{\text{hel}}$ in the regions below and above the $\rho(770)$ resonance pole (left) and $m_{\pi^+\pi^-}$ distribution in the $f_2(1270)$ mass region with the corresponding CP asymmetry (right).



of S -wave give consistent results, and show large CP violation in S -wave at low $m_{\pi^+\pi^-}$. CP violation generated due to S -wave and P -wave interference is also observed for the first time. In addition, a first observation of CP violation in tensor particle ($f_2(1270)$) is obtained. The newly observed CP violation sources are clearly shown in Fig. 4.

The Isobar model is used in the amplitude analysis of $B^+ \rightarrow K^+K^-\pi^+$, with contributions from $\phi(1020)$, $K^*(892)^0$, $K^*(1430)^0$, $\rho(1450)^0$, $f_2(1270)$, a non-resonant contribution in $m_{K^+\pi^-}$ and a non-resonant contribution in $m_{K^+K^-}$ described by a re-scattering model [80]. CP violation is found to be as large as $(-66.4 \pm 3.8 \pm 1.9)\%$ for the K^+K^- non-resonant contribution, which responses for almost all the CP violation observed in the $B^+ \rightarrow K^+K^-\pi^+$ decays.

Branching fractions are important inputs for understanding three-body charmless B decays and are needed to obtain absolute branching fractions of each quasi-two-body decays. Using full data sets collected by the LHCb experiment, the most precise branching fraction ratios of these decays has been obtained where the uncertainties are at 1% level or less [81].

3.4 New opportunities

The angle α is a key input to the CKM global fit and is important for new physics searches. It is currently one of the least known CKM parameters, new ideas are essential to improve its sensitivity, in addition to add more data samples. To fully use the information from the $B \rightarrow 4\pi$ system, ideas are proposed to use the resonant behaviour to solve the ambiguities on α [82], to reduce bias and systematic uncertainties [83] and to open the possibility for precision SU(3) measurement in $B^0 \rightarrow a_1^\pm\pi^\mp$ [84].

3.5 Rediscovery of charmless B decays from Belle II

The Belle II experiment has started to collect data in 2019 and using the new data samples, it can already make re-discovery of the channels seen before in the Belle detector.

To test the Isospin sum rules in the $B \rightarrow K\pi$ system, branching fraction and CP violation have been measured for all the four decays, $B^0 \rightarrow K^+\pi^-$, $B^+ \rightarrow K^0\pi^+$, $B^+ \rightarrow K^+\pi^0$ and $B^0 \rightarrow K^0\pi^0$, where a 5σ re-observation of the $B^0 \rightarrow K^0\pi^0$ decay has been obtained [85, 86]. The branching fraction of $B^0 \rightarrow K^0\pi^0$ is determined to be $(8.5^{+1.7}_{-1.6} \pm 1.2) \times 10^{-6}$ and CP asymmetry to be $-0.40^{+0.46}_{-0.44} \pm 0.04$, where the first uncertainties are statistical and the second systematic. Using these early results, the sensitivity for the Isospin test is at a precision of 0.13, which is expected to be reduced to around 0.03 with the full data samples collected by Belle II till 2030.

The Belle II experiment is also designed to play a major role in the measurement of the angle α . Previous results of $B^0 \rightarrow \pi^0\pi^0$ from B-factories show large discrepancy from theoretical predictions though with large uncertainties, it is thus important to confirm this discrepancy or deny it. Belle II has looked into this channel and found $14^{+6.8}_{-5.6}$ events, not significant yet [87]. However, the results show that the Belle-II detector works as expected and have large potential to improve the precision on the angle α in the future.

