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Extracting bigravity from string amplitudes

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The origin of the graviton from string theory is well understood: it corresponds to a massless state in closed string spectra, whose low–energy effective action, as extracted from string scattering amplitudes, is that of Einstein–Hilbert. In this talk, we will discuss our recent published work on the possibility of such a string–theoretic emergence of ghost–free bimetric theory, a recently proposed extension of general relativity that involves two dynamical metrics and which around particular backgrounds propagates the graviton and a massive spin–2 field, with the latter having been put forward as a viable dark matter candidate.

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1. Introduction

Our current understanding of gravity largely relies on Einstein's theory of General Relativity (GR), that is a geometric framework in which matter and radiation source spacetime's curvature. In its core of assumptions lie Lorentz invariance and locality, which, under the condition that no ghost degrees of freedom propagate, yield a theory invariant under diffeomorphisms, namely general coordinate transformations. Alternatively, GR can be viewed as the field theory that uniquely describes the kinematics of a massless helicity–2 field, the graviton.

A microscopically more fundamental look at the graviton reveals that it also appears as an on–shell state of closed string spectra. For example, the massless level of the bosonic string is built out of combinations of string oscillators that give rise to a symmetric and an asymmetric rank–2 tensor, as well as a scalar; the former further enjoys self–interactions that are identical to those of the GR graviton, as was shown by means of extracting efffective Lagrangians at first from three– and four–point string amplitudes involving this symmetric state as an external state [1–3].

Despite the fact that GR predictions can account for numerous observational and experimental data with fine precision, questions such as the enigmas of the cosmological constant, dark matter (DM) and dark energy, remain unanswered and motivate the investigation of GR extensions. A promising example of the kind is ghost–free bimetric theory, a theory that involves two distinct but interacting dynamical rank–2 symmetric tensors [4, 5]. Its structure was inspired by ghost–free massive gravity [6] and, crucially, it contains all GR solutions as well as, around particular backgrounds, a massive spin–2 field with features similar to those of DM [7–9]. It is, therefore, a minimal GR modification that may further shed light on the nature of DM.

In the absence of sources and in four spacetime dimensions, the Lagrangian of ghost–free bimetric theory is given by [5]

$$\mathcal{L}_{\rm HR} = m_g^2\,\sqrt{g}R(g) + m_f^2\,\sqrt{f}R(f) - 2\,m_g^2 m_f^2\,\sqrt{g}\,V(S;\beta_n)\,, \tag{1}$$

where $g_{\mu\nu}$ and $f_{\mu\nu}$ are the two symmetric rank–2 tensors, with m_g and m_f their respective Planck masses. Each of the two tensors is namely governed by its own Einstein–Hilbert action and they further interact via a non–derivative potential that reads

$$V(S;\beta_n) = \sum_{n=0}^{4}\beta_n e_n(S) \quad , \quad S^{\mu}{}_{\nu} = \left(\sqrt{g^{-1}f}\right)^{\mu}{}_{\nu}\,, \tag{2}$$

where β_n are dimensionless, a priori arbitrary, parameters and e_n elementary symmetric polynomials of order n; they involve the trace of the square–root of the matrix $(g^{-1}f)^{\mu}{}_{\nu}$ appearing in various powers up to the n–th, as well as traces of products of the square–root in question with itself.

The action corresponding to (1) was motivated by [6], the first construction of a healthy theory describing a massive graviton around arbitrary backgrounds. In particular, the linear theory of Fierz–Pauli (FP), describing the five degrees of freedom of a massive graviton propagating in Minkowski spacetime [10], had long been thought of as lacking a nonlinear extension, as any such seemed to inevitably excite an additional but unhealthy degree of freedom that renders the (classical) Hamiltonian unbounded from below, the so–called Boulware–Deser ghost [11]. Nevertheless, it has been shown that the ghost can be evaded if the mass term has a very specific structure that involves elementary symmetric polynomials built out of the square–root of the product of the (inverse) metric with another, non–dynamical, reference metric [5, 6, 12].

