PROCEEDINGS OF SCIENCE

Entanglement Islands, AdS-Massive Gravity and Holography

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Studies on entanglement islands and Page curves in braneworld models realising systems of black holes in 4d gravity coupled to an external bath and their string theory uplifts, are recent developments with profound physical significance. In this contribution, we focus on a special family of the above uplifts, where the IIB solution allows for a graviton with a small mass. In this context, we focus on the emergence of the islands and their behaviour on special limits of the geometry and we comment on the significance of the graviton mass.

Corfu Summer Institute 2021 "School and Workshops on Elementary Particle Physics and Gravity"
29 August - 9 October 2021
Corfu, Greece

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https://pos.sissa.it/

PoS(CORFU2021)208

1. Introduction

One of the recent major breakthroughs, is the development of methods for the calculation of the time evolution of the entanglement entropy of the Hawking radiation emitted from an evaporating black hole, in consistency with unitarity [1, 2]. These are mostly semi-classical analyses and share a common feature: large corrections to the entanglement entropy of the emitted radiation coming from special regions of the gravitational system, referred to as *islands*. These constitute regions of the gravitational system, spatially disconnected from the region where the emitted radiation is gathered. The significance of the islands relies on the fact that their contribution dominates at late times and leads to Page curves for the time evolution of the entaglement entropy. [3–5]

Most of the important results were initially obtained in holographic context in two dimensions, with the study of black holes in JT gravity. Later analyses in higher dimensional setups are mostly based on Karch-Randall brane configurations [6, 7]. A series of studies on braneworld configurations modelling black holes on anti-de Sitter (AdS) coupled to external non-gravitating and gravitating baths [8, 9] has offered significant insights on various aspects of these systems, ranging from the role of the mass of the graviton [10] and the critical parameters of the setups, to the role of dynamical gravity on the bath subsystem and the behaviour of the island extremal surfaces [11].

The uplift of these setups to type IIB string theory is introduced in [12] and provides a description of the quantum extremal surfaces and Page curves in terms of the parameters of the string theory background. In special limits of these backgrounds the lowest-lying spin-2 mode acquires a parametrically small mass which can be explicitly computed in terms of the microscopic parameters of the solution.

These short proceedings are organized in three parts. The first part discusses the aforementioned Karch-Randall brane setup realizing AdS black hole coupled to a non-gravitating external bath and the recent contruction realizing its UV-embedding. Special focus is given in the details of the solutions realizing the string theory embedding. The second part is devoted to the main objective, which is the study of the above setup in the case where the background is appropriately engineered in order to allow for a graviton with a parametrically small mass. This is based in the corresponding string theory embeddings of AdS_4 massive gravity and bimetric gravity introduced in [13, 14], the main attributes of which are presented. In this context, we elaborate on the behaviour of the islands in this special limit background and also on the effect of the variation of the dilaton field. Finally we close with a discussion on a potentially interesting question, which regards a special limit of the gravitating bath solution embedded in string theory, which additionally to a massless graviton, may include a graviton with a parametrically small mass.

This contribution was presented in the 2021 Humboldt Kolleg meeting on Quantum Gravity and Fundamental Interactions, in the context of the Corfu Summer Institute, and described work (at that time) in progress, in collaboration with S. Demulder A. Gnecchi and D. Lüst. The talk given at the conference was focused on the structure of the problem and on some preliminary work. The complete analysis and the results of the work are now found in the article [15].

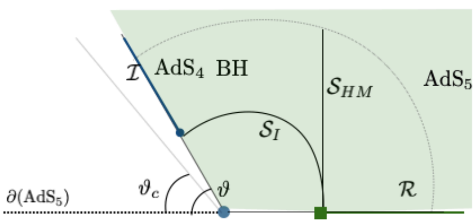
2. AdS Black Holes coupled to external baths: Karch-Randall branes and string theory realization

In this section we review the braneworld models realizing a black hole in AdS_4, coupled to an external non-gravitating bath, and its recent embedding in type IIB string theory. In order to consistently present the embedding, we first give a brief review of the corresponding string backgrounds.