4. Other CP violation and polarisation measurements

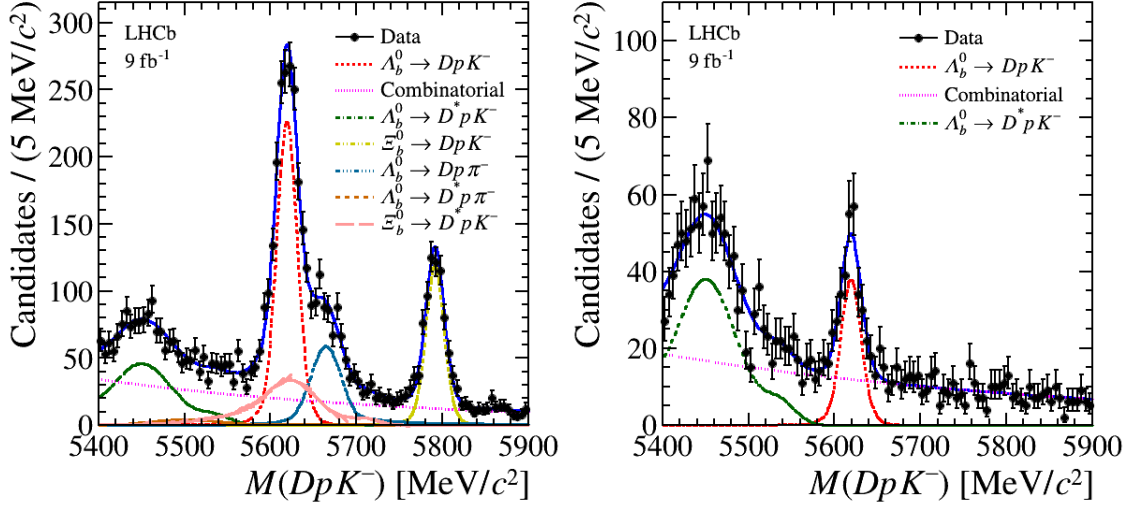
The CP violation and polarisation measurements in some interesting decay modes aid the searches for new physics. There are coherent deviations, up to 5.3σ , in the measured and theoretical branching fractions in hadronic two-body B meson decays, as reported here [88, 89]. The discrepancy can be explained only by a downward shift from the SM amplitude of the order of 10%. The possible new physics contributions to these deviations are studied by building a new model that contains heavy vector-like quarks and heavy SU(2) gauge multiplet. This analysis suggest possibilities of W' from an additional SU(2)_L that partially cancel the SM color allowed amplitude. A more dedicated analysis at collider experiments for low-dijet mass and broad width regime is important to exclude any such possibility.

4.1 CP violation in baryon decays at LHCb

The measurements of CP violating observables are established in K , B and D meson decays and are consistent with the SM predictions; but it is not observed in baryonic decays yet. CP violation searches in baryonic decays are carried out at LHCb in the modes $\Xi_b^- \rightarrow pK^-K^-$ [90], $\Lambda_b^0 \rightarrow DpK^-$ [91], and $\Lambda_b^0 \rightarrow p\pi^-\pi^+\pi^-$ [92]. The search for CP violation in $\Xi_b^- \rightarrow pK^-K^-$ decays is facilitated via an amplitude analysis allowing for CP violation effects. This is the first of such measurements in any baryon decays. The model includes sufficiently well established resonances by adding them iteratively to maximize the change in negative log likelihood. There are no significant CP violation in any of the contributing components.

In $\Lambda_b^0 \rightarrow DpK^-$ decays, the favoured and suppressed D decays to a pair of changed kaon and pion are analysed. The presence of interference between $b \rightarrow c$ and $b \rightarrow u$ amplitudes give access to the CKM angle γ . Hence, b -baryon decays to final states involving a single charm meson are promising for measurements of CP violation effects. The suppressed decay mode

Figure 5: Invariant-mass distributions of the selected candidates with fits overlaid for $\Lambda_b^0 \rightarrow [K^- \pi^+]_D p K^-$ (left) and $\Lambda_b^0 \rightarrow [K^+ \pi^-]_D p K^-$ (right).



$\Lambda_b^0 \rightarrow [K^+ \pi^-]_D p K^-$ is observed for the first time with a yield of 241 ± 22 . There are 1437 ± 92 events from the favoured decay mode; the fit projections are shown in Fig. 5. The efficiency corrections are determined as a function of the phase space variables, $M^2(Dp)$ and $M^2(pK^-)$ from simulation. The CP asymmetry is measured in the integrated phase space as well as in the region involving excited Λ^* states. There are no significant CP violation observed.

The CP asymmetry in $\Lambda_b^0 \rightarrow p \pi^- \pi^+ \pi^-$ decays is measured using triple product correlations as well as energy test method. The T -odd observables are built using the momenta \vec{p}_i of three final state particles in the centre-of-mass frame of Λ_b^0 . There is a rich resonant structure in the decay, with the dominant contributions proceeding through N^* , a_1^- , Δ^{++} and ρ^0 . Asymmetries are measured in the integrated phase space as well as in bins of the phase space to enhance sensitivity to local CP violation effects. There are no evidence of local CP asymmetry in any of the phase space bins.

4.2 Polarisation measurement in $B \rightarrow VV$ decays at LHCb

The decays of B meson into two vector particles are described in terms of three angular variables: helicity angles of the two vector decays and the angle between their decay planes. The observed deviations on the longitudinal polarisation fraction, f_L from the SM expectations using helicity counting rules by Belle and BaBar experiments motivated to have extensive theoretical studies including QCD effects and new physics. The main approaches in QCD have been through QCD factorisation, perturbative QCD, and soft-collinear effective theory.