It is this second arbitrary metric that was promoted to a dynamical field, in what has come to be known as ghost–free bimetric theory. It has also been shown that this theory propagates seven healthy degrees of freedom around generic backgrounds [4, 13], that do not necessarily split into those of the massless (2) and of the massive graviton (5), with the exception of maximally symmetric backgrounds. With the unique structure of $V(S;\beta_n)$ serving as the guarantee of ghost–freedom, and given that both GR and bimetric theory are non–renormalisable effective field theories, while string theory accommodates the massless graviton, we wondered: can we extract information on $V(S;\beta_n)$ and its parameters from string scattering amplitudes, the physical observables of string theory?

To attack this problem, we formulated the following logical plan:

1. the Lagrangian (1) is given in terms of the two full rank–two tensors, that are not well–defined mass eigenstates. We thus need to treat bimetric theory perturbatively, namely choose suitable backgrounds for the two tensors and expand (1) around these.

2. The resulting mass eigenstates around the chosen backgrounds have to be appropriately identified with string states with the same mass and spin properties in four dimensions.

3. States belonging to string spectra are by construction on–shell and we only consider on-shell string amplitudes. Consequently, the on–shell conditions reflecting the properties of the chosen string states have to be imposed on the bimetric expansion.

4. With the first nontrivial interactions in the bimetric expansion being cubic vertices, we can compute three–point amplitudes with the chosen string states as asymptotic states, extract the corresponding effective actions and compare them with the bimetric expansion.

In the present proceedings material, we will review our recent original published work [14] on the matter, to which we refer the reader for further technical details.

2. The bimetric expansion

Among the solutions of bimetric theory, of particular importance are the proportional backgrounds, namely those for which the background values of the two rank–2 tensors are proportional to each other [4, 15]. For simplicity, we choose the subclass of Minkowski backgrounds for the two tensors, namely expand the fields as

$$g_{\mu\nu} = \eta_{\mu\nu} + \delta g_{\mu\nu} \quad , \quad f_{\mu\nu} = \eta_{\mu\nu} + \delta f_{\mu\nu} \, . \tag{3}$$

It can then be shown that the mass matrix of (1) can be diagonalised around these backgrounds, with the two mass eigenstates being

$$G_{\mu\nu} \equiv m_g \left(\delta g_{\mu\nu} + \alpha^2 \delta f_{\mu\nu}\right) \quad , \quad M_{\mu\nu} \equiv \alpha m_g \left(\delta f_{\mu\nu} - \delta g_{\mu\nu}\right) , \tag{4}$$

where $\alpha \equiv m_f/m_g$. In particular, at order quadratic in the fields, (1) contains the following set of terms

$$
\begin{aligned}
\mathcal{L}^{(2)}(G) &= \tfrac{1}{2} G^{\mu\nu}\, \hat{\mathcal{E}}^{\rho\sigma}_{\mu\nu} G_{\rho\sigma} \\
\mathcal{L}^{(2)}(M) &= \tfrac{1}{2} M^{\mu\nu}\, \hat{\mathcal{E}}^{\rho\sigma}_{\mu\nu} M_{\rho\sigma} - \tfrac{m^2_{\rm FP}}{4}\left([M^2]-[M]^2\right),
\end{aligned} \tag{5}
$$

where $\hat{\mathcal{E}}^{\rho\sigma}_{\mu\nu}$ is the Lichnerowicz operator that arises from the linearisation of the Einstein–Hilbert action around Minkowski and which contains two–derivative terms. Brackets denote the trace of the respective field.

At quadratic order, ghost–free bimetric theory namely propagates the standard massless graviton of GR, denoted by $G_{\mu\nu}$, as well as a massive spin–2 field, $M_{\mu\nu}$, whose kinematics are identical to those of the GR graviton and whose mass is given by [4, 15].

$$
m^2_{\rm FP} \equiv m^2_g (1+\alpha^2)\,(\beta_1 + 2\beta_2 + \beta_3)\,. \tag{6}
$$

The Lagrangian of $M_{\mu\nu}$ is thus that of FP, as should the case be, given that the latter is the unique ghost–free Lagrangian for a massive graviton propagating in Minkowski spacetime. Notice that we have here the explicit split of the total seven physical degrees of freedom into two and five. Upon considering matter couplings, it can be shown that $M_{\mu\nu}$ interacts with the graviton but very weakly so with matter (at least for a large region of the parameter space of bimetric theory); it is precisely because of this feature of the massive spin–2 that it has been put forward as a viable DM candidate [7–9].