2.1 Brane-world models and Page curves

Karch-Randall braneworlds play an absolutely prominent role in the study of black hole evaporation in AdS (among other numerous questions), for which have offered important results and insights. The attribute of these configurations rendering them unique in these analyses is their *doubly holographic* nature: Indeed, starting from such a configuration, one can deduce equivalent ones by applying twice the holographic duality, partially. For the case of interest, which is the one of four dimensional gravity coupled to an external bath, the typical configuration is comprised by gravity on an AdS_5 bulk sliced by an AdS_4 brane, serving as an end of the world (ETW) brane. This is holographically dual to AdS_4 gravity on the brane coupled to a CFT_4 on the half-space ($\mathbb{M}^4/\mathbb{Z}_2$). In turn, applying holographic duality for the AdS_4 brane, this is equivalent to a CFT_4 on the held-space coupled to a CFT_3 on the boundary ($\mathcal{B}_{3d}$).

Among the aforementioned dual descriptions, the one realizing AdS_4 gravity on the brane coupled to a CFT_4 on the boundary is the one used in the central analysis: it describes a black hole on the brane coupled to an external, non-gravitating bath. In the above figure, more details can be observed. The emitted radiation is gathered on the region R of the half-space, referred to as the *radiation region*.



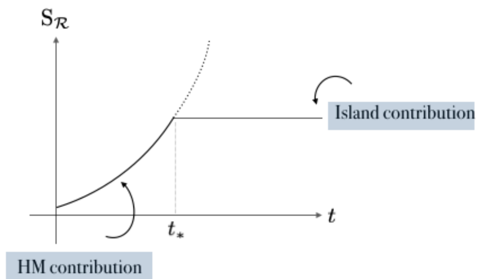


Figure 1: 4d black hole coupled to en external bath using Karch-Randall braneworlds and the Page curve for the time evolution of the entanglement entropy of the black hole radiation.

There are two kinds of extremal surfaces that contribute to the Page curve, corresponding to the black curves in the figure above. The trivial surface or equivalently the *Hartman-Maldacena* (HM)surface ($\mathcal{S}_{\text{HM}}$) is the one which connects, by crossing the horizon, the boundary of the radiation region with its corresponding point in the thermofield double [16]. The second one, is the Island surface ($\mathcal{S}_I$), connecting the boundary of the radiation region to the boundary of the island, which corresponds to the blue segment on the brane. At early times, the dominating surface is the HM one, whose area is increasing with time, resulting to a Page curve which grows linearly in time.

After some characteristic time, the area of the HM surface becomes larger than the one of the island surface and the latter dominates. The fact that the area of the island surface remains constant, results to saturation of the entropy of the radiation.

A crucial ingredient of these configurations, is the angle formed between the AdS_4-brane and the conformal boundary of the bulk AdS_5. As displayed in [8], at zero temperature the islands seize to contribute to the entanglement entropy once the brane is bent towards the boundary below a certain *critical* value ϑ_c due to the fact that the anchoring point of the island surface on the AdS_4 brane diverges towards the Poincaré horizon as $\vartheta \to \vartheta_c$.

Finally, a distinguishing feature of these configurations, is the fact that the 4d graviton is massive. Indeed, in the above description, the coupling of the black hole to the external non-gravitating bath is realized via transparent boundary conditions, which result to a non-vanishing graviton mass. The massless limit then corresponds then to the limit of vanishing angle ($\vartheta \to 0$), in which the islands disappear. The current work is mainly focused on the bahaviour of the islands in string backgrounds allowing for gravitons with a parametrically small mass and therefore we will return to this discussion in the context of string theory, after having reviewed the corresponding UV embedding.

