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Running Vacuum and the ΛCDM tensions

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In the last few years a lot of work has provided significant support to the possibility that the vacuum energy density (VED), $\rho_{\rm vac}$, is a running quantity throughout the cosmological history. Recent theoretical studies have shown that the properly renormalized $\rho_{\rm vac}$ in FLRW spacetime adopts the 'running vacuum model' (RVM) form, in which the scaling with the renormalization point turns into dependence on the Hubble rate, H. The late time VED evolves as an additive term plus a dynamical component $\mathcal{O}(H^2)$. Higher (even) powers $\mathcal{O}(H^{2n})$ are also predicted, which can trigger inflation in the early universe, although we shall not discuss this part here. In addition, the VED running is free from the quartic powers of the masses of the fields ($\sim m^4$) and hence the cosmic evolution of $\rho_{\rm vac}$ is really smooth. On the phenomenological side, the RVM fits the cosmological data remarkably well and it may help to reduce the H_0 and σ_8 tensions afflicting the ΛCDM. Overall, the RVM is sound since its theoretical structure can be derived from quantum field theory in curved spacetime and the model is phenomenologically consistent.

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PoS(CORFU2021)106

1. Introduction

The so-called standard or concordance model of cosmology (aka ΛCDM) became consolidated only in the mid nineties, [1–4]. Apart from the unverified existence of dark matter (DM), the theoretical status of the cosmological constant (CC) term, Λ, in Einstein's equations is no less worrisome. The notion of vacuum energy density (VED) in cosmology has been considered by theoretical physics and cosmology for more than half of a century, specially with the breakthrough of Quantum Theory and in general with the development of the formal aspects of QFT. The relation with the cosmological terms reads $\rho_{\rm vac} = \Lambda/(8\pi G_N)$ [5], where G_N is Newton's constant. Measurements of the CC term performed in the last decades using data from distant type Ia supernovae (SnIa) [6] and from the anisotropies of the cosmic microwave background (CMB) [7], have put the foundations of the ΛCDM [2].

While the Λ-term in the gravitational field equations was introduced by A. Einstein 105 years ago [8], the "cosmological constant problem" (CCP) as such was first formulated 50 years later by Y. B. Zeldovich [9]. The CCP is the chocking realization that the manyfold successes of QFT in the world of the elementary particles turn into a blatant fiasco in the realm of gravity. This is because the usual QFT treatment predicts a value for $\rho_{\rm vac}$ which is excruciatingly much larger than that of the current critical density of the universe and that of matter at present, ρ_m^0 [10–16].

Long after the discovery of the accelerated expansion of the universe a plethora of new models invaded the cosmology market, namely the 'dark energy' (DE) models, which appeared in different contexts and formulations, see e.g. [17] and references therein. The idea was to substitute the Λ-term in Einstein's equations by a new entity behaving in a similar way and possibly enjoying of better theoretical properties. This is how the vacuum energy option as a possible explanation for the speeding up of the universe became undermined and partly abandoned for a long time as if it were the only proposal afflicted by the fine tuning problem. However, the criticisms against the vacuum option usually have nothing better to offer. The vacuum is in fact the most fundamental concept of QFT and hence we should expect that a proper description of the CCP using the machinery of QFT in curved spacetime must eventually provide the clue to solving the cosmological constant problem from fundamental physics [16]. It is usually mentioned that Quantum Gravity (QG) should have the clue to the CCP. However, QG does not exist as a well defined theory yet, as it is well-known. However, our point of view is that it should be possible to tackle the CCP already at the level of quantum fields in a classical gravitational background [18], and this is the point of view put forward in [19, 20], which will be reviewed here – see also [16] for more details. For related studies about renormalization in curved spacetime, see e.g. [21–26].