At order cubic in the fields, (1) contains various kinds of vertices, with and without derivatives. For example, the terms involving one massless graviton and two massive spin–2 fields are [8]

$$
\begin{aligned}
\mathcal{L}_{\rm GM^2} \;=\;& \tfrac{m_{\rm Pl}}{8}(\beta_1+2\beta_2+\beta_3)\left[[G][M]^2 - 4[M][GM] - [G][M^2] + 4[GM^2]\right] \\
&+\tfrac{1}{4m_{\rm Pl}}\Big[G^{\mu\nu}\big(\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - \partial_\mu[M]\partial_\nu[M] + 2\partial_\nu[M]\partial_\rho M^\rho_\mu \\
&+2\partial_\nu M^\rho_\mu \partial_\rho[M] - 2\partial_\rho[M]\partial^\rho M_{\mu\nu} + 2\partial_\rho M_{\mu\nu}\partial_\sigma M^{\rho\sigma} - 4\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu \\
&-2\partial_\rho M_{\nu\sigma}\partial^\sigma M^\rho_\mu + 2\partial_\sigma M_{\nu\rho}\partial^\sigma M^\rho_\mu\big) + \tfrac{1}{2}[G]\big(\partial_\rho[M]\partial^\rho[M] \\
&-\partial_\rho M_{\mu\nu}\partial^\rho M^{\mu\nu} - 2\partial_\rho[M]\partial_\mu M^{\mu\rho} + 2\partial_\rho M_{\mu\nu}\partial^\nu M^{\mu\rho}\big)\Big] \\
&+\tfrac{1}{2m_{\rm Pl}}\Big[M^{\mu\nu}\big(\partial_\mu G_{\rho\sigma}\partial_\nu M^{\rho\sigma} - \partial_\mu[G]\partial_\nu[M] + \partial^\rho G_{\rho\mu}\partial_\nu[M] \\
&+\partial_\nu G_{\mu\rho}\partial^\rho[M] - \partial_\rho G_{\mu\nu}\partial^\rho[M] + \partial_\rho G^{\rho\sigma}\partial_\sigma M_{\mu\nu} - 2\partial_\mu G^{\rho\sigma}\partial_\sigma M_{\nu\rho} \\
&+\partial_\mu[G]\partial^\rho M_{\rho\nu} + \partial^\rho G_{\mu\nu}\partial^\sigma M_{\rho\sigma} - 2\partial_\rho G_{\mu\sigma}\partial_\nu M^{\rho\sigma} - 2\partial^\rho G_{\mu\sigma}\partial^\sigma M_{\nu\rho} \\
&+2\partial^\rho G_{\mu\sigma}\partial_\rho M^\sigma_\nu + \partial^\rho[G]\partial_\nu M_{\mu\rho} - \partial^\rho[G]\partial_\rho M_{\mu\nu}\big) + \tfrac{1}{2}[M]\big(\partial_\rho[G]\partial^\rho[M] \\
&-\partial_\rho G_{\mu\nu}\partial^\rho M^{\mu\nu} - \partial_\rho[G]\partial_\sigma M^{\rho\sigma} - \partial_\rho G^{\rho\sigma}\partial_\sigma[M] + 2\partial_\rho G_{\mu\nu}\partial^\nu M^{\mu\rho}\big)\Big]\,,
\end{aligned} \tag{7}
$$

where $m^2_{\rm Pl} \equiv m^2_g(1+\alpha^2)$. In regard to the structure of the vertices, let us emphasise that the two–derivative self–interactions of $G_{\mu\nu}$ and $M_{\mu\nu}$ are identical and due to the linearisation of the Einstein–Hilbert action around Minkowski, while, unlike $G_{\mu\nu}$, $M_{\mu\nu}$ further enjoys non–derivative self–interactions that can be traced back to the bimetric potential. $G_{\mu\nu}$ can thus be identified with the GR graviton also at cubic level and it is clear that the expansion respects the theorem according to which the allowed self–interactions of the graviton are strictly of the Einstein–Hilbert kind [16]. Let us also highlight that no terms involving two gravitons and one massive spin–2 field appear at cubic order; the absence of such interactions can be thought of as a discriminating feature of ghost–free bimetric theory opposite other theories involving two metrics [8].