2.2 String theory uplift

Warped AdS_4 solutions The solutions on which the embedding of [12] is built, are type IIB supergravity solutions on $AdS_4 \times_w \mathcal{M}_6$ preserving $\mathcal{N} = 4$, where the internal manifold $\mathcal{M}_6 = S^2 \times \hat{S}^2 \ltimes \Sigma_{(2)}$ is a warped product of two two-spheres over a Riemann surface, which here is the infinite strip:

$$\Sigma_{(2)} = \left\{ z = x + iy \mid x \in \mathbb{R},\ y \in \left[0, \frac{\pi}{2}\right] \right\} , \tag{1}$$

The local form of these solutions was constructed in [17] while the global form and the holographic dictionary were developed in [18].

The metric ansatz describing these solutions is given below:

$$ds^2_{10} = L_4^2 ds^2_{AdS_4} + f^2 ds_{S_1^2} + f_2{}^2 ds_{S_2{}^2} + 4\rho^2 dz d\bar{z}, \tag{2}$$

where the metric factors corresponding to the AdS_4 part, the two spheres and the Riemann surface, are expressed in terms of two harmonic functions $(h, \hat{h}) = f(z, \bar{z})$ which parametrize the solution on the strip and on which we will elaborate more below. The expressions for the metric factors are the following:

$$L_4^8 = 16\frac{U\hat{U}}{W^2} , \qquad \rho^8 = \frac{U\hat{U}W^2}{h^4\hat{h}^4} , \qquad f^8 = 16h^8\frac{\hat{U}W^2}{N^3} , \qquad \hat{f}^8 = 16\hat{h}^8\frac{UW^2}{\hat{U}^3} , \tag{3}$$

where:

$$W = \partial h \bar{\partial}\hat{h} + \bar{\partial}h\partial\hat{h} = \partial\bar{\partial}(h\hat{h}) , \qquad U = 2h\hat{h}|\partial h|^2 - h^2 W , \qquad \hat{U} = 2h\hat{h}|\partial\hat{h}|^2 - \hat{h}^2 W . \tag{4}$$

Regularity of the solutions is guaranteed by imposing corresponding conditions for the harmonic functions. In particular, in order for the solutions to be regular, one of the harmonic functions should vanish on each of the boundaries of the Riemann surface, along with the normal derivative of the other harmonic function. As a consequence, one of the two spheres vanishes on each boundary. Having imposed these regularity solutions, the general form of the harmonic functions parametrizing this background, is given below:

$$h = -\alpha \mathrm{sh}(z-\beta) - \sum_{i=1}^{p} \gamma_i \mathrm{logth}\Big(\frac{i\pi}{4} - \frac{z}{2} + \frac{\delta_i}{2}\Big) + \mathrm{c.c} \tag{5}$$

$$\hat{h} = \hat{\alpha} \mathrm{ch}(z-\hat{\beta}) - \sum_{j=1}^{\hat{p}} \hat{\gamma}_j \mathrm{logth}\Big(\frac{z}{2} - \frac{\hat{\delta}_j}{2}\Big) + \mathrm{c.c} \tag{6}$$

Let's comment on the structure of these functions and on their behaviour, describing a regular solution. Both functions are labeled by a set of real parameters, each with a distinct physical significance: α, β, γ and δ and the hatted set labelling the second harmonic function, respectively. We will gradually explain their meaning as they play an important role in the embedding. The first thing to notice here is that the harmonic functions have logarithmic singularities at the positions δ_i and $\hat{\delta}_j$, respectively, which are located on the boundaries of the strip. We underline here that these points are interior points of the geometry: the only boundary here is the conformal boundary of the AdS_4.

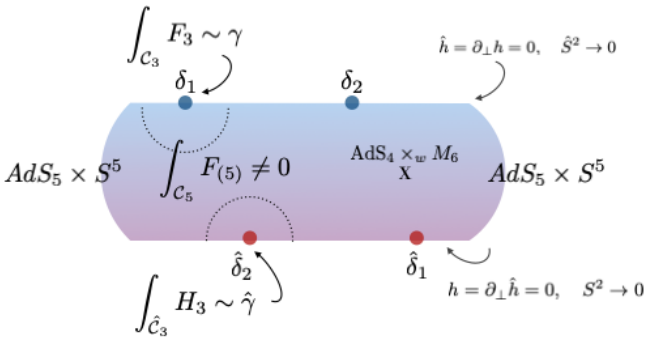


Figure 2: A simplified version of the $\mathrm{AdS}_4 \times_w \mathcal{M}_6$ IIB background with two point singularities on each boundary of the Riemann surface.

