PROCEEDINGS OF SCIENCE

Very Special Relativity Axial Anomaly

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We study the axial anomaly in Very Special Relativity Electrodynamics using various regularizations: Pauli-Villars and dimensional regularization of ultraviolet divergences and Mandelstam-Leibbrandt regularization of infrared divergences. We get the 2 and 4 dimensional anomaly. We explicitly show that this procedure preserves charge conservation. VSR corrections are absent from axial anomaly in 2 and 4 dimensions. Moreover, the anomaly in the path integral approach is obtained.

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1. Introduction

The discovery of neutrino oscillations showed that the neutrinos have mass whereas in the SM they are massless[2].

Although the Standard Model of Particle Physics(SM) has been verified by the discovery of the Higgs boson at CERN[1], neutrino's masses remain as one of the most important problems of Particle Physics. Since neutrinos appear to have left handed chirality, this is not a simple task.

The seesaw mechanism is a popular scheme to obtain massive neutrinos[3]. However, new interactions and particles are required.

One possibility to have massive chiral neutrinos is Very Special Relativity(VSR)[4]

VSR assumes that the true symmetry of Nature is not the full Lorentz group, but some of its subgroups. The most interesting of these subgoups are $Sim(2)$ and $Hom(2)$. Using these subgroups new terms are allowed such that the neutrino get a mass, preserving its chirality[5].

Various applications of VSR have been considered, like the inclusion of supersymmetry [6, 7], curved spaces [8, 9], noncommutativity [10, 11] , dark matter [12] and also in cosmology [13].

Some time ago, we proposed the SM with VSR[14] (VSRSM).Its particle composition and interactions are the same as in the SM, but neutrinos can have a VSR mass without lepton number violation.

Loop computations in VSR are non trivial though. New infrared divergences appear and they have to be regularized. We studied how to do so using the calculation of integrals in the Mandelstam-Leibbrandt (ML) prescription[15],[16] introduced in [17], in [18] and [19]. The Ward identities corresponding to the gauge and the $Sim(2)$ symmetry of the model are preserved.

Two years ago, we applied these techniques to the Schwinger model in VSR [20] and to the photon mass in VSR [21].

As the SM, the VSRSM is a chiral gauge symmetry theory,so the presence of chiral anomalies may destroy the consistency of the model, because the gauge symmetry will be lost and renormalizability and unitarity could not be simultaneously realized.Therefore a very important test that it has to satisfy is the cancellation of axial anomalies.

In [20] we did a computation of the two dimensional axial anomaly. We obtained that the vector current is conserved and the axial anomaly get a correction from VSR in the form of a multiplicative factor.

The authors of [22] tried to compute the axial anomaly in four dimensions using the prescription to treat γ^5 introduced in [23]. They claim that there is an anomaly in the vector current as well as in the axial vector current. However their computation missed two important graphs.(Please see chapter IV).

In this work we review the calculations contained in [24]. We explain how to compute the axial anomaly in two and four dimensions using Pauli-Villars (PV) and dimensional(DR) regularization of ultraviolet divergences and ML prescription for infrared divergences. Extra graphs appear, due to the non-locality of the currents. They must be there to preserve the Schwinger-Dyson equations(chapter VIII). We show explicitly that the vector current is conserved and that the axial anomaly is the same we get in Lorentz invariant Electrodynamics, without any correction from VSR. Our result relies on two properties of the ML prescription: First, it allows shifting of the loop momentum variable(which implies gauge invariance) and second, it respects naive power counting.

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One important conclusion to be drawn from this is that VSRSM is consistent because is free from axial anomalies.

A gauge invariant mass term for the gauge field is possible in VSR[14, 25, 26]. Here, we did not include such a mass term for the photon because it will not affect the axial anomaly, since the axial anomaly is due to a loop of fermions.

The paper is written as follows. In chapter II we define the lagrangian of VSR Electrodynamics and derive the Feynman rules that will be used to compute the anomalies.In chapter III we compute the axial anomaly in two dimensional space time. In chapter IV we study the axial anomaly in four dimensions. In chapter V we study the axial anomaly in 2d using DR. In chapter VI, we derive the axial anomaly in 4d, using DR. In chapter VII we present the derivation of the axial anomaly using the path integral. In chapter VIII we derive the Schwinger-Dyson identity for the product of three currents. Extra terms appear. This is an important new result of our calculation. Finally in chapter IX we draw some conclusions.

2. VSRSM Electrodynamics

The Electrodynamics sector of the VSRSM in the Feynman gauge.

$$\mathcal{L} = \bar{\psi}\left(i\left(\not{D} + \frac{1}{2}\not{n}m^2(n\cdot D)^{-1}\right) - M\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{(\partial_\mu A_\mu)^2}{4} \tag{1}$$

$$D_\mu = \partial_\mu - ieA_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

The vector current(electric charge conservation) is:

$$j^\mu = \bar{\psi}\gamma^\mu\psi + \frac{1}{2}m^2\left(\frac{1}{n\cdot D^\dagger}\bar{\psi}\right)\not{n}n^\mu\left(\frac{1}{n\cdot D}\psi\right)$$

The axial vector current is:

$$j^{\mu 5} = \bar{\psi}\gamma^\mu\gamma^5\psi + \frac{1}{2}m^2\left(\frac{1}{n\cdot D^\dagger}\bar{\psi}\right)\not{n}n^\mu\gamma^5\left(\frac{1}{n\cdot D}\psi\right)$$

Both currents are conserved at the classical level[20]. We are interested in computing expectation values of these currents.

To get the Feynman rules we use the expansion of $(n.D)^{-1}$ both in the currents and the lagrangian.

$$(n.D)^{-1} = (1 + ie(n.\partial)^{-1}(n.A) + (ie)^2(n.\partial)^{-1}(n.A)(n.\partial)^{-1}(n.A) + (ie)^3(n.\partial)^{-1}(n.A)(n.\partial)^{-1}(n.A)(n.\partial)^{-1}(n.A))(n.\partial)^{-1} + \ldots$$

The Feynman rules are listed in Appendix A.