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As previously explained, the string states we consider and, therefore, the string amplitudes we have computed, are on–shell. For the case of on–shell states corresponding to symmetric rank–2 tensors, this means that the respective masses are on–shell and the polarisation tensors transverse and traceless. Consequently, these conditions have to be considered in position space and imposed on the field theory side; for the two mass eigenstates in the bimetric expansion around Minkowski, they thus take the form

$$\begin{array}{c} \Box G_{\mu\nu} = 0 \quad , \quad \partial^\mu G_{\mu\nu} = 0 \quad , \quad [G] = 0 \\ (\Box - m_{\rm FP}^2) M_{\mu\nu} = 0 \quad , \quad \partial^\mu M_{\mu\nu} = 0 \quad , \quad [M] = 0 \, . \end{array} \tag{8}$$

These are namely nothing other than the massless and massive versions of the Klein–Gordon equation for transverse and traceless spin–2 fields. To facilitate the comparison between the bimetric expansion and effective actions extracted from on–shell string amplitudes, we thus have to impose (8) in the former. Employing partial integrations, the totality of the cubic vertices surviving then these constraints is [14]

$$\begin{aligned} \mathcal{L}_{\rm G^3} &= \tfrac{1}{m_g\sqrt{1+\alpha^2}}\, G^{\mu\nu}\big(\partial_\mu G_{\rho\sigma}\partial_\nu G^{\rho\sigma} - 2\partial_\nu G_{\rho\sigma}\partial^\sigma G^\rho_\mu\big) \\ \mathcal{L}_{\rm GM^2} &= \tfrac{1}{m_g\sqrt{1+\alpha^2}}\, \Big[G^{\mu\nu}\big(\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - 4\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu\big) \\ &\qquad + 2M^{\mu\nu}\big(\partial_\mu G_{\rho\sigma}\partial_\nu M^{\rho\sigma} - \partial_\rho G_{\mu\sigma}\partial_\nu M^{\rho\sigma}\big)\Big] \\ \mathcal{L}_{\rm M^3} &= \tfrac{(-\beta_1+\beta_3)\,(1+\alpha^2)^{3/2} m_g}{6\,\alpha}\,[M^3] \\ &\quad + \tfrac{(1-\alpha^2)}{m_g\,\alpha\sqrt{1+\alpha^2}}\, M^{\mu\nu}\big(\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - 2\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu\big)\,. \end{aligned} \tag{9}$$

What is crucial to observe here is that the terms given in (9) are, up to partial integrations, *all* possible Lorentz invariant terms at cubic order in transverse and traceless symmetric rank–2 tensors that can be written, *excluding* interactions involving two gravitons and one massive spin–2 field. This means that what is unique in (9) is not the structure of vertices, but the precise dependence of the couplings on the bimetric parameters. It is information on these coefficients that we aimed at extracting from string theory, which we now turn to.

3. Identification of string states

With the graviton universally identified with a massless state of closed string spectra as previously explained, the first step is to scan open and closed string spectra of ten–dimensional superstring theory for a massive spin–2 state. As is well known, string spectra can be thought of as infinite towers of states of increasing mass and spin, so at first glance it is not obvious which state it would be most suitable to choose and no argument seems to present itself against either of the two kinds of spectra. Drawing motivation from [17], where the massive graviton was thought of as being a brane state, we identified $M_{\mu\nu}$ with the first massive spin–2 that appears in the open superstring spectrum and are currently investigating closed string possibilities. With the mass of this string state being $1/\alpha'$, the state identification in question implies the following relation between the bimetric parameters and the string scale α':

$$m_g^2(1+\alpha^2)\,(\beta_1 + 2\beta_2 + \beta_3) \overset{!}{=} \frac{1}{\alpha'} \tag{10}$$

due to (6), whose validity is subject to the result of the amplitudes' computation [14].