A set of f_L measurements are presented from the LHCb experiment. The f_L in $B_s^0 \rightarrow J/\psi (\rightarrow \mu^+ \mu^-) K^+ K^-$ decays is measured [93]. The decay proceeds via mainly $\phi(1020)$ with modest presence of $f_0(980)$. The result obtained is $f_L = 0.5186 \pm 0.0029 \pm 0.0023$. A similar measurement is performed for the $J/\psi \rightarrow e^+ e^-$ decay channel [94]. Here, partially reconstructed events form one of the dominant sources of background along with the random combinatorial events. The result

$f_L = 0.530 \pm 0.029 \pm 0.013$ is compatible with the SM prediction. The measurement in the decay mode $B^0 \rightarrow D^{*-} D_s^{*+} (\rightarrow D_s^+ \gamma)$ results in $f_L = 0.578 \pm 0.010 \pm 0.011$ [95]. The polarisations in these decays involving $b \rightarrow c$ transitions is relatively low and in agreement with QCD factorisation predictions. Any $e - \mu$ lepton anomaly observed in f_L measurements are ruled out at a very good precision level.

Polarisation measurements different charmless B decays are also presented from LHCb [96–99]. There is potential good agreement with different theoretical predictions in these decay modes. It is observed that the polarisations are different between $B^0 \rightarrow K^{*0} \bar{K}^{*0}$ and $B_s^0 \rightarrow K^{*0} \bar{K}^{*0}$. This could be due to any possible deficit in $b \rightarrow s$ compared to $b \rightarrow c$ in new physics framework.

5. Summary

Measurements of the CKM angle γ provide a stringent test of CP violation in the SM. More and more precise measurements are carried out by LHCb and Belle experiments. With the start of Run 3 at LHCb and more data at Belle II, the precision on γ is expected to go below 1° . Inputs on D meson decays from BESIII plays an important role in achieving this goal. Novel ideas and new decay modes will aid for further improvements. Charmless B decays are sensitive to the CKM angles and B mixing phases. A set of new measurements in two- and three-body modes are presented by LHCb and Belle. The precision is expected to improve further with more such measurements at Belle II.

Multibody decays are interesting probes to search for CP violation due to their rich resonant substructure. The searches in baryonic decays at LHCb don't provide any evidence for CP violation at the moment. However, the sensitivity is expected to improve with the larger dataset anticipated at the Run 3 of LHCb experiment. Polarisation measurements in $B \rightarrow VV$ decays also provide tests of new physics scenarios. There are no disagreements observed in the measurements presented from LHCb.

References

- [1] M. Gronau and D. London, *How to determine all the angles of the unitarity triangle from $B_d^0 \rightarrow DK_S$ and $B_s^0 \rightarrow D\phi$* , *Phys. Lett. B* **253** (1991) 483–488.
- [2] M. Gronau and D. Wyler, *On determining a weak phase from CP asymmetries in charged B decays*, *Phys. Lett. B* **265** (1991) 172–176.
- [3] D. Atwood, I. Dunietz and A. Soni, *Enhanced CP violation with $B \rightarrow KD^0(\bar{D}^0)$ modes and extraction of the CKM angle γ* , *Phys. Rev. Lett.* **78** (1997) 3257–3260, [[hep-ph/9612433](#)].
- [4] A. Giri, Y. Grossman, A. Soffer and J. Zupan, *Determining γ using $B^\pm \rightarrow DK^\pm$ with multibody D decays*, *Phys. Rev. D* **68** (2003) 054018, [[hep-ph/0303187](#)].
- [5] Y. Grossman, Z. Ligeti and A. Soffer, *Measuring γ in $B^\pm \rightarrow K^\pm(KK^*)_D$ decays*, *Phys. Rev. D* **67** (2003) 071301, [[hep-ph/0210433](#)].
- [6] A. Bondar and A. Poluektov, *Feasibility study of model-independent approach to ϕ_3 measurement using Dalitz plot analysis*, *Eur. Phys. J. C* **47** (2006) 347–353, [[hep-ph/0510246](#)].