3. Two dimensional axial anomaly

In this case we have to compute the expectation value of the axial vector current in a background field A_ν. We use the convention of [27],$\epsilon^{01} = +1$.

$$< j^{5\nu}(q) >= \int d^2x < j^{5\nu}(x) > e^{iqx} = (-ie)^{-1}i\Pi^{5\mu\nu}(q)A_\mu \tag{2}$$

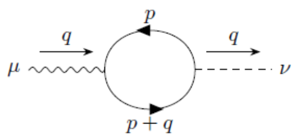

Figure 1

The contribution to the two dimensional anomaly in VSR Electrodynamics is given by the two graphs (Figure 1 and Figure 2):

$$i\Pi^{15\mu\nu} = -(-ie)^2 \int dp\, \mathrm{Tr}\{\left[\gamma^\mu + \frac{1}{2}n^\mu\,(\not{n})\,m^2(n.(p+q))^{-1}(n.p)^{-1}\right] \frac{i\left(\not{p} + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot p}\right)}{p^2 - M^2 - m^2 + i\varepsilon}\left[\gamma^\nu + \frac{1}{2}n^\nu\,(\not{n})\,m^2(n.(p+q))^{-1}(n.p)^{-1}\right]\gamma^5 \frac{i\left((\not{p}+\not{q}) + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot(p+q)}\right)}{(p+q)^2 - M^2 - m^2 + i\varepsilon}\} \quad (3)$$

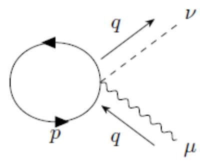

Figure 2

$$i\Pi^{25\mu\nu} = (-1)(ie)^2 n^\mu n^\nu i \int dp (n.p)^{-1}(n.p)^{-1}[(n.(q+p))^{-1} + (n.(-q+p))^{-1}]\,\mathrm{Tr}\{\frac{1}{2}\not{n}m^2 \frac{i\left(\not{p} + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot p}\right)}{p^2 - M^2 - m^2 + i\varepsilon}\gamma^5\} \quad (4)$$

Notice that Figure 2 is absent from the standard computation of the axial anomaly. It is the result of the non-locality of the currents. (Please see chapter VIII).

To compute the axial anomaly we will use Pauli-Villars regularization and Mandelstam-Leibbrandt prescription to treat infrared divergences. We will follow reference [28].

Notice that equation (3) is logarithmically divergent and equation (4) is finite.

It is easy to check that formally:

$$q_\mu(\Pi^{15\mu\nu} + \Pi^{25\mu\nu}) = 0$$

if shift of the integration variable $p \to p + k$ is allowed. Here k is a constant vector. This would be true if the integral (3) would be finite.(Appendix B).

Introduce a Pauli-Villars particle of mass $\bar{M}$ and define the regularized amplitude:

$$\Pi^{5R\mu\nu}(M,\bar{M},q) = \Pi^{15\mu\nu}(M,q) + \Pi^{25\mu\nu}(M,q) - \Pi^{15\mu\nu}(\bar{M},q) - \Pi^{25\mu\nu}(\bar{M},q)$$

Since $\Pi^{5R\mu\nu}(M, \bar{M}, q)$ is finite, it satisfies the naive Ward identity(electric charge conservation):

$$q_\mu \Pi^{5R\mu\nu}(M, \bar{M}, q) = 0$$

On the other hand, the axial Ward identity is, formally:

$$i(\Pi^{15\mu\nu} + \Pi^{25\mu\nu})q_\nu = 2M\mathcal{A}(M, q)^\mu$$

$$= 2M(-ie)^2 \int dp \, \mathrm{Tr} \left\{ \left[\gamma^\mu + \tfrac{1}{2} n^\mu \, (\not{n}) \, m^2 (n.(p+q))^{-1} (n.p)^{-1}\right] \frac{i\left(\not{p}+M-\frac{m^2}{2}\frac{\not{n}}{n\cdot p}\right)}{p^2-M^2-m^2+i\varepsilon} \gamma^5 \frac{i\left((\not{p}+\not{q})+M-\frac{m^2}{2}\frac{\not{n}}{n\cdot(p+q)}\right)}{(p+q)^2-M^2-m^2+i\varepsilon} \right\} \tag{5}$$

if shift of the integration variable $p \to p + k$ is allowed.

Therefore the regularized amplitude satisfies:

$$i\Pi^{5R\mu\nu}(M, \bar{M}, q)q_\nu = 2M\mathcal{A}(M, q)^\mu - 2\bar{M}\mathcal{A}(\bar{M}, q)^\mu$$

Since the original amplitude is obtained formally as $lim_{\bar{M}\to\infty}$, the axial anomaly is given by:

$$B^\mu = lim_{\bar{M}\to\infty}(-2\bar{M}\mathcal{A}(\bar{M}, q)^\mu)$$

Now, we compute (5). First notice that after computing the trace, the integral is finite. A tipical term containing the vector n^μ is of the form:

$$C^\mu = 2M^2 m^2 (-ie)^2 \varepsilon^{\mu\alpha} n_\alpha \int dp \frac{1}{p^2 - M^2 - m^2 + i\varepsilon} \frac{1}{(p+q)^2 - M^2 - m^2 + i\varepsilon} \frac{1}{n.p}$$

Now we recall an important property of ML prescription. It preserves naive power counting. According to this, $C^\mu \sim M^{-1}$ for large M.

In the same way, we can check easily that all terms containing n^μ vanish when $M \to \infty$.

It remains the Lorentz invariant term:

$$i\Pi^{5\mu\nu}(q)q_\nu = lim_{\bar{M}\to\infty}(-4e^2)\bar{M}^2 \varepsilon^{\alpha\mu} q_\alpha \int dp \frac{1}{p^2 - \bar{M}^2 - m^2 + i\varepsilon} \frac{1}{(p+q)^2 - \bar{M}^2 - m^2 + i\varepsilon} = -i\frac{e^2}{\pi}\varepsilon^{\alpha\mu} q_\alpha \tag{6}$$

$$q_\nu < j^{5\nu} >= \frac{1}{-ie} i\Pi^{5\mu\nu}(q) A_\mu q_\nu = \frac{e}{\pi}\varepsilon^{\alpha\mu} q_\alpha A_\mu \tag{7}$$

Equation (7) is the standard Lorentz invariant result[27].

We want to comment on a previous computation of the anomaly in [20]. There and here, the vector current is conserved, but a different axial anomaly is obtained. This difference may be a result of different normalization conditions[28] or the extra freedom we have when Lorentz symmetry is broken[29] .

It is clear though that the procedure used in [20] does not respect naive power counting of the loop integrals.

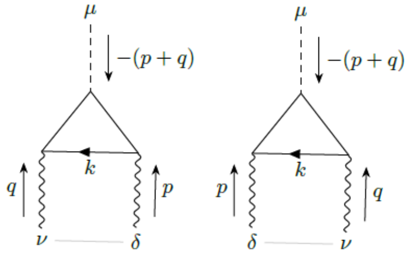

Figure 3: $\Pi^{5\mu\nu\delta}$

4. Four dimensional axial anomaly

We compute:

$$\int d^4x e^{-irx} < p,q|j^{\mu 5}(x)|0 >= (2\pi)^4 \delta(-r+p+q)\varepsilon^*_\nu(q)\varepsilon^*_\delta(p) i\Pi^{\mu\nu\delta}$$

There are four graphs that contribute to the axial anomaly in four dimensions (Figure 3-6). Notice that in [22] Figure 5,6 are missing. They are fundamental to satisfy the Ward identity for the vector current(charge conservation) as well as the right computation of the axial anomaly.