As the massless $G_{\mu\nu}$ and massive graviton $M_{\mu\nu}$ are identified with closed and open string states respectively in our setup, the simplest relevant worldsheet topology is that of a disk. At tree–level, the master formula [18, 19] for such a scattering amplitude is essentially given by a suitable integral (over the worldsheet) of the correlator of (normal–ordered) primary conformal fields of the worldsheet conformal field theory that the string defines. Each of these fields, the so–called vertex operators [20], is thought of as creating each of the string states that are taken to be external states of the amplitude. The vertex operators are built using the string coordinates X^μ and ψ^μ, that are spacetime vectors and worldsheet superpartners, and the correlator is evaluated using their correlators on the disk and Wick's theorem. To treat divergences of the amplitude associated with reparametrisation invariance, one also has to to divide the result with the volume V_{CKG} of the conformal Killing group before integration.

In the late '90s, several string amplitudes describing the scattering of massless external states where computed [21–24], while more recently scattering involving a single massive and several massless ones was investigated [25]. Our novelty consists in two points:

1. *all* external states of the amplitudes we have computed are either helicity–2 or spin–2
2. at least *two* external states are massive .

Interestingly, these considerations significantly complicate both the calculations as well as the derivation of the respective effective actions.

The vertex operators for the massless and massive spin–2 states we have used are valid in both ten and four dimensions and are respectively given by (see for example [26])

$$\begin{aligned} V_G^{(0,0)}(z,\bar{z},\varepsilon,q) &= -\tfrac{2g_c}{\alpha'}\,\varepsilon_{\mu\nu}\left[i\bar{\partial}X^\mu + \tfrac{\alpha'}{2}(q\widetilde{\psi})\widetilde{\psi}^\mu(\bar{z})\right] \\ &\quad\times\left[i\partial X^\nu + \tfrac{\alpha'}{2}(q\psi)\psi^\nu(z)\right] e^{iqX(z,\bar{z})} \end{aligned} \tag{11}$$

subject to the on–shell conditions

$$\varepsilon_{\mu\nu}q^\mu = \varepsilon_{\mu\nu}q^\nu = 0 \quad , \quad q^2 = 0 \quad , \quad \varepsilon_{\mu\nu} = \varepsilon_{\nu\mu} \quad , \quad \varepsilon_{\mu\nu}\eta^{\mu\nu} = 0 \tag{12}$$

and [27–29]

$$\begin{aligned} V_M^{(-1)}(x,\alpha,k) &= \tfrac{g_o}{(2\alpha')^{1/2}}T^a\, e^{-\phi(x)}\,\alpha_{\mu\nu}\, i\partial X^\mu(x)\psi^\nu(x)\, e^{ikX(x)} \\ V_M^{(0)}(x,\alpha,k) &= \tfrac{g_o}{(2\alpha')}T^a\,\alpha_{\mu\nu}\big[i\partial X^\mu(x)\partial X^\nu(x) - 2i\alpha'\partial\psi^\mu(x)\psi^\nu(x) \\ &\quad +2\alpha'\,(k\psi)(x)\,\psi^\nu(x)\partial X^\mu(x)\big]e^{ikX(x)} \end{aligned} \tag{13}$$

subject to the on–shell conditions

$$\alpha_{\mu\nu}k^\mu = 0 \quad , \quad k^2 = -\tfrac{1}{\alpha'} \quad , \quad \alpha_{\mu\nu} = \alpha_{\nu\mu} \quad , \quad \alpha_{\mu\nu}\eta^{\mu\nu} = 0\,. \tag{14}$$

In the above, z is the worldsheet coordinate and x that of its boundary, $\varepsilon_{\mu\nu}$ and $\alpha_{\mu\nu}$ the polarisation tensors of the two states and q and k their momenta, g_c and g_o the closed and open string coupling respectively and T^a the generator of the open string gauge group. A superstring vertex operator may appear in different ghost "pictures" [20], denoted by the parentheses next to the symbols of the vertex operators; all such pictures describe the same physical state but several thereof may be needed in order to cancel the worldsheet background ghost charge, depending on the exact scattering process.