- [7] B. Kayser and D. London, *Exploring CP violation with $B_d^0 \rightarrow DK_S$ decays*, *Phys. Rev. D* **61** (2000) 116013, [[hep-ph/9909561](#)].
- [8] M. Gronau, Y. Grossman, N. Shuhmaher, A. Soffer and J. Zupan, *Using untagged $B^0 \rightarrow DK_S$ to determine γ* , *Phys. Rev. D* **69** (2004) 113003, [[hep-ph/0402055](#)].
- [9] R. Aleksan, T. C. Petersen and A. Soffer, *Measuring the weak phase γ in color allowed $B \rightarrow DK\pi$ decays*, *Phys. Rev. D* **67** (2003) 096002, [[hep-ph/0209194](#)].
- [10] T. Gershon, *On the measurement of the unitarity triangle angle γ from $B^0 \rightarrow DK^{*0}$ decays*, *Phys. Rev. D* **79** (2009) 051301, [[0810.2706](#)].
- [11] T. Gershon and A. Poluektov, *Double Dalitz plot analysis of the decay $B^0 \rightarrow DK^+\pi^-$, $D \rightarrow K_S^0\pi^+\pi^-$* , *Phys. Rev. D* **81** (2010) 014025, [[0910.5437](#)].
- [12] T. Gershon and M. Williams, *Prospects for the measurement of the unitarity triangle angle γ from $B^0 \rightarrow DK^+\pi^-$ decays*, *Phys. Rev. D* **80** (2009) 092002, [[0909.1495](#)].
- [13] A. Bondar and T. Gershon, *On ϕ_3 measurements using $B^- \rightarrow D^*K^-$ decays*, *Phys. Rev. D* **70** (2004) 091503, [[hep-ph/0409281](#)].
- [14] N. Sinha, *Determining γ using $B \rightarrow D^{**}K$* , *Phys. Rev. D* **70** (2004) 097501, [[hep-ph/0405061](#)].
- [15] P. Naik, *Novel sources of correlated $D^0\bar{D}^0$ for c/b physics and tests of T/CPT* , [2102.07729](#).
- [16] J. P. Silva and A. Soffer, *Impact of $D^0-\bar{D}^0$ mixing on the experimental determination of γ* , *Phys. Rev. D* **61** (2000) 112001, [[hep-ph/9912242](#)].
- [17] A. Bondar, A. Poluektov and V. Vorobiev, *Charm mixing in the model-independent analysis of correlated $D^0\bar{D}^0$ decays*, *Phys. Rev. D* **82** (2010) 034033, [[1004.2350](#)].
- [18] M. Rama, *Effect of $D^0-\bar{D}^0$ mixing in the extraction of γ with $B^- \rightarrow D^0K^-$ and $B^- \rightarrow D^0\pi^-$ decays*, *Phys. Rev. D* **89** (2014) 014021, [[1307.4384](#)].
- [19] Y. Grossman, A. Soffer and J. Zupan, *The Effect of $D - \bar{D}$ mixing on the measurement of γ in $B \rightarrow DK$ decays*, *Phys. Rev. D* **72** (2005) 031501, [[hep-ph/0505270](#)].
- [20] Y. Grossman and M. Savastio, *Effects of $K^0 - \bar{K}^0$ mixing on determining γ from $B^\pm \rightarrow DK^\pm$* , *JHEP* **03** (2014) 008, [[1311.3575](#)].
- [21] M. Gronau, Y. Grossman, Z. Surujon and J. Zupan, *Enhanced effects on extracting γ from untagged B^0 and B_s decays*, *Phys. Lett. B* **649** (2007) 61–66, [[hep-ph/0702011](#)].
- [22] LHCb collaboration, R. Aaij et al., *Measurement of the CKM angle γ from a combination of $B^\pm \rightarrow Dh^\pm$ analyses*, *Phys. Lett. B* **726** (2013) 151–163, [[1305.2050](#)].
- [23] W. Wang, *CP Violation Effects on the Measurement of the Cabibbo-Kobayashi-Maskawa Angle γ from $B \rightarrow DK$* , *Phys. Rev. Lett.* **110** (2013) 061802, [[1211.4539](#)].