$$i\Pi^{15\mu\nu\delta} = -(-ie)^2 \int dk \, \mathrm{Tr}\{\left[\gamma^\mu + \frac{1}{2} n^\mu \, (\not{n}) \, m^2 (n.(k+q))^{-1} (n.(k-p))^{-1}\right] \gamma^5 \frac{i\left((\not{k}+\not{q}) + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot(k+q)}\right)}{(k+q)^2 - M^2 - m^2 + i\varepsilon}$$
$$\left[\gamma^\nu + \frac{1}{2} n^\nu \, (\not{n}) \, m^2 (n.(k+q))^{-1} (n.k)^{-1}\right] \frac{i\left(\not{k} + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot k}\right)}{k^2 - M^2 - m^2 + i\varepsilon}$$
$$\left[\gamma^\delta + \frac{1}{2} n^\delta \, (\not{n}) \, m^2 (n.(k-p))^{-1} (n.k)^{-1}\right] \frac{i\left(\not{k} - \not{p} + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot(k-p)}\right)}{(k-p)^2 - M^2 - m^2 + i\varepsilon}\} + (p,\delta) \to (q,\nu) \quad (8)$$

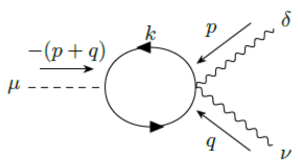

Figure 4: $\Pi^{25\mu\nu\delta}$

$$i\Pi^{25\mu\nu\delta} = (-1)(ie)^2 n^\delta n^\nu i \int dk (n.k)^{-1} (n.(k-p-q))^{-1} [(n.(k-q))^{-1} + (n.(k-p))^{-1}]$$
$$\mathrm{Tr}\{\frac{1}{2}\not{n} m^2 \frac{i\left(\not{k} - \not{p} - \not{q} + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot(k-p-q)}\right)}{(k-p-q)^2 - M^2 - m^2 + i\varepsilon} \left[\gamma^\mu + \frac{1}{2} n^\mu \, (\not{n}) \, m^2 (n.k)^{-1} (n.(k-p-q))^{-1}\right] \gamma^5$$
$$\frac{i\left(\not{k} + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot k}\right)}{k^2 - M^2 - m^2 + i\varepsilon}\} \quad (9)$$

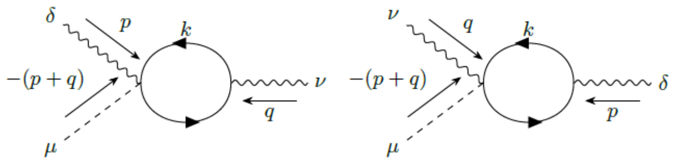

Figure 5: $\Pi^{35\mu\nu\delta}$

$$i\Pi^{35\mu\nu\delta} = (-1)(ie)^2 n^\delta n^\mu i \int dk (n.k)^{-1}(n.(k-q))^{-1}[(n.(k-q-p))^{-1} + (n.(k+p))^{-1}]$$
$$\mathrm{Tr}\left\{\frac{1}{2}\not{n} m^2\gamma^5 \frac{i\left(\not{k}+M-\frac{m^2}{2}\frac{\not{n}}{n\cdot k}\right)}{k^2-M^2-m^2+i\varepsilon}\left[\gamma^\nu + \frac{1}{2}n^\nu\,(\not{n})\,m^2(n.k)^{-1}(n.(k-q))^{-1}\right]\frac{i\left(\not{k}-\not{q}+M-\frac{m^2}{2}\frac{\not{n}}{n\cdot(k-q)}\right)}{(k-q)^2-M^2-m^2+i\varepsilon}\right\}$$
$$+(p,\delta)\to(q,\nu)\quad(10)$$

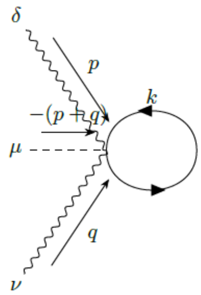

Figure 6: $\Pi^{45\mu\nu\delta}$

$$i\Pi^{45\mu\nu\delta} = (-1)(ie)^2 n^\nu n^\mu n^\delta i\int dk\{\frac{1}{n.k}\frac{1}{n.k}[\frac{1}{n.(k+p+q)}\frac{1}{n.(k+p)} + \frac{1}{n.(k+p+q)}\frac{1}{n.(k+q)} +$$
$$\frac{1}{n.(k-p)}\frac{1}{n.(k-p-q)} + \frac{1}{n.(k-q)}\frac{1}{n.(k+p)} + \frac{1}{n.(k-p)}\frac{1}{n.(k+q)} + \frac{1}{n.(k-q)}\frac{1}{n.(k-p-q)}]\}$$
$$\mathrm{Tr}\left\{\frac{1}{2}\not{n} m^2\gamma^5[i]\frac{i\left(\not{k}+M-\frac{m^2}{2}\frac{\not{n}}{n\cdot(k)}\right)}{(k)^2-M^2-m^2+i\varepsilon}\right\}\quad(11)$$

Notice that $\Pi^{25\mu\nu\delta}, \Pi^{35\mu\nu\delta}, \Pi^{45\mu\nu\delta}$ are ultraviolet finite. Only $\Pi^{15\mu\nu\delta}$ is linearly divergent as in the Lorentz invariant electrodynamics.

Figures (4-6) are new additions, due to the non-locality of the currents(chapter VIII).

To compute the axial anomaly we will use Pauli-Villars regularization and Mandelstam-Leibbrandt prescription to treat infrared divergences. We will follow reference [28].

It is easy to check that formally:

$$(\Pi^{15\mu\nu\delta} + \Pi^{25\mu\nu\delta} + \Pi^{35\mu\nu\delta} + \Pi^{45\mu\nu\delta})p_\delta = 0$$

if shift of the integration variable $k \to k + Q$ is allowed. Here Q is a constant vector.[1]

Introduce a Pauli-Villars particle of mass $\bar{M}$ and define the regularized amplitude:

$$\Pi^{5R\mu\nu\delta}(M, \bar{M}, p, q) = (\Pi^{15\mu\nu\delta} + \Pi^{25\mu\nu\delta} + \Pi^{35\mu\nu\delta} + \Pi^{45\mu\nu\delta})(M, p, q) - (\Pi^{15\mu\nu\delta} + \Pi^{25\mu\nu\delta} + \Pi^{35\mu\nu\delta} + \Pi^{45\mu\nu\delta})(\bar{M}, p, q)$$