4. String scattering

Wa have thus brought the amplitude describing the scattering of one massless and two massive spin–2 states to the form

$$\begin{aligned}\mathcal{A}(2,1) &= \int_{\mathcal{R}}\int_{\mathcal{H}_+}\frac{\mathrm{d}x_1\mathrm{d}x_2\,\mathrm{d}^2z}{V_{\text{CKG}}} \\ &\quad \langle :V_M^{(-1)}(x_1,\alpha_1,k_1)::V_M^{(-1)}(x_2,\alpha_2,k_2)::V_G^{(0,0)}(z,\bar{z},\varepsilon,q):\rangle_{\mathbb{D}_2}\,. \end{aligned} \tag{15}$$

Moreover, because the brane is superheavy, momentum conservation is relevant only along the brane directions and takes the form

$$(k_1+k_2+q_{\parallel})^\mu = 0\,, \tag{16}$$

where the closed string momentum q splits into $q_{\parallel}$ along the brane and a transverse component. Using further the fact that the closed string can be thought of as a copy of two open strings, one may treat this process as a four–point scattering and define Mandelstam variables, to finally conclude that there exists a single kinematic invariant, which we take to be

$$s = -2 + 2\alpha' k_1 k_2\,. \tag{17}$$

By appropriately treating the worldsheet reparametrisation invariance and performing all contractions, we have brought the full amplitude to the form of an integral over the real line x and grouped the various terms into four sets A_i for convenience. The formulas are lengthy and we have given them all explicitly in [14]. Here we only give an example

$$\begin{aligned}\mathbf{A}_4 &= 4^s\,\alpha^1_{\kappa\lambda}\alpha^2_{\rho\sigma}\varepsilon_{\mu\nu}g^{\lambda\sigma}\int_{-\infty}^{\infty}dx\,|x|^{s+2}(x^2+1)^{-s}\frac{(x-i)(x+i)}{(2x)^4}\Big\{A^{\mu\nu\kappa\rho} \\ &\quad +\frac{B^{\mu\nu\kappa\rho}}{(x-i)(x+i)}+\frac{C^{\mu\nu\kappa\rho}}{(x+i)^2}+\frac{\widetilde{\Delta}^{\mu\nu\kappa\rho}}{(x-i)^2}+i\frac{E^{\mu\nu\kappa\rho}}{x+i}+i\frac{F^{\mu\nu\kappa\rho}}{x-i}\Big\}\,,\end{aligned} \tag{18}$$

where for instance

$$A^{\mu\nu\kappa\rho} = -\tfrac{1}{16}\alpha'^3\,qDq\,D^{\mu\nu}k_2^\kappa k_1^\rho + \tfrac{1}{8}\alpha'^2\,D^{\mu\nu}\Big(k_1^\rho k_2^\kappa + \tfrac{1}{4}g^{\kappa\rho}\,qDq\Big) - \tfrac{1}{16}\alpha' D^{\mu\nu}g^{\kappa\rho}\,, \tag{19}$$

where D is a matrix capturing the properties of the worldsheet boundary. We have computed all 36 kinematic packages such as $A^{\mu\nu\kappa\rho}$; they depend on the momenta of the external states, are exact expressions and we have arranged them in orders of α'.

After partial fractioning and evaluation of the integrals, we have brought the involved expressions to forms such as

$$\begin{aligned}\mathbf{A}_4 &= \tfrac{1}{16}\,4^s\Big\{2A\,\frac{\sqrt{\pi}2^{-s}s\,\Gamma\left(\frac{s-1}{2}\right)}{\Gamma\left(\frac{s}{2}+1\right)} - (C+\widetilde{\Delta})\,\frac{\sqrt{\pi}2^{-s}\Gamma\left(\frac{s-1}{2}\right)}{\Gamma\left(\frac{s}{2}+1\right)} \\ &\quad +(E-F)\,\frac{(s-1)\left[\Gamma\left(\frac{s-1}{2}\right)\right]^2}{4\Gamma(s)}\Big\}\,,\end{aligned} \tag{20}$$

where, for example,

$$E = \alpha^1_{\kappa\lambda}\alpha^2_{\rho\sigma}\varepsilon_{\mu\nu}\,g^{\lambda\sigma}\left[2E^{\mu\nu\kappa\rho}+B^{\mu\nu\kappa\rho}\right]. \tag{21}$$

We have namely computed the *full* amplitude that is valid in both ten and four dimensions and which schematically takes the form

$$\mathbf{A}_i = \mathcal{K}(k_1, k_2, q; \alpha') \times \mathcal{I}(s)\,, \tag{22}$$

where $\mathcal{K}(k_1, k_2, q; \alpha')$ are objects depending on the momenta of the external states and on α' explicitly and $\mathcal{I}(s)$ are the values of the integrals, whose dependence on α' is obscure.