The supergravity solution has non-vanishing R-R (F_3) and NS-NS (H_3) three-form fluxes as well as non-vanishing R-R five-form fluxes (F_5) threading non-contractible three-cycles and five-cycles supported on the singularities, as seen in the figure 2 above. On the point singularities on the upper boundary of the string we have non-vanishing F_3 flux while on the corresponding points on the lower boundary, non-vanishing H_3 flux, indicating the presence of stacks of D5 and NS5 branes, respectively, the number (charge) of which is given in terms of the parameters $(\gamma, \hat{\gamma})$. Moreover, in both cases we also have non-vanishing F_5 flux, indicating the presence of D3 branes, connecting the stacks of the aforementioned five-branes. In the limit $z \to \pm\infty$ the geometry asymptotes to $\mathrm{AdS}_5 \times S^5$, corresponding to semi-infinite D3 brane throats. Therefore the geometry of the solution corresponds to the near horizon geometry of intersecting D3-D5-NS5 brane configurations. In particular, these are Gaiotto-Hanany-Witten configurations, preserving $\mathcal{N} = 4$ and give rise to a particular class of 3d $\mathcal{N} = 4$ theories. The brane configuration is given in figure 3 below.

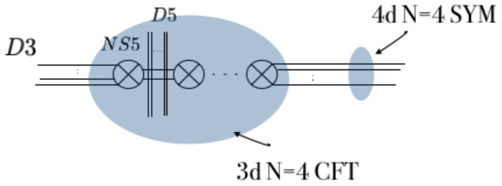


Figure 3: The Gaiotto-Hanany-Witten brane configuration described in the text. These intersecting brane configurations realize 3d $\mathcal{N} = 4$ defect theories in 4d $\mathcal{N} = 4$ super Yang-Mills.

The last ingredient of the solution is the dilaton field, also expressed in terms of the harmonic functions. The parameters that encode its variation between the two asymptotic regions are $(\beta, \hat{\beta})$ where $\beta = -\hat{\beta} = \frac{\delta\phi}{2}$. In the case of vanishing dilaton variation, the geometry is asymptotically $\mathrm{AdS}_5 \times S^5$, while for varying dilaton the geometry asymptotes to the supersymmetric Janus, which is a one-parameter deformation of the $\mathrm{AdS}_5 \times S^5$.

Finally, the $(\alpha, \hat{\alpha})$ parameters are related to the the asymptotic regions of the geometry. In the smooth limit $\alpha, \hat{\alpha} \to 0$ the two asymptotic regions cap-off and become interior regions of the full ten-dimensional geometry. In this way we obtain a compact internal manifold and the $\mathrm{AdS}_4 \times_w \mathcal{M}_6$ solutions are holographically dual to a three-dimensional $\mathcal{N} = 4$ superconformal field theories [18]. Another interesting limit, is the one discussed in detail below, in which only one of the two asymptotic regions caps off.

Black holes and Islands in type IIB string theory The IIB backgrounds presented above, constitute the basis upon the UV-embedding of the Karch-Randall braneworld models is performed. In this part, we are presenting the basic attributes of the embedding, originally implemented in [12] while focusing on the aspects which will be central for our analysis.