Since $\Pi^{5R\mu\nu\delta}(M, \bar{M}, p, q)$ is finite, it satisfies the naive Ward identity(electric charge conservation):

$$\Pi^{5R\mu\nu\delta}(M, \bar{M}, p, q)p_\delta = 0$$

Besides, the axial Ward identity formally is,if shift of the integration variable $k \to k+Q$ is allowed:

$$-(p+q)_\mu i(\Pi^{15\mu\nu\delta} + \Pi^{25\mu\nu\delta} + \Pi^{35\mu\nu\delta} + \Pi^{45\mu\nu\delta}) = 2M\mathcal{A}(M,p,q)^{\nu\delta} =$$
$$-2M(-ie)^2 \int dk\{\mathrm{Tr}\{\gamma^5 \frac{i\left((\not{k}+\not{p}+\not{q}) + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot(k+p+q)}\right)}{(k+p+q)^2 - M^2 - m^2 + i\varepsilon}\left[\gamma^\nu + \frac{1}{2}n^\nu\,(\not{n})\,m^2(n.(k+p+q))^{-1}(n.(k+p))^{-1}\right]$$
$$\frac{i\left(\not{k}+\not{p}+M - \frac{m^2}{2}\frac{\not{n}}{n\cdot(k+p)}\right)}{(k+p)^2 - M^2 - m^2 + i\varepsilon}\left[\gamma^\delta + \frac{1}{2}n^\delta\,(\not{n})\,m^2(n.(k+p))^{-1}(n.k)^{-1}\right]\frac{i\left(\not{k}+M - \frac{m^2}{2}\frac{\not{n}}{n\cdot(k)}\right)}{(k)^2 - M^2 - m^2 + i\varepsilon}\} + (p,\delta) \to (q,\nu)\} \quad (12)$$

$$(-2M)(ie)^2 n^\delta n^\nu i \int dk(n.k)^{-1}(n.(k-p-q))^{-1}[(n.(k-q))^{-1} + (n.(k-p))^{-1}]$$
$$\mathrm{Tr}[\frac{1}{2}\not{n}m^2\frac{i\left(\not{k}-\not{p}-\not{q}+M - \frac{m^2}{2}\frac{\not{n}}{n\cdot(k-p-q)}\right)}{(k-p-q)^2 - M^2 - m^2 + i\varepsilon}\gamma^5$$
$$\frac{i\left(\not{k}+M - \frac{m^2}{2}\frac{\not{n}}{n\cdot k}\right)}{k^2 - M^2 - m^2 + i\varepsilon}] \quad (13)$$

The term (13) is convergent and has zero trace in four dimensions. So it vanishes.

Therefore the regularized amplitude satisfies:

$$-(p+q)_\mu i\Pi^{5R\mu\nu\delta}(M, \bar{M}, p, q) = 2M\mathcal{A}(M,p,q)^{\nu\delta} - 2\bar{M}\mathcal{A}(\bar{M},p,q)^{\nu\delta}$$

Since the original amplitude is obtained formally as $lim_{\bar{M}\to\infty}$, the axial anomaly is given by:

$$A^{\nu\delta} = lim_{\bar{M}\to\infty}(-2\bar{M}\mathcal{A}(\bar{M},p,q)^{\nu\delta})$$

After computing the trace, we use ML prescription to regulate the infrared divergences. $\mathcal{A}(M,p,q)^{\nu\delta}$ is ultraviolet finite

A remarkable property of ML prescription is that preserve naive power counting. Using this property, we can easily show that all terms containing n^μ in $\mathcal{A}(M,p,q)^{\nu\delta}$ are smaller than M^{-2} for large M, so they do not contribute to the axial anomaly.

$\mathcal{A}^{\nu\delta} = \lim_{\bar{M}\to\infty}$

$$8\bar{M}^2\varepsilon^{\nu\delta\alpha\beta}p_\alpha q_\beta(-ie)^2\int d^4k\frac{1}{(k+p)^2 - m^2 - \bar{M}^2}\frac{1}{k^2 - m^2 - \bar{M}^2}\frac{1}{(k+p)^2 - m^2 - \bar{M}^2}$$

[1]This is true if we use DR as in chapter V and VI.

That is:

$$\mathcal{A}^{\nu\delta} = -(ie)^2 \frac{i}{2\pi^2} \varepsilon^{\nu\delta\beta\mu} p_\beta q_\mu$$

This is the standard result [28][27].

We see that Pauli-Villars regularization of ultraviolet divergences and Mandelstam-Leibbrandt regularization of infrared divergences preserve the Ward identity for the vector current(electric charge conservation) as well as the standard anomaly for the axial current, without modification from VSR terms.

5. Two dimensional axial anomaly in dimensional regularization

To treat γ^5 we follow the prescription of [23]. That is, in any number of dimensions

$$\begin{aligned} \gamma^5 &= i\gamma^0\gamma^1 \\ \{\gamma^5, \gamma^\mu\} = 0.\mu = 0, 1; &\quad [\gamma^5, \gamma^\mu] = 0, \mu = 2, 3 \ldots, d \\ q_\mu, n^\mu \text{ are two dimensional vectors .} &\quad p_\mu \text{ is } d - \text{dimensional} \end{aligned}$$

$$\begin{aligned} i\Pi^{15\mu\nu} q_\nu = -(-ie)^2 \int dp \operatorname{Tr}\{ &\left[\gamma^\mu + \frac{1}{2} n^\mu \, (\not{n}) \, m^2 (n.(p+q))^{-1} (n.p)^{-1}\right] \frac{i\left(\not{p} + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot p}\right)}{p^2 - M^2 - m^2 + i\varepsilon} \\ &(\not{q} + \frac{1}{2} n.q \, (\not{n}) \, m^2 (n.(p+q))^{-1} (n.p)^{-1}) \gamma^5 \frac{i\left((\not{p} + \not{q}) + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot(p+q)}\right)}{(p+q)^2 - M^2 - m^2 + i\varepsilon} \} \end{aligned}$$

Write

$$\not{p} = \not{p}_1 + \not{p}_2; \text{1 lives in two dimensions, 2 lives in } d-2 \text{ dimensions}$$

Now we use the identity:

$$\begin{aligned} &\left[\not{q} + \frac{1}{2} n.q \, (\not{n}) \, m^2 (n.(p+q))^{-1} (n.p)^{-1}\right] = \\ &\left[\not{p} + \not{q} - \frac{1}{2}\not{n} m^2 (n.(p+q))^{-1} - M - \left(\not{p} - \frac{m^2 \not{n}}{2n.p} - M\right)\right] \end{aligned} \tag{14}$$

$$\begin{aligned} \Pi^{15\mu\nu} q_\nu = -(-ie)^2 \int dp \operatorname{Tr}\{ &-\left[\gamma^\mu + \frac{1}{2} n^\mu \, (\not{n}) \, m^2 (n.(p+q))^{-1} (n.p)^{-1}\right] \gamma_5 \frac{i\left((\not{p} + \not{q}) + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot(p+q)}\right)}{(p+q)^2 - M^2 - m^2 + i\varepsilon} + \\ &\left[\gamma^\mu + \frac{1}{2} n^\mu \, (\not{n}) \, m^2 (n.(p+q))^{-1} (n.p)^{-1}\right] \frac{i\left(\not{p} + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot p}\right)}{p^2 - M^2 - m^2 + i\varepsilon} \gamma_5 (-) \\ &\left[\left(\not{p}_1 - \not{p}_2 + \not{q} - \frac{1}{2}\not{n} m^2 (n.(p+q))^{-1} - M\right) + 2M\right] \frac{i\left((\not{p} + \not{q}) + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot(p+q)}\right)}{(p+q)^2 - M^2 - m^2 + i\varepsilon} \} \end{aligned} \tag{15}$$