- [24] M. Martone and J. Zupan, $B^\pm \rightarrow DK^\pm$ with direct CP violation in charm, *Phys. Rev. D* **87** (2013) 034005, [1212.0165].
- [25] B. Bhattacharya, D. London, M. Gronau and J. L. Rosner, Shift in weak phase γ due to CP asymmetries in D decays to two pseudoscalar mesons, *Phys. Rev. D* **87** (2013) 074002, [1301.5631].
- [26] A. Bondar, A. Dolgov, A. Poluektov and V. Vorobiev, Effect of direct CP violation in charm on γ extraction from $B \rightarrow DK^\pm$, $D \rightarrow K_S^0 \pi^+ \pi^-$ Dalitz plot analysis, *Eur. Phys. J. C* **73** (2013) 2476, [1303.6305].
- [27] J. Brod and J. Zupan, The ultimate theoretical error on γ from $B \rightarrow DK$ decays, *JHEP* **01** (2014) 051, [1308.5663].
- [28] J. Brod, Electroweak effects in the extraction of the CKM angle γ from $B \rightarrow D\pi$ decays, *Phys. Lett. B* **743** (2015) 56–60, [1412.3173].
- [29] J. Brod, A. Lenz, G. Tetlalmatzi-Xolocotzi and M. Wiebusch, New physics effects in tree-level decays and the precision in the determination of the quark mixing angle γ , *Phys. Rev. D* **92** (2015) 033002, [1412.1446].
- [30] A. Lenz and G. Tetlalmatzi-Xolocotzi, Model-independent bounds on new physics effects in non-leptonic tree-level decays of B -mesons, *JHEP* **07** (2020) 177, [1912.07621].
- [31] BELLE collaboration, E. Waheed et al., Study of $\bar{B}^0 \rightarrow D^+ h^-$ ($h = K/\pi$) decays at Belle, *Phys. Rev. D* **105** (2022) 012003, [2111.04978].
- [32] BELLE-II collaboration, F. Abudinén et al., Study of $B \rightarrow D^{(*)} h$ decays using 62.8 fb^{-1} of Belle II data, in *55th Rencontres de Moriond on QCD and High Energy Interactions*, 4, 2021, 2104.03628.
- [33] BELLE, BELLE-II collaboration, F. Abudinén et al., Combined analysis of Belle and Belle II data to determine the CKM angle ϕ_3 using $B^+ \rightarrow D(K_S^0 h^- h^+) h^+$ decays, *JHEP* **02** (2022) 063, [2110.12125].
- [34] BELLE collaboration, P. K. Resmi et al., First measurement of the CKM angle ϕ_3 with $B^\pm \rightarrow D(K_S^0 \pi^+ \pi^- \pi^0) K^\pm$ decays, *JHEP* **2019** (2019) 178.
- [35] LHCb collaboration, R. Aaij et al., Measurement of CP observables in $B^\pm \rightarrow D^{(*)} K^\pm$ and $B^\pm \rightarrow D^{(*)} \pi^\pm$ decays using two-body D final states, *JHEP* **04** (2021) 081, [2012.09903].
- [36] LHCb collaboration, R. Aaij et al., Measurement of the CKM angle γ in $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ decays with $D \rightarrow K_S^0 h^+ h^-$, *JHEP* **02** (2021) 169, [2010.08483].
- [37] LHCb collaboration, R. Aaij et al., Simultaneous determination of CKM angle γ and charm mixing parameters, *JHEP* **12** (2021) 141, [2110.02350].
- [38] BESIII collaboration, M. Ablikim et al., Determination of Strong-Phase Parameters in $D \rightarrow K_{S,L}^0 \pi^+ \pi^-$, *Phys. Rev. Lett.* **124** (2020) 241802.