Introducing a black hole solution in the above backgrounds is implemented by substituting the AdS_4 factor in the metric ansatz (2) with the metric of an AdS_4 black hole:

$$ds^2_{\mathrm{AdS}_4} \to ds^2_{\mathrm{AdS}_4-\mathrm{BH}} = \frac{dr^2}{b(r)} + e^{2r}(ds^2_{\mathbb{R}^2} - b(r)dt^2), b(r) = 1 - e^{3(r_H - r)}, \tag{7}$$

where r stands for the AdS_4 radial coordinate, with the black hole horizon at $r = r_H$. There is a particular limit of the $\mathrm{AdS}_4 \times_w \mathcal{M}_6$ background which is important for the UV embedding of braneworld models describing an AdS black hole coupled to an external non-gravitating bath. In that limit, one of the two asymptotic regions caps off and is substituted by a regular interior point of the geometry, while the other asymptotic remains intact. The region including the five-branes and the capped-off part of the geometry in this limit is the region of the AdS_4 black hole, while the asymptotic $\mathrm{AdS}_5 \times S^5$ plays the role of the non-gravitating bath, in which the black hole radiates.

The form of the harmonic functions parametrizing this solution is given below:

$$\begin{aligned} h &= -\frac{1}{2} e^z K - N \log \mathrm{th}\left(\frac{i\pi}{4} - \frac{z}{2}\right) + \mathrm{c.c}\,, \\ \hat{h} &= \frac{1}{2} e^z K - N \log \mathrm{th}\left(\frac{z}{2}\right) + \mathrm{c.c}\,. \end{aligned} \tag{8}$$

and describes the simplified case where the solution has one point singularity on each boundary of Σ_2, with equal number of N-fivebranes, found at $(x, y) = (0, 0)$ and $(x, y) = (0, \frac{\pi}{2})$ respectively. The five-branes are connected by D3 branes, as explained in the previous subsection, while the asymptotic region corresponds to a semi-infinite D3 brane throat of charge $2NK$, with $AdS_5 \times S^5$ geometry. In figure 4, we demonstrate this background and also the corresponding D3-D5-NS5 brane configuration.

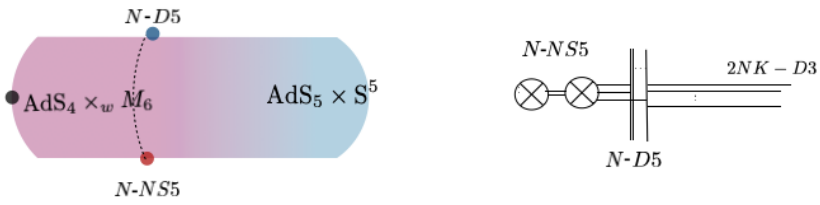


Figure 4: The IIB background realizing the coupling of an AdS_4 black hole to a non-gravitating bath and the corresponding intersecting brane configuration.

Extremal surfaces The island surfaces in this background are minimal surfaces extending from the $AdS_4 \times_w \mathcal{M}_6$ region to the $AdS_5 \times S^5$ asymptotic. These are eight dimensional Ryu-Takayanagi surfaces embedded in the full ten dimensional geometry, wrapping the two S^2's while partially wrapping Σ and the AdS_4 black hole. Focusing on the Riemann surface, the embedding surfaces are realized in terms of the AdS_4 radial coordinate $r(x, y)$. The extremality condition for this surface is expressed in terms of the harmonic functions in the above form.

The most important component of the construction of the embedding surfaces is the fixing of the boundary conditions: it is crucial to know how the surfaces behave close to the singularities and in the bath region. The surfaces are smooth in the limit $x \to -\infty$ which corresponds to the capped-off part of the geometry. At location of the point singularities on the strip boundaries the surfaces fall into the horizon. The surfaces continue in the bath region, where they anchor at a specific positive value of the AdS_4 radial coordinate, which corresponds to a Dirichlet boundary condition at $x \to +\infty$ where the embedding function $r(x, y)$ takes a constant value.

As presented in detail [12], the behaviour of the surface is obtained by numerically solving the extremality conditions. The analysis then shows that the behaviour of the island surfaces strongly dependends on the parameter $\alpha = N/K$. This parameter is a ratio of the number of the finite-length D3 branes suspended between the five-branes of the solution, over the number of semi-infinite ones.