$$= -(-ie)^2 \int dp \, \mathrm{Tr} \left\{ \gamma^5 \left[\gamma^\mu + \frac{1}{2} n^\mu \left(\not{n} \right) m^2 (n.(p+q))^{-1} (n.p)^{-1} \right] \frac{i \left((\not{p}+\not{q}) + M - \frac{m^2}{2} \frac{\not{n}}{n\cdot(p+q)} \right)}{(p+q)^2 - M^2 - m^2 + i\varepsilon} - \right.$$

$$\left. \gamma^5 \left[\gamma^\mu + \frac{1}{2} n^\mu \left(\not{n} \right) m^2 (n.(p+q))^{-1} (n.p)^{-1} \right] \frac{i \left(\not{p} + M - \frac{m^2}{2} \frac{\not{n}}{n\cdot p} \right)}{p^2 - M^2 - m^2 + i\varepsilon} \right\} +$$

$$2M(-ie)^2 \int dp \, \mathrm{Tr} \left\{ \left[\gamma^\mu + \frac{1}{2} n^\mu \left(\not{n} \right) m^2 (n.(p+q))^{-1} (n.p)^{-1} \right] \frac{i \left(\not{p} + M - \frac{m^2}{2} \frac{\not{n}}{n\cdot p} \right)}{p^2 - M^2 - m^2 + i\varepsilon} \gamma_5 \frac{i \left((\not{p}+\not{q}) + M - \frac{m^2}{2} \frac{\not{n}}{n\cdot(p+q)} \right)}{(p+q)^2 - M^2 - m^2 + i\varepsilon} \right\}$$

$$-(-ie)^2 \int dp \, \mathrm{Tr} \left\{ \left[\gamma^\mu + \frac{1}{2} n^\mu \left(\not{n} \right) m^2 (n.(p+q))^{-1} (n.p)^{-1} \right] \frac{i \left(\not{p} + M - \frac{m^2}{2} \frac{\not{n}}{n\cdot p} \right)}{p^2 - M^2 - m^2 + i\varepsilon} \gamma^5 2 \not{p}_2 \frac{i \left((\not{p}+\not{q}) + M - \frac{m^2}{2} \frac{\not{n}}{n\cdot(p+q)} \right)}{(p+q)^2 - M^2 - m^2 + i\varepsilon} \right\}$$

In dimensional regularization we can shift variable $p \to p - q$ in the term (16). Then the addition of terms (16) and (17)is canceled by the contribution of Figure 2.

The anomaly is:

$$A^\mu = -(-ie)^2$$

$$\int dp \, \mathrm{Tr} \left\{ \left[\gamma^\mu + \frac{1}{2} n^\mu \left(\not{n} \right) m^2 (n.(p+q))^{-1} (n.p)^{-1} \right] \frac{i \left(\not{p} + M - \frac{m^2}{2} \frac{\not{n}}{n\cdot p} \right)}{p^2 - M^2 - m^2 + i\varepsilon} \gamma_5 2 \not{p}_2 \frac{i \left((\not{p}+\not{q}) + M - \frac{m^2}{2} \frac{\not{n}}{n\cdot(p+q)} \right)}{(p+q)^2 - M^2 - m^2 + i\varepsilon} \right\} \quad (18)$$

That is:

$$A^\mu = 4(-ie)^2 \int dp \frac{\left[-p_2^2 \varepsilon^{\mu\nu} q_\nu - p_2^2 \frac{1}{2} n^\mu m^2 (n.(p+q))^{-1} (n.p)^{-1} \varepsilon^{\alpha\beta} n_\alpha q_\beta \right]}{(p^2 - M^2 - m^2 + i\varepsilon)((p+q)^2 - M^2 - m^2 + i\varepsilon)}$$

$p_2^2 \sim (d-2)p^2$ when $d \to 2$. The VSR part of the integral is convergent, using ML prescription, so it is zero, when we take $d = 2$.

So only the Lorentz invariant part of the integral contributes to the anomaly.

$$A^\mu = 4e^2 \varepsilon^{\mu\nu} q_\nu \int dp \frac{p_2^2}{p^2 - M^2 - m^2 + i\varepsilon} \frac{}{(p+q)^2 - M^2 - m^2 + i\varepsilon} = e^2 \varepsilon^{\mu\nu} q_\nu \frac{i}{\pi} \quad (19)$$

That is:

$$q_\mu < j^{5\mu}(q) >= \frac{e^2}{-ie} \varepsilon^{\mu\nu} q_\nu \frac{i}{\pi} A_\mu = -\frac{e}{\pi} \varepsilon^{\mu\nu} q_\nu A_\mu = \frac{e}{\pi} \varepsilon^{\nu\mu} q_\nu A_\mu$$

which is the standard result[27].

In Appendix B we study the vector Ward identity. If we use dimensional regularization there, then shifting the integration variable $p- > p + Q$ is allowed. So the naive Ward identity for the vector current is satisfied without anomaly.

6. 4d axial anomaly. Dimensional regularization

In this section we compute the axial anomaly using dimensional regularization. The contribution of Figure 3 is:

$$-(p+q)_\mu i\Pi^{15\mu\nu\delta} = -(-ie)^2\int dk\,\mathrm{Tr}\{\left[-\left(\not p+\not q\right) - \frac{1}{2}(p+q).n\,(\not n)\,m^2(n.(k+q))^{-1}(n.(k-p))^{-1}\right]\gamma^5$$
$$\frac{i\left(\left(\not k+\not q\right)+M-\frac{m^2}{2}\frac{\not n}{n\cdot(k+q)}\right)}{(k+q)^2-M^2-m^2+i\varepsilon}\left[\gamma^\nu+\frac{1}{2}n^\nu\,(\not n)\,m^2(n.(k+q))^{-1}(n.k)^{-1}\right]\frac{i\left(\not k+M-\frac{m^2}{2}\frac{\not n}{n\cdot k}\right)}{k^2-M^2-m^2+i\varepsilon}$$
$$\left[\gamma^\delta+\frac{1}{2}n^\delta\,(\not n)\,m^2(n.(k-p))^{-1}(n.k)^{-1}\right]\frac{i\left(\not k-\not p+M-\frac{m^2}{2}\frac{\not n}{n\cdot(k-p)}\right)}{(k-p)^2-M^2-m^2+i\varepsilon}\}$$

Write $\not k = \not k_1 + \not k_2$

$$\left(\not k_1+\not k_2+\not p+\not q+M\right)\gamma_5 = -\gamma_5\left(\not k_1+\not k_2+\not p+\not q+M\right)+2\gamma_5\not k_2+2M\gamma_5$$