- [39] BESIII collaboration, M. Ablikim et al., *Model-independent determination of the relative strong-phase difference between D^0 and $\bar{D}^0 \rightarrow K_{S,L}^0 \pi^+ \pi^-$ and its impact on the measurement of the CKM angle γ/ϕ_3* , *Phys. Rev. D* **101** (2020) 112002.
- [40] BESIII collaboration, M. Ablikim et al., *Improved model-independent determination of the strong-phase difference between D^0 and $\bar{D}^0 \rightarrow K_{S,L}^0 K^+ K^-$ decays*, *Phys. Rev. D* **102** (2020) 052008.
- [41] BESIII collaboration, M. Ablikim et al., *Measurement of the $D \rightarrow K^- \pi^+ \pi^+ \pi^-$ and $D \rightarrow K^- \pi^+ \pi^0$ coherence factors and average strong-phase differences in quantum-correlated $D\bar{D}$ decays*, *JHEP* **2021** (2021) 164.
- [42] A. Poluektov, *Unbinned model-independent measurements with coherent admixtures of multibody neutral D meson decays*, *Eur. Phys. J. C* **78** (2018) 121, [1712.08326].
- [43] D. Craik, T. Gershon and A. Poluektov, *Optimising sensitivity to γ with $B^0 \rightarrow DK^+ \pi^-$, $D \rightarrow K_S^0 \pi^+ \pi^-$ double Dalitz plot analysis*, *Phys. Rev. D* **97** (2018) 056002, [1712.07853].
- [44] A. K. Giri, R. Mohanta and M. P. Khanna, *Possibility of extracting the weak phase gamma from $\Lambda_b \rightarrow \Lambda D^0$ decays*, *Phys. Rev. D* **65** (2002) 073029, [hep-ph/0112220].
- [45] LHCb collaboration, R. Aaij et al., *Observation of the suppressed $\Lambda_b^0 \rightarrow D p K^-$ decay with $D \rightarrow K^+ \pi^-$ and measurement of its CP asymmetry*, *Phys. Rev. D* **104** (2021) 112008, [2109.02621].
- [46] C. Bobeth, M. Misiak and J. Urban, *Photonic penguins at two loops and m_t dependence of $BR[B \rightarrow X_s l^+ l^-]$* , *Nucl. Phys. B* **574** (2000) 291–330, [hep-ph/9910220].
- [47] M. Misiak and M. Steinhauser, *Three loop matching of the dipole operators for $b \rightarrow s \gamma$ and $b \rightarrow s g$* , *Nucl. Phys. B* **683** (2004) 277–305, [hep-ph/0401041].
- [48] M. Gorbahn and U. Haisch, *Effective Hamiltonian for non-leptonic $|\Delta F| = 1$ decays at NNLO in QCD*, *Nucl. Phys. B* **713** (2005) 291–332, [hep-ph/0411071].
- [49] M. Gorbahn, U. Haisch and M. Misiak, *Three-loop mixing of dipole operators*, *Phys. Rev. Lett.* **95** (2005) 102004, [hep-ph/0504194].
- [50] M. Czakon, U. Haisch and M. Misiak, *Four-Loop anomalous dimensions for radiative flavour-changing decays*, *JHEP* **03** (2007) 008, [hep-ph/0612329].
- [51] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, *QCD factorization for $B \rightarrow \pi\pi$ decays: Strong phases and CP violation in the heavy quark limit*, *Phys. Rev. Lett.* **83** (1999) 1914–1917, [hep-ph/9905312].
- [52] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, *QCD factorization for exclusive, nonleptonic B meson decays: General arguments and the case of heavy light final states*, *Nucl. Phys. B* **591** (2000) 313–418, [hep-ph/0006124].

- [53] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, *QCD factorization in $B \rightarrow \pi K$, $\pi\pi$ decays and extraction of Wolfenstein parameters*, *Nucl. Phys. B* **606** (2001) 245–321, [[hep-ph/0104110](#)].
- [54] Y. Y. Keum, H.-N. Li and A. I. Sanda, *Penguin enhancement and $B \rightarrow K\pi$ decays in perturbative QCD*, *Phys. Rev. D* **63** (2001) 054008, [[hep-ph/0004173](#)].
- [55] Y.-Y. Keum, H.-n. Li and A. I. Sanda, *Fat penguins and imaginary penguins in perturbative QCD*, *Phys. Lett. B* **504** (2001) 6–14, [[hep-ph/0004004](#)].
- [56] C.-D. Lu, K. Ukai and M.-Z. Yang, *Branching ratio and CP violation of $B \rightarrow \pi\pi$ decays in perturbative QCD approach*, *Phys. Rev. D* **63** (2001) 074009, [[hep-ph/0004213](#)].
- [57] D. Zeppenfeld, *SU(3) relations for B meson decays*, *Z. Phys. C* **8** (1981) 77.
- [58] Q. Qin, H.-n. Li, C.-D. Lü and F.-S. Yu, *Branching ratios and direct CP asymmetries in $D \rightarrow PV$ decays*, *Phys. Rev. D* **89** (2014) 054006, [[1305.7021](#)].
- [59] H.-n. Li, C.-D. Lu and F.-S. Yu, *Branching ratios and direct CP asymmetries in $D \rightarrow PP$ decays*, *Phys. Rev. D* **86** (2012) 036012, [[1203.3120](#)].
- [60] H.-Y. Jiang, F.-S. Yu, Q. Qin, H.-n. Li and C.-D. Lü, *D^0 - \bar{D}^0 mixing parameter γ in the factorization-assisted topological-amplitude approach*, *Chin. Phys. C* **42** (2018) 063101, [[1705.07335](#)].
- [61] G. Bell, *NNLO vertex corrections in charmless hadronic B decays: Imaginary part*, *Nucl. Phys. B* **795** (2008) 1–26, [[0705.3127](#)].
- [62] G. Bell, *NNLO vertex corrections in charmless hadronic B decays: Real part*, *Nucl. Phys. B* **822** (2009) 172–200, [[0902.1915](#)].
- [63] G. Bell, M. Beneke, T. Huber and X.-Q. Li, *Two-loop non-leptonic penguin amplitude in QCD factorization*, *JHEP* **04** (2020) 055, [[2002.03262](#)].
- [64] M. Beneke, P. Böer, J.-N. Toelstede and K. K. Vos, *QED factorization of non-leptonic B decays*, *JHEP* **11** (2020) 081, [[2008.10615](#)].
- [65] M. Bordone, N. Gubernari, T. Huber, M. Jung and D. van Dyk, *A puzzle in $\bar{B}_{(s)}^0 \rightarrow D_{(s)}^+ \{\pi^-, K^-\}$ decays and extraction of the f_s/f_d fragmentation fraction*, *Eur. Phys. J. C* **80** (2020) 951, [[2007.10338](#)].
- [66] M. Beneke, P. Böer, G. Finauri and K. K. Vos, *QED factorization of two-body non-leptonic and semi-leptonic B to charm decays*, *JHEP* **10** (2021) 223, [[2107.03819](#)].
- [67] M. Endo, S. Iguro and S. Mishima, *Revisiting rescattering contributions to $\bar{B}_{(s)} \rightarrow D_{(s)}^{(*)} M$ decays*, *JHEP* **01** (2022) 147, [[2109.10811](#)].
- [68] S. Iguro and T. Kitahara, *Implications for new physics from a novel puzzle in $\bar{B}_{(s)}^0 \rightarrow D_{(s)}^{(*)+} \{\pi^-, K^-\}$ decays*, *Phys. Rev. D* **102** (2020) 071701, [[2008.01086](#)].