As presented in the previous subsection, in the Karch-Randall setups, the angle ϑ between the ETW brane and the conformal boundary of AdS_5 determines at zero temperature the contribution of the islands to the entropy. In the above IIB embedding, the parameter α is actually the quantity corresponding to the angle ϑ. Practically this quantity is a ratio of "boundary" over "bulk" degrees of freedom and can be interpreted as the "tension" of the composite D3-D5-NS5 brane. Therefore, large values of α- namely for larger tension of the composite brane- are expected to correspond to small values of the angle ϑ: $\alpha \leftrightarrow 1/\vartheta$. Indeed, this is verified in the respective analysis.

In particular, the analysis of [12] indicates that at zero temperature, there exists a critical value of this parameter, which we refer to as α_{crit}, below which the island surfaces dominate over the

Hartmann-Maldacena surface and the islands contribute to the entropy leading to a Page curve. The value of this critical parameter is determined numerically to be $\alpha_{crit} \approx 4$, by studying the divergence of the distance between the anchoring points of the island surface on the black hole region and on the external bath. This behavior is analoguous to the one in the subcritical braneworld setups: for angles above the critical one, islands contribute to the entanglement entropy the evolution of which follows a Page curve.

At this point, we underline the fact that the existence of a critical value for the geometric ratio α for which the islands stop to contribute is an attribute of the zero temperature solution. On the contrary, at finite temperature this effect is regulated by the black hole horizon and islands continue to contribute even for values of the geometric ratio higher than the critical one determined by the zero temperature solution. This is completely analogous to the behaviour of entanglement islands in the low-dimensional braneworld models.

A point to be noted is that, as in the low dimensional models, in the above solution the lowest-lying graviton is massive, as a result of the non-compactness of the internal manifold [19]. Nevertheless an explicit computation of its mass is only possible on the special limit of the geometry which we now present.

3. Islands in AdS$_4$ Massive gravity

The goal of our work in progress, is to study the above in a special limit of the introduced IIB background, in which it admits a slightly massive low-lying graviton.

The AdS$_4$ vacuum where the internal manifold $\mathcal{M}_6$ is compact, includes a massless low-lying graviton and a KK tower of massive modes. The geometry in which the graviton acquires a mass, is the one where the internal manifold is instead non-compact. The spin-2 spectrum can be numerically computed by the corresponding eigenvalue problem for the mass-squared (Laplace-Beltrami) operator on the internal manifold [19]. An analytical result for the mass eigenvalue of the lowest lying graviton can be obtained by solving the variational problem giving the minimal eigenvalue of the mass squared operator [13]. The background allowing for a small mass for the lowest-lying spin-2 is exactly the one introduced in the previous section, but with the constraint that the radius of the semi-infinite throat is much smaller than the size of the region in the vicinity of the fivebranes, referred to as the *bag* region ($L_5 << L_{\text{bag}} \sim L_4$), as can be seen in figure 5. Note that, due to non-separation of scales of AdS vacua, the AdS$_4$ radius L_4 is parametrically bound to the size of the internal manifold L_{bag}.

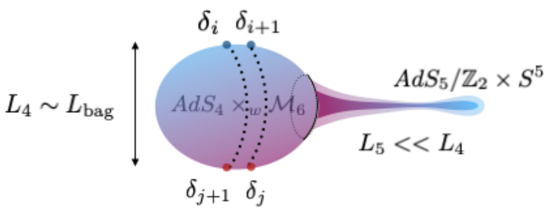


Figure 5: The background of interest, where the lowest-tlying spin-2 mode acquires a parametrically small mass.