In the context under study one finds that $\rho_{\rm vac}$ appears (in the current universe) as a dominant term plus a dynamical component which varies as $\sim \nu_{\rm eff} H^2 m_{\rm Pl}^2$, with $\nu_{\rm eff}$ a small (dimensionless) and computable coefficient (playing the role of β-function coefficient of the VED running) and $m_{\rm Pl}$ is the usual Planck mass ($m_{\rm Pl} = G_N^{-1/2}$). Such a structure constitutes the Running Vacuum Model (RVM), see [14–16] and references therein. A model with the aforementioned features remains close to the ΛCDM for long periods of the cosmic evolution and hence it can provide essentially the same results as in the ΛCDM, up to some small dynamical features. These, however, turn out to be crucial to lessen the H_0 and σ_8 tensions afflicting the latter [27–35].

2. Scalar field energy-momentum tensor in a curved background

The Einstein-Hilbert (EH) action for gravity plus matter reads

$$S_{\rm EH+m} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\,R - \int d^4x\sqrt{-g}\,\rho_\Lambda + S_{\rm m}\,. \tag{2.1}$$

The matter action $S_{\rm m}$ will be specified shortly. The term ρ_Λ is at this point just a bare parameter of the EH action, as the gravitational coupling G_N itself. The gravitational field equations emerging from the variation of the action (2.1) can be conveniently written as follows:

$$\frac{G_{\mu\nu}}{8\pi G_N} = -\rho_\Lambda g_{\mu\nu} + T^{\rm m}_{\mu\nu}\,, \tag{2.2}$$

As usual, $G_{\mu\nu} = R_{\mu\nu} - (1/2)g_{\mu\nu}R$ is Einstein's tensor and $T^{\rm m}_{\mu\nu}$ is the energy-momentum tensor (EMT for short) of matter. At this point we assume that there is only one matter quantum field contribution to the EMT defined by a real scalar field, ϕ. We call it $T^{\phi}_{\mu\nu}$. Thus,

$$S[\phi] = -\int d^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}(m^2+\xi R)\phi^2\right)\,. \tag{2.3}$$

Here ξ is the non-minimal coupling of ϕ to gravity. We recall that for the particular value $\xi = 1/6$, the massless ($m = 0$) action is conformally invariant. We will, however, keep ξ unspecified. In addition, we assume that ϕ has no classical potential, except the mass term. This implies that ϕ has no self-interactions. In this study, we wish to focus on the zero-point energy (ZPE) of ϕ only. Even with this simplification, the vacuum problem is nontrivial at all in curved spacetime.

The classical EMT follows immediately from the action (2.3):

$$\begin{aligned} T^{\phi}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S[\phi]}{\delta g^{\mu\nu}} &= (1-2\xi)\partial_\mu\phi\partial_\nu\phi + \left(2\xi - \frac{1}{2}\right)g_{\mu\nu}\partial^\sigma\phi\partial_\sigma\phi \\ &- 2\xi\phi\nabla_\mu\nabla_\nu\phi + 2\xi g_{\mu\nu}\phi\Box\phi + \xi G_{\mu\nu}\phi^2 - \frac{1}{2}m^2 g_{\mu\nu}\phi^2. \end{aligned} \tag{2.4}$$

The Klein-Gordon (KG) equation for ϕ in curved spacetime follows from varying the above action with respect to ϕ:

$$(\Box - m^2 - \xi R)\phi = 0\,, \tag{2.5}$$

where $\Box\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = (-g)^{-1/2}\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\phi\right)$ is the d'Alembertian operator in curved coordinates.

3. Renormalization of the VED in FLRW spacetime: absence of $\sim m^4$ contributions

To address the vacuum energy problem in Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime we assume a spatially flat three-dimensional geometry. In order to compute the VED we must quantize the matter field ϕ and then determine the vacuum expectation value (VEV) of the EMT, Eq. (2.4). However, because of the expanding curved background in the FLRW context an exact solution of the field modes of the theory is not possible since it corresponds to an anharmonic oscillator with time-dependent frequencies. Still, the VEV can be computed with respect to the adiabatic vacuum [18]. The latter is an approximation to the true vacuum in which an asymptotic solution of the field modes at high frequency is possible. It is a situation similar to geometrical optics, where solutions of the wave equation of an inhomogeneous medium can be found at high frequency, hence at short lengths, through a WKB expansion of the solution. In the present case, we have to find a WKB expansion of the Klein-Gordon equation (2.5). The expansion is formally divergent and must be renormalized by an