In a similar fashion, we have computed the amplitude corresponding to the scattering of three massive spin–2 states; the result is [14]

$$\begin{aligned}\mathcal{A}(3,0) \;=\; & \tfrac{g_o}{4\alpha'^3}\,\mathrm{Tr}(T^{a_1}\{T^{a_2},T^{a_3}\})\Big\{3\,(2\alpha')^2\,\mathrm{Tr}(\alpha^1\cdot\alpha^2\cdot\alpha^3) + (2\alpha')^3\times \\ & \Big[(k_1\cdot\alpha^2\cdot k_1)(\alpha^3\cdot\alpha^1) + (k_2\cdot\alpha^3\cdot k_2)(\alpha^2\cdot\alpha^1) + (k_3\cdot\alpha^1\cdot k_3)(\alpha^2\cdot\alpha^3) \\ & +3\,k_1\cdot\alpha^2\cdot\alpha^1\cdot\alpha^3\cdot k_2 + 3\,k_2\cdot\alpha^3\cdot\alpha^2\cdot\alpha^1\cdot k_3 + 3\,k_3\cdot\alpha^1\cdot\alpha^3\cdot\alpha^2\cdot k_1\Big] \\ & +(2\alpha')^4\Big[(k_1\cdot\alpha^2\cdot k_1)(k_2\cdot\alpha^3\cdot\alpha^1\cdot k_3) + (k_2\cdot\alpha^3\cdot k_2)(k_3\cdot\alpha^1\cdot\alpha^2\cdot k_1) \\ & +(k_3\cdot\alpha^1\cdot k_3)(k_1\cdot\alpha^2\cdot\alpha^3\cdot k_2)\Big]\Big\}\,.\end{aligned} \tag{23}$$

At this point let us recall that the derivative cubic self–interactions of $G_{\mu\nu}$ and $M_{\mu\nu}$ are identical and can be traced back to the linearisation of the Einstein–Hilbert action around Minkowski. This implies that the two–momenta terms in (23) should be the same as those in the three–graviton amplitude. However, upon comparing (23) with the universal three–graviton string amplitude [3, 30–34], we observed a coefficient discrepancy: while the same set of terms appear in both cases, the coefficients do not match. This is already a sign that the open string state we have used does not interact as the massive spin–2 of bimetric theory does.

5. Effective actions and comparison

To compare with bimetric theory, we have considered $D^{\mu\nu} = g^{\mu\nu}$ and $U(1)$ as the brane gauge group and made the following replacements in our results for $\mathcal{A}(2,1)$ and $\mathcal{A}(3,0)$:

$$\varepsilon_{\mu\nu} \to G_{\mu\nu} \quad , \quad \alpha^{1,2}_{\mu\nu} \to M_{\mu\nu} \quad , \quad k_\mu\,, q_\mu \to i\partial_\mu\,. \tag{24}$$

Our aim being the extraction of Lagrangians valid at low–energy scales, the standard limit to be taken at the level of amplitudes is

$$\alpha' \to 0\,. \tag{25}$$

However, in the case of $\mathcal{A}(2,1)$, the dependence of $\mathcal{I}(s)$ on α' is given through s, which contains the term $\alpha' k_1 \cdot k_2$: this object becomes of order 1 in the limit (25), since the mass of the external states is $1/\alpha'$. Expanding, therefore, the expressions $\mathcal{I}(s)$ in small s and substituting via (17) in the amplitudes' results cannot yield a meaningful low–energy truncation at an arbitrary order in α' [14].