The fact that the lowest-lying graviton in the above background obtains a small mass, has a very interesting holographic interpretation [13]: the AdS_4 solutions introduced in the previous subsection, admit a holographic dual description as three dimensional $\mathcal{N} = 4$ superconformal quiver theories. Such a theory has a conserved stress-tensor $T^{\mu\nu}_{(3)}$, which has canonical conformal dimension ($\Delta = 3$), as a result of the conservation. Once this three-dimensional theory is coupled to a four dimensional $\mathcal{N} = 4$ super Yang-Mills bulk theory, the conservation of the 3d stress tensor is violated: the three-dimensional stress tensor is no longer conserved but rather leaks in the bulk direction. In particular, we consider that the stress tensor is dissipating weakly in the extra bulk direction, a fact that implies that its (canonical) dimension acquires a small anomalous dimension. Holographically, the anomalous dimension of the stress tensor corresponds to a small correction to the AdS_4 graviton mass:

$$m_g^2 L_4^2 = \Delta(\Delta - 3) \xrightarrow{\Delta=3+\varepsilon} m_g^2 L_4^2 \sim \varepsilon \tag{9}$$

The weak dissipation is ensured by the fact that the number of degrees of freedom of the 3d boundary theory is much greater than the one of the bulk 4d theory. Note that the dual $CFT_3 - CFT_4$ system, can be read off the intersecting brane configuration corresponding to the the AdS_4 background we are considering (see figure 4): indeed, these are the theories living in the world volume of the two kinds of D3 branes in the configuration. The scarcity of the bulk degrees of freedom corresponds to a small number of semi-infinite D3 branes as opposed to the large number of D3 branes suspended between the five-branes. This will be very useful in adressing the question of the behaviour of the entanglement islands in this background.

After solving the aforementioned variational problem, the graviton mass is given by the following expression, in terms of the geometric parameters of the background [13]:

$$m_g^2 L_4^2 \equiv \bar{m}_g^2 \simeq \frac{3\pi^3}{4}\left(\frac{L_5}{L_{\rm bag}}\right)^8 \mathcal{J}(\mathrm{ch}(\delta\varphi)) \tag{10}$$

at leading order in the ratio of the sizes of the internal manifold and of the semi-infinite throat. It is important to note the meaning of the last factor in the above expression: recall from the previous section that the solution at hand has a dilaton field, expressed in terms of the harmonic functions $(h, \hat{h})$ and its value in the *bag* region depends on the number of the five-branes of the solution. However, in the above case, the dilaton is allowed to vary through the semi infinite throat and its value at infinity determines the coupling of the four dimensional bulk theory to the three dimensional theory. If the dilaton field remains constant $\delta\phi = 0$ the geometry of the throat is $AdS_5 \times S^5$ while if it varies the geometry of the throat is the Janus generalization of the $AdS_5 \times S^5$. This is encoded in the *Janus correction function* $\mathcal{J}(\delta\phi)$, which reduces the graviton mass for increasing dilaton variation, while its contribution becomes trivial in the case of a constant dilaton, see figure 6.

The above background is realized as a special limit of the bimetric gravity embedding [14], where the throat connects two compact regions, with the spectrum including also a massless graviton, while it is related under an appropriate deformation to solutions with multiple massive gravitons [20].

Islands, light gravitons and the dilaton In this work, we aspire to analyse how the tuning of the parameters of the geometry leading to a background where the graviton is slightly massive affects

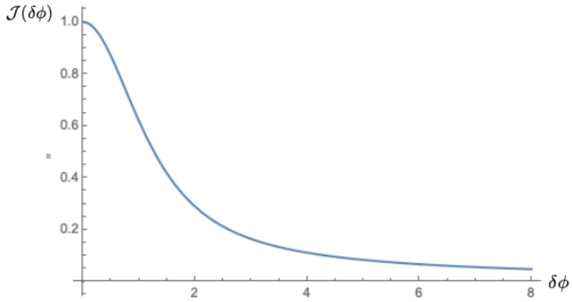


Figure 6: The Janus correction function, encoding the dilaton variation.

the properties of the island surfaces as extracted from the analysis of [12]. In particular, we would like to examine whether islands can still contribute in that limit.