That is the anomaly is:

$$\Gamma^{5\nu\delta}(p,q) =$$
$$2(-ie)^2\int dk\,\mathrm{Tr}\{\gamma^5\not k_2\frac{i\left(\left(\not k+\not p+\not q\right)+M-\frac{m^2}{2}\frac{\not n}{n\cdot(k+p+q)}\right)}{(k+p+q)^2-M^2-m^2+i\varepsilon}\left[\gamma^\nu+\frac{1}{2}n^\nu\,(\not n)\,m^2(n.(k+p+q))^{-1}(n.(k+p))^{-1}\right]$$
$$\frac{i\left(\not k+\not p+M-\frac{m^2}{2}\frac{\not n}{n\cdot(k+p)}\right)}{(k+p)^2-M^2-m^2+i\varepsilon}\left[\gamma^\delta+\frac{1}{2}n^\delta\,(\not n)\,m^2(n.(k+p))^{-1}(n.k)^{-1}\right]\frac{i\left(\not k+M-\frac{m^2}{2}\frac{\not n}{n\cdot(k)}\right)}{(k)^2-M^2-m^2+i\varepsilon}\}\quad(20)$$

To compute the trace, we notice that there must be an even number of $\not k_2$ otherwise the trace vanishes. Assume there are four $\not k_2$

$$\mathrm{Tr}\left\{\gamma^5\not k_2\not k_2\gamma^\nu\not k_2\gamma^\delta\not k_2\right\} = (k_2^2)^2\,\mathrm{Tr}\{\gamma^5\gamma^\nu\gamma^\delta\} = 0$$

That is, only two $\not k_2$ contribute to the trace.

The trace can be written as $\mathrm{Tr} = k_2^2 S$

But $k_2^2 S \sim (d-4)k^2 S$. So if $k^2 S$ is convergent in $d=4$ the contribution of this S vanishes.If we use ML prescription to regularize the infrared divergences we can show that $k^2 S$ is convergent in $d=4$ for all VSR S's, since ML preserves naive power counting. Therefore only Lorentz invariant terms contribute to the anomaly.

Finally the anomaly is:

$$\Gamma^{5\nu\delta}(p,q) = 2(-ie)^2 i^3\int dk k_2^2\frac{\mathrm{Tr}\left\{\gamma_5\left(\not q\right)\gamma_\nu\left(\not p\right)\gamma_\delta\right\}}{(k+p+q)^2-M^2-m^2+i\varepsilon}\frac{1}{(k+p)^2-M^2-m^2+i\varepsilon}\frac{1}{(k)^2-M^2-m^2+i\varepsilon}$$
$$= -i\frac{e^2}{2\pi^2}\varepsilon^{\nu\delta\alpha\mu}p_\alpha q_\mu\quad(21)$$

Therefore

$$< p,q|\partial_\mu j^{5\mu}(0)|0 >= -\frac{e^2}{2\pi^2}\varepsilon^{\mu\nu\alpha\delta}(-iq_\mu)\varepsilon^*_\nu(q)(-ip_\alpha)\varepsilon^*_\delta(p)\quad(22)$$

which is the standard result[27].

Following the same reasoning as in Appendix B, we can study the vector Ward identity in four dimensions. If we use dimensional regularization there, then shifting the integration variable $k->k+Q$ is allowed. So the naive Ward identity for the vector current is satisfied without anomaly.

7. Path integral derivation of the axial anomaly

We use the approach of [30].

The generating functional in the presence of an external field A_μ is;

$$Z = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}e^{i\int d^4x\bar{\psi}i\tilde{\not{D}}\psi}$$

where the gauge invariant and $Sim(2)$ invariant Dirac operator is

$\tilde{\not{D}} = \not{D} + \frac{1}{2}\not{n}m^2(n\cdot D)^{-1}, D_\mu = \partial_\mu - ieA_\mu$

Introduce a basis of eigenvectors of $\tilde{\not{D}}$

$$\tilde{\not{D}}\phi_m = \lambda_m\phi_m, \tag{23}$$

$$\int d^4x\phi_n^\dagger(x)\phi_m(x) = \delta_{nm}, \tag{24}$$

$$\sum_n \phi_n(x)\phi_n^\dagger(y) = \delta(x-y) \tag{25}$$

We can expand

$$\psi(x) = \sum_m a_m\phi_m(x), \quad \bar{\psi}(x) = \sum_m \bar{a}_m{\phi^\dagger}_m(x)$$

The integration measure is defined by:

$$\mathcal{D}\psi\mathcal{D}\bar{\psi} = \prod_m da_m d\bar{a}_m$$

Under the change of variables:

$$\psi'(x) = (1 + i\alpha(x)\gamma^5)\psi(x)$$

we get:

$$\mathcal{D}\psi'\mathcal{D}\bar{\psi}' = \mathcal{J}^{-2}\mathcal{D}\psi\mathcal{D}\bar{\psi} \tag{26}$$

where the jacobian $\mathcal{J}$ is given by:

$$\log\mathcal{J} = i\int d^4x\alpha(x)\sum_n \phi_n^\dagger(x)\gamma^5\phi_n(x) \tag{27}$$

To evaluate it we introduce a gauge invariant and Sim(2) invariant regularization:

$$\sum_n \phi_n^\dagger(x)\gamma^5\phi_n(x) = \lim_{M\to\infty}\sum_n \phi_n^\dagger(x)\gamma^5\phi_n(x)e^{-\frac{\lambda_n^2}{M^2}} =$$

$$\lim_{M\to\infty} < x\left|\mathrm{Tr}\left\{\gamma^5 e^{-\frac{(\tilde{\not{D}})^2}{M^2}}\right\}\right|x >$$

Tr traces over Dirac indices.

Since this expression is finite, we can evaluate the trace in a plane wave basis.

$$\lim_{M\to\infty} < x\left|\mathrm{Tr}\left\{\gamma^5 e^{-\frac{(\not{\mathcal{D}})^2}{M^2}}\right\}\right|x >= \lim_{M\to\infty}\mathrm{Tr}\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\gamma^5 e^{-\frac{(\not{\mathcal{D}})^2}{M^2}}e^{ikx} =$$

$$\lim_{M\to\infty}\mathrm{Tr}\int\frac{d^4k}{(2\pi)^4}\gamma^5 e^{\frac{-\left(i\not{k}+\not{D}+\frac{1}{2}\not{n}m^2(n\cdot(k+D))^{-1}\right)^2}{M^2}}, \quad k_\mu\to Mk_\mu$$

$$= \lim_{M\to\infty} M^4\,\mathrm{Tr}\int\frac{d^4k}{(2\pi)^4}\gamma^5 e^{-\left(i\not{k}+\not{D}/M+\frac{1}{2}\frac{1}{M^2}\not{n}m^2(n\cdot(ik+D/M))^{-1}\right)^2} \tag{28}$$

We have used that ML preserve scaling.