- [69] F.-M. Cai, W.-J. Deng, X.-Q. Li and Y.-D. Yang, *Probing new physics in class-I B-meson decays into heavy-light final states*, *JHEP* **10** (2021) 235, [2103.04138].
- [70] X.-G. He and W. Wang, *Flavor SU(3) Topological Diagram and Irreducible Representation Amplitudes for Heavy Meson Charmless Hadronic Decays: Mismatch and Equivalence*, *Chin. Phys. C* **42** (2018) 103108, [1803.04227].
- [71] T. Huber and G. Tetlalmatzi-Xolocotzi, *Estimating QCD-factorization amplitudes through SU(3) symmetry in $B \rightarrow PP$ decays*, *Eur. Phys. J. C* **82** (2022) 210, [2111.06418].
- [72] HFLAV collaboration, Y. S. Amhis et al., *Averages of b-hadron, c-hadron, and τ -lepton properties as of 2018*, *Eur. Phys. J. C* **81** (2021) 226, [1909.12524].
- [73] M. Gronau, *A precise sum rule among four $B \rightarrow K\pi$ CP asymmetries*, *Phys. Lett. B* **627** (2005) 82–88, [hep-ph/0508047].
- [74] LHCb collaboration, R. Aaij et al., *Measurement of CP Violation in the Decay $B^+ \rightarrow K^+\pi^0$* , *Phys. Rev. Lett.* **126** (2021) 091802, [2012.12789].
- [75] LHCb collaboration, R. Aaij et al., *LHCb Detector Performance*, *Int. J. Mod. Phys. A* **30** (2015) 1530022, [1412.6352].
- [76] LHCb collaboration, R. Aaij et al., *Measurement of CP violation in the phase space of $B^\pm \rightarrow K^\pm\pi^+\pi^-$ and $B^\pm \rightarrow K^\pm K^+K^-$ decays*, *Phys. Rev. Lett.* **111** (2013) 101801, [1306.1246].
- [77] LHCb collaboration, R. Aaij et al., *Measurement of CP violation in the phase space of $B^\pm \rightarrow K^+K^-\pi^\pm$ and $B^\pm \rightarrow \pi^+\pi^-\pi^\pm$ decays*, *Phys. Rev. Lett.* **112** (2014) 011801, [1310.4740].
- [78] LHCb collaboration, R. Aaij et al., *Observation of several sources of CP violation in $B^+ \rightarrow \pi^+\pi^+\pi^-$ decays*, *Phys. Rev. Lett.* **124** (2020) 031801, [1909.05211].
- [79] LHCb collaboration, R. Aaij et al., *Amplitude analysis of the $B^+ \rightarrow \pi^+\pi^+\pi^-$ decay*, *Phys. Rev. D* **101** (2020) 012006, [1909.05212].
- [80] LHCb collaboration, R. Aaij et al., *Amplitude analysis of $B^\pm \rightarrow \pi^\pm K^+K^-$ decays*, *Phys. Rev. Lett.* **123** (2019) 231802, [1905.09244].
- [81] LHCb collaboration, R. Aaij et al., *Measurement of the relative branching fractions of $B^+ \rightarrow h^+h'^+h'^-$ decays*, *Phys. Rev. D* **102** (2020) 112010, [2010.11802].
- [82] J. Dalseno, *Resolving the $\phi_2(\alpha)$ ambiguity in $B \rightarrow \rho\rho$* , *JHEP* **11** (2018) 193, [1808.09391].
- [83] J. Dalseno, *Impact of systematic and amplitude model correlations within and between systems of combined input: a case study with $\phi_2(\alpha)$* , *JHEP* **10** (2021) 110, [2108.06182].
- [84] J. Dalseno, *Resolving the $\phi_2(\alpha)$ ambiguity in $B^0 \rightarrow a_1^\pm\pi^\mp$* , *JHEP* **10** (2019) 191, [1907.09237].