Some initial insights can be obtained in the simple case of the $AdS_5 \times S^5$ bath, namely where the dilaton variation vanishes. In that case, the radii of the internal manifold and of the semi-infinte troat are given by $L_5^4 \sim NK$ and $L_{\text{bag}}^4 \sim N^2$ and hence the expression for the graviton mass reads:

$$m_g^2 L_4^2 \equiv \bar{m}_g^2 \simeq \frac{3\pi^3}{4}\left(\frac{K}{N}\right)^2 = \frac{3\pi^3}{4}\frac{1}{\alpha^2} \tag{11}$$

from this, we see that parametrically small values of the mass, require sufficiently large values of the parameter α. Nevertheless, recall from the previous subsection, that there is a critical value of this parameter above which at zero temperature the islands do not contribute to the entropy. It can be seen from the mass formula that parametrically small ($\bar{m}_g^2 << 1$) values for the graviton mass can be obtained for values of the parameter α way higher than the indicated critical value $\alpha_{crit} \approx 4$. Therefore from this arguement we can expect that islands would not seem to contribute at this limit. This is something rather expected as, even in the low-dimensional braneworld analysis, the islands do not contribute close to the massless limit. Moreover, in the case of finite temperature, due to the presence of the horizon, the islands contribute also for values of the parameter $\alpha > \alpha_{crit}$ and therefore it is expected that they would also contribute in the limit where the mass of the graviton is parametrically small. However, the interesting problem is to understand the role of the dilaton variation in this context. Indeed, in the above analysis we have considered $\delta\phi = 0$: since we already know that an increasing dilaton variation supresses the graviton mass, it would be crucial to understand how it would affect the α-parameter. This can be done as follows: we need to fix the data of the solution, namely the number of five-branes, which determine the value of the dilaton in the *bag* region and fix the value of the dilaton at infinity, which can be chosen freely. We can then numerically extract how the α parameter behaves for increasing values of the dilaton variation and see how it affects the contribution of the islands in this special limit of the geometry. This analysis was completed recently in [15].

4. Discussion: beyond the non-gravitating bath

In this short contribution, we have reviewed the recent UV-embedding of the Karch-Randall configuration realizing a black hole in AdS_4 coupled to an external, non-gravitating bath and we have presented the motivation and the first steps of our work. We have reviewed how the extremal surfaces behave in that case, both in the low-dimensional braneworld models and in the IIB string theory embedding and we have commented on the analogies between these two setups.

One thing left for a future analysis, is the case of the gravitating bath: in the context of the Karch-Randall braneworlds, the gravitating bath is realized from a setup similar to the one of the non-gravitating bath, with the difference that the radiation region is found on a second brane introduced in the setup, forming an angle with the boundary of the bulk AdS_5. In that case both the island-brane and the bath-brane have dynamical gravity (and hence the term *gravitating* bath) and this affects the island surface anchored on these two branes: the island surface settles on the horizon and leads to a constant entropy curve. However, an interesting extremal surface leading to a Page curve, is the one connecting the boundary of the island to the 3d defect on the boundary, in the junction of the two branes [9]. In the IIB embedding of [12] the string theory realization of the gravitating bath is implemented in the limit where both asymptotic regions cap-off, in the way we have already described in the corresponding review section. The extremal surface anchored at $x = \pm\infty$ settles on the black hole horizon and does not give a Page curve, in agreement with the low-dimensional result. Surfaces leading to Page curves, appropriately split the internal space and the Page curve behaviour depends on the distance between the two sets of point singularities separated by the surface.

It would be interesting to extend the above in the case of [14]. The difference with the aforementioned gravitating bath solution, is that apart from the massless graviton, this setup includes also a massive graviton of parametrically small mass. In this construction, by appropriately separating two sets of point singularities on the strip, we obtain two *bag* regions connected by a throat of small radius. It would be interesting to consider two black hole regions - analogously with the presented embedding- connected by the throat and study the possible extremal surfaces.

Acknowledgements I would like to thank S. Demulder, A. Gnecchi and D. Lüst for the excellent collaboration on this project and Costas Bachas for fruitful discussions. I would also like to thank G. Zoupanos and the organizers of the 2021 Humboldt Kolleg meeting on Quantum Gravity and Fundamental Interactions, in the context of the Corfu Summer institute, in which I had the opportunity to present this work. This work is supported by the “Origins" Excellence Cluster.

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