The only term that survives the limit is:

$$= \frac{1}{2}\int\frac{d^4k}{(2\pi)^4}e^{k^2}\,\mathrm{Tr}(\gamma^5\not{D}^4) \tag{29}$$

We have that:

$$(\not{D})^2 = D_\mu D_\nu g^{\mu\nu} - \frac{e}{2}F_{\mu\nu}\sigma^{\mu\nu}, \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu,\gamma^\nu]$$

We use:

$$< x\left|e^{-\partial^2}\right|x >= \lim_{x\to y}\int\frac{d^4k}{(2\pi)^4}e^{-ik(x-y)}e^{k^2} =$$
$$i\int\frac{d^4k_E}{(2\pi)^4}e^{-k_E^2} = i\frac{1}{16\pi^2}$$

Then:

$$\lim_{M\to\infty} < x\left|\mathrm{Tr}\left\{\gamma^5 e^{-\frac{(\not{\mathcal{D}}^2)}{M^2}}\right\}\right|x >= -\frac{e^2}{32\pi^2}\varepsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}(x)F_{\mu\nu}(x)$$

That is:

$$\mathcal{J} = \exp\left(-i\int d^4x\alpha(x)\frac{e^2}{16\pi^2}\varepsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}(x)F_{\mu\nu}(x)\right)$$

Then the Adler-Bell-Jackiw anomaly follows.

Notice that we could get this result assuming that the infrared regulator of $\frac{1}{n.\partial}$ preserves scaling(naive power counting). To garanty this property we work with the ML prescription, as in the perturbative approach.

8. Schwinger-Dyson (SD)identity for the product of three currents in VSR

We follow [27] page 311.
The vector current is:

$$j^\mu = \bar\psi\gamma^\mu\psi + \frac{1}{2}m^2 n^\mu((n\cdot\partial)^{-1}\bar\psi)\ \not{n}(n\cdot\partial)^{-1}\psi$$

The axial vector current is:

$$j^{\mu 5} = \bar\psi\gamma^\mu\gamma^5\psi + \frac{1}{2}m^2\left(\frac{1}{n\cdot\partial}\bar\psi\right)\ \not{n}n^\mu\gamma^5\left(\frac{1}{n\cdot\partial}\psi\right)$$

Consider the path integral, where S is the action (1) with $A_\mu = 0$

$$Z = \int \mathcal{D}\psi\mathcal{D}\bar\psi e^{iS} j^{\alpha 5}(y)j^\beta(z)$$

Make the following local transformations

$$\delta\psi(x) = i\alpha(x)\psi(x); \qquad \delta\bar\psi(x) = -i\alpha(x)\bar\psi(x)$$

The integration measure is invariant under this transformation.
We get the following Ward identity:

$$\partial^x_\mu < 0|T(j^\mu(x)j^{\alpha 5}(y)j^\beta(z))|0> - <0|T\delta_x j^{\alpha 5}(y)j^\beta(z))|0> - <0|T(j^{\alpha 5}(y)\delta_x j^\beta(z))|0> = 0 \quad (30)$$

where:

$$\delta_x j^\mu(y) = \frac{1}{2}m^2 n^\mu\left[\bar\psi\ \not{n}(n\cdot\partial)^{-1}(n\cdot\partial)^{-1}\psi\delta(x-y) + \bar\psi\ \not{n}(n\cdot\partial)^{-1}\psi(n\cdot\partial)^{-1}\delta(x-y) - \right.$$
$$\left.((n\cdot\partial)^{-1}(n\cdot\partial)^{-1}\bar\psi)\ \not{n}\psi\delta(x-y) - ((n\cdot\partial)^{-1}\bar\psi)\ \not{n}\psi(n\cdot\partial)^{-1}\delta(x-y\right]$$

$$\delta_x j^{\mu 5}(y) = \frac{1}{2}m^2 n^\mu\left[\bar\psi\ \not{n}\gamma^5(n\cdot\partial)^{-1}(n\cdot\partial)^{-1}\psi\delta(x-y) + \bar\psi\ \not{n}\gamma^5(n\cdot\partial)^{-1}\psi(n\cdot\partial)^{-1}\delta(x-y) - \right.$$
$$\left.((n\cdot\partial)^{-1}(n\cdot\partial)^{-1}\bar\psi)\ \not{n}\gamma^5\psi\delta(x-y) - ((n\cdot\partial)^{-1}\bar\psi)\ \not{n}\gamma^5\psi(n\cdot\partial)^{-1}\delta(x-y\right]$$

The non-locality of the action and currents modify the SD identity for the triangle graph. It is easy to check that the graphs in Figure 3 satisfy (30) if shifting of the loop integration variable is allowed.

Notice that (30) is not given by the addition of the graphs in Figure 1 of [22].

In a similar way we can derive the Ward identity for the divergence of the axial vector current.

9. Conclusions

We have reexamined the appearance of axial anomalies in VSR electrodynamics, using Pauli-Villars and dimensional regularization of ultraviolet divergences and Mandelstam-Leibbrandt regularization of infrared divergences.

Since ML preserves naive power counting in loop integrals, we have verified that the usual form for the anomaly of the axial current appears, without corrections from VSR terms. No anomaly is present in the vector current conservation. This computation is at variance from a previous result for the axial anomaly in two dimensions [20],where corrections from VSR terms were found. This difference may be due to different normalization conditions for the anomaly term[28] or some extra freedom that occurs when Lorentz invariance is violated[29]. In any case, our result implies that the procedure of [20] destroys the naive power counting of loop integrals.

The anomaly is produced because the loop integral is ultraviolet (UV) divergent, so a regulator must be introduced. To fix the renormalized quantities we must impose normalization conditions (Please see [28], chapter 13.1). These normalization conditions reflects the symmetries to be satisfied by the model. Different regulators may produce correspondingly different normalization conditions and therefore the anomaly could appear in the vector current, axial vector current or in a combination of both(This is important for a chiral theory like the SM). See for example [31] where a regulator is able to interpolate between different forms of the anomaly.

In this work we are reviewing a theory that has infrared(IR) divergences as well. So a new ambiguity in the value of the loop integral appears. The result depends on the IR regulator we choose. Beside the normalization conditions on the renormalized quantities are different, because we have a new fixed vector n_μ and non-local terms are allowed.

The results contained in [20] and in [24] correspond to different normalization conditions. In this sense, both results are right. But naive power counting of loop integrals is such an important tool in Quantum Field Theory that the normalization conditions of the present paper should be preferred.

In four dimension we find a completely different result compared to [22]. They claim that the conservation for the vector current has an anomaly and VSR corrections should appear in the anomaly of the axial current. We notice also that Figure 5,6 are lacking in the computation of both anomalies in [22]. Figure 5,6 are crucial to satisfy the Ward identity for the vector current as a procedure in 4d similar to the one explained in Appendix B shows.

In chapter VIII we derived the Ward identity for the product of two vectors and one axial vector current in VSR. The non-locality of the model introduces new contact terms.

We study also the axial anomaly from the point of view of the path integral method. Again ML property of preserving scaling(naive power counting) permits to show that the axial anomaly is the Lorentz invariant one, without corrections from VSR.