- [85] BELLE-II collaboration, F. Abudinén et al., *Measurements of branching fractions and direct CP-violating asymmetries in $B^+ \rightarrow K^+\pi^0$ and $\pi^+\pi^0$ decays using 2019 and 2020 Belle II data*, [2105.04111](#).
- [86] BELLE-II collaboration, F. Abudinén et al., *First search for direct CP-violating asymmetry in $B^0 \rightarrow K^0\pi^0$ decays at Belle II*, [2104.14871](#).
- [87] BELLE-II collaboration, F. Abudinén et al., *Measurement of the branching fraction for $B^0 \rightarrow \pi^0\pi^0$ decays reconstructed in 2019-2020 Belle II data*, in *55th Rencontres de Moriond on QCD and High Energy Interactions*, 7, 2021, [2107.02373](#).
- [88] S. Iguro and T. Kitahara, *Implications for new physics from a novel puzzle in $\bar{B}_{(s)}^0 \rightarrow D_{(s)}^{(*)+}\pi^-$, K^- decays*, *Phys. Rev. D* **102** (2020) 071701.
- [89] S. I. Motoi Endo and S. Mishima, *Revisiting rescattering contributions to $\bar{B}_{(s)}^0 \rightarrow D_{(s)}^{(*)}M$ decays*, *JHEP* **2022** (2022) 147.
- [90] LHCb collaboration, R. Aaij et al., *Search for CP violation in $\Xi_b^- \rightarrow pK^-K^-$ decays*, *Phys. Rev. D* **104** (2021) 052010.
- [91] LHCb collaboration, R. Aaij et al., *Observation of the suppressed $\Lambda_b^0 \rightarrow DpK^-$ decay with $D \rightarrow K^+\pi^-$ and measurement of its CP asymmetry*, *Phys. Rev. D* **104** (2021) 112008.
- [92] LHCb collaboration, R. Aaij et al., *Search for CP violation and observation of P violation in $\Lambda_b^0 \rightarrow p\pi^-\pi^+\pi^-$ decays*, *Phys. Rev. D* **102** (2020) 051101.
- [93] LHCb collaboration, R. Aaij et al., *Updated measurement of time-dependent CP-violating observables in $B_s^0 \rightarrow J/\psi K^+K^-$ decays*, *EPJC* **79** (2019) 706.
- [94] LHCb collaboration, R. Aaij et al., *First measurement of the CP-violating phase in $B_s^0 \rightarrow J/\psi(\rightarrow e^+e^-)\phi$ decays*, *EPJC* **81** (2021) 1026.
- [95] LHCb collaboration, R. Aaij et al., *Angular analysis of $B^0 \rightarrow D^{*-}D_s^{*+}$ with $D_s^{*+} \rightarrow D_s^+\gamma$ decays*, *JHEP* **2021** (2021) 177.
- [96] R. Aaij et al., *Observation of the $B^0 \rightarrow \rho^0\rho^0$ decay from an amplitude analysis of $B^0 \rightarrow (\pi^+\pi^-)(\pi^+\pi^-)$ decays*, *Phys Lett B* **747** (2015) 468.
- [97] LHCb collaboration, R. Aaij et al., *Measurement of CP violation in the $B_s^0 \rightarrow \phi\phi$ decay and search for the $B^0 \rightarrow \phi\phi$ decay*, *JHEP* **2019** (2019) 155.
- [98] LHCb collaboration, R. Aaij et al., *Amplitude analysis of the $B_{(s)}^0 \rightarrow K^{*0}\bar{K}^{*0}$ decays and measurement of the branching fraction of the $B^0 \rightarrow K^{*0}\bar{K}^{*0}$ decay*, *JHEP* **2019** (2019) 32.
- [99] LHCb collaboration, R. Aaij et al., *Study of the $B^0 \rightarrow \rho(770)^0K^{*0}$ (892) decay with an amplitude analysis of $B^0 \rightarrow (\pi^+\pi^-)(K^+\pi^-)$ decays*, *JHEP* **2019** (2019) 26.