Instead, we have used momentum conservation to expand $\mathcal{I}(s)$ in small

$$s = -2\alpha'\, k_1 \cdot q\,, \tag{26}$$

which is well–behaved as it involves the massless closed string momentum; we have obtained [14]

$$\begin{aligned}\mathcal{A}(2,1) \;=\;& g_c \Big\{ -2\,\mathrm{Tr}(\alpha^1\cdot\alpha^2)\,\varepsilon_{\mu\nu}k_1^\mu k_2^\nu + 2(\varepsilon\cdot\alpha^2\cdot\alpha^1)_{\mu\nu}k_1^\mu k_2^\nu \\ & +2(\varepsilon\cdot\alpha^1\cdot\alpha^2)_{\mu\nu}k_1^\nu k_2^\mu + 2(\varepsilon\cdot\alpha^2\cdot\alpha^1)_{\mu\nu}\,k_2^\mu q^\nu + 2(\varepsilon\cdot\alpha^1\cdot\alpha^2)_{\mu\nu}\,k_1^\mu q^\nu \\ & +2\,\mathrm{Tr}(\varepsilon\cdot\alpha^1\cdot\alpha^2)\,(k_1\cdot q) + \big[\,\mathrm{Tr}(\varepsilon\cdot\alpha^2)\alpha^1_{\mu\nu} - 2(\alpha^1\cdot\varepsilon\cdot\alpha^2)_{\mu\nu} \\ & +\mathrm{Tr}(\varepsilon\cdot\alpha^1)\alpha^2_{\mu\nu}\big]\,q^\mu q^\nu\Big\} + \mathcal{O}\left(\alpha'^3\right)\,. \end{aligned} \tag{27}$$

Finally, the corresponding effective actions up to second order in α' we have extracted (as well as the universal cubic graviton self–interactions) are given by [14]

$$\begin{aligned}\mathcal{L}^{\mathrm{eff}}_{\mathrm{G}^3} &= g_c\, G^{\mu\nu}\big[\partial_\mu G_{\rho\sigma}\partial_\nu G^{\rho\sigma} - 2\partial_\nu G_{\rho\sigma}\partial^\sigma G^\rho_\mu\big] \\ \mathcal{L}^{\mathrm{eff}}_{\mathrm{GM}^2} &= g_c\,\Big[G^{\mu\nu}\big(\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - 4\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu\big) \\ &\qquad +M^{\mu\nu}\big(\partial_\mu G_{\rho\sigma}\partial_\nu M^{\rho\sigma} - \partial_\rho G_{\mu\sigma}\partial_\nu M^{\rho\sigma}\big)\Big] \\ \mathcal{L}^{\mathrm{eff}}_{\mathrm{M}^3} &= \frac{g_o}{\alpha'}\Big\{\big[M^3\big] + 2\alpha' M^{\mu\nu}\big[\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - 3\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu\big] \\ &\qquad +4\alpha'^2\,\partial^\mu\partial^\nu M_{\rho\sigma}\partial^\rho M^\kappa_\nu\partial^\sigma M_{\mu\kappa}\Big\}\,. \end{aligned} \tag{28}$$

Upon comparing with (9), we have thus observed that, while ghost–free bimetric theory and the superstring provide the same set of cubic terms for the string states we have used, there appear mismatches at the level of coefficients in both the mixed and self–interactions. In the case of the massive spin–2 self–interactions, the numerical discrepancy 2 vs 3 is strikingly reminiscent of the van Dam, Veltman, Zakharov discontinuity [35, 36], in the form that the latter appears in the massless limit of the massive spin–2 FP propagator; nevertheless, that is related to the mass term of the graviton in a straightforward manner, while our own discrepancy is associated with derivative interactions.

6. Conclusions

In our original published work [14] reviewed in the present proceedings material, we have computed for the first time tree–level string scattering amplitudes describing the interactions of massless and massive spin–2 string states, as well as showed that is possible to extract the corresponding low–energy effective actions, despite the fact that the massives states' mass strongly depends on the string scale. We have, moreover, showed that massive spin–2 states belonging to the open string spectrum do not interact at low–energies as the massive spin–2 field of ghost–free bimetric theory does and we cannot, therefore, extract information on the relation between the bimetric parameters and the string scale using the chosen open string state for the massive spin–2 field. The reasons as to why this is the case remain a puzzle, but let us highlight that the conclusion to be drawn is that graviton derivative interactions are very different from those of brane spin–2 states; the kinematics of the latter at low energies seem to not respect diffeomorphism invariance, even for the simple case of the $U(1)$ brane gauge group that we have considered. With this in mind, we are currently investigating the already promising scenario of the massive spin–2 field identified with a bulk string state, much like the graviton.

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