Using Dimensional(or PV)regularization of UV divergences and ML regularization of IR divergences ensues the VSRSM must be free from local chiral anomalies, since the same anomalies as in the SM are obtained , so the usual mechanism of cancellation of anomalies within families of leptons and quarks should work.

Lastly we recall that M is not the mass of the particle. So if the fermion acquires a VSR mass m even if $M = 0$, the divergence of the axial current will contain the anomalous term only.

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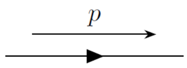
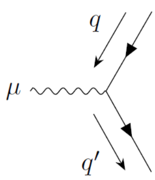
Appendix A:Feynman rules



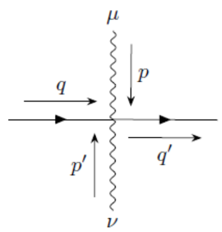
$$= i\frac{\not{p} + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot p}}{p^2 - M^2 - m^2 + i\varepsilon}$$

Figure 7: Electron propagator



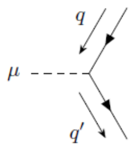
$$= -ie(\gamma_\mu + \frac{1}{2}m^2\frac{\not{n}n_\mu}{n\cdot q\; n\cdot q'})$$

Figure 8: $e - e - A_\mu$ vertex



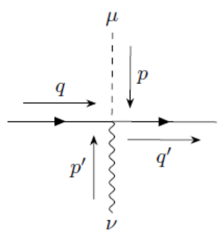
$$= -(ie)^2 \frac{1}{2}\, \not{n} m^2 \frac{n_\mu n_\nu}{n \cdot q \; n \cdot q'} [\frac{1}{n \cdot (p+q)} + \frac{1}{n \cdot (p'+q)}]$$

Figure 9: $e - e - A_\mu - A_\nu$ vertex



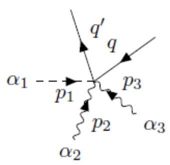
$$= -ie(\gamma_\mu + \frac{1}{2} m^2 \frac{\not{n} n_\mu}{n \cdot q \; n \cdot q'})\gamma_5$$

Figure 10: axial-e-e vertex



$$= -(ie)^2 \frac{1}{2}\, \not{n} m^2 \frac{n_\mu n_\nu}{n \cdot q \; n \cdot q'} [\frac{1}{n \cdot (p+q)} + \frac{1}{n \cdot (p'+q)}]\gamma_5$$

Figure 11: $axial - A_\nu - e - e$ vertex



$$= V(p_1, p_2, p_3, q)$$

Figure 12: axial $-A_{\alpha_2} - A_{\alpha_3} - e - e$ vertex

$$
\begin{aligned}
V(p_1,p_2,p_3,q) &= i(ie)^3 \frac{m^2}{2} \not{n} n^{\alpha_1} n^{\alpha_2} n^{\alpha_3} \frac{1}{n.(q+p_1+p_2+p_3)} \\
&(\frac{1}{n.(q+p_1+p_2)} \frac{1}{n.(q+p_1)} + \frac{1}{n.(q+p_1+p_2)} \frac{1}{n.(q+p_2)} + \\
&\frac{1}{n.(q+p_3+p_2)} \frac{1}{n.(q+p_3)} + \frac{1}{n.(q+p_1+p_3)} \frac{1}{n.(q+p_1)} + \\
&\frac{1}{n.(q+p_2+p_3)} \frac{1}{n.(q+p_2)} + \frac{1}{n.(q+p_3+p_1)} \frac{1}{n.(q+p_3)})\gamma^5
\end{aligned}
$$

Appendix B:Formal proof of the Ward identities in 2d

In this appendix we want to show in some detail how to obtain the Ward identities in 2d. In 4d we have more graphs, but the procedure is essentially the same.

$$q_\mu \Pi^{15\mu\nu} = -(-ie)^2 \int dp\,\mathrm{Tr}\{\left[\not{q} + \frac{1}{2} n.q\,(\not{n})\,m^2 (n.(p+q))^{-1}(n.p)^{-1}\right] \frac{i\left(\not{p} + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot p}\right)}{p^2 - M^2 - m^2 + i\varepsilon}$$
$$\left[\gamma^\nu + \frac{1}{2} n^\nu\,(\not{n})\,m^2 (n.(p+q))^{-1}(n.p)^{-1}\right]\gamma^5 \frac{i\left((\not{p}+\not{q}) + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot(p+q)}\right)}{(p+q)^2 - M^2 - m^2 + i\varepsilon}\}$$

Now we use the identity:

$$\left[\not{q} + \frac{1}{2} n.q\,(\not{n})\,m^2 (n.(p+q))^{-1}(n.p)^{-1}\right] =$$
$$\left[\not{p} + \not{q} - \frac{1}{2}\not{n} m^2 (n.(p+q))^{-1} - M - \left(\not{p} - \frac{m^2 \not{n}}{2n.p} - M\right)\right] \tag{31}$$

and the cyclic property of the trace to get:

$$q_\mu \Pi^{15\mu\nu} = (-ie)^2 \int dp\,\mathrm{Tr}\{\frac{\left(\not{p} + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot p}\right)}{p^2 - M^2 - m^2 + i\varepsilon}\left[\gamma^\nu + \frac{1}{2} n^\nu\,(\not{n})\,m^2 (n.(p+q))^{-1}(n.p)^{-1}\right]\gamma^5 -$$
$$\left[\gamma^\nu + \frac{1}{2} n^\nu\,(\not{n})\,m^2 (n.(p+q))^{-1}(n.p)^{-1}\right]\gamma^5 \frac{\left((\not{p}+\not{q}) + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot(p+q)}\right)}{(p+q)^2 - M^2 - m^2 + i\varepsilon}\} \tag{32}$$

Besides:

$$q_\mu \Pi^{25\mu\nu} =$$
$$2(ie)^2 n.q n^\nu \int dp (n.p)^{-1}\left[(n.(q+p))^{-1}(n.(-q+p))^{-1}\right]\mathrm{Tr}\,\frac{1}{2}\not{n} m^2 \frac{\left(\not{p} + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot p}\right)}{p^2 - M^2 - m^2 + i\varepsilon}\gamma^5$$

In the second term of(32) shift $p \to p - q$ to get: [2]

$$q_\mu \Pi^{15\mu\nu} =$$
$$-(-ie)^2 \int dp\,\mathrm{Tr}\left\{\frac{1}{2} n^\nu\,(\not{n})\,m^2 \frac{\left(\not{p} + M - \frac{m^2}{2}\frac{\not{n}}{n\cdot p}\right)}{p^2 - M^2 - m^2 + i\varepsilon}\gamma^5 (n.p)^{-1}(-2n.q)\left[(n.(p+q))^{-1}(n.(p-q))^{-1}\right]\right\}$$

That is $\Pi^{\mu\nu} = \Pi^{1\mu\nu} + \Pi^{2\mu\nu}$ is transverse.

The axial Ward identity is obtained in the same way.

[2] This is justified if we use DR as in chapter V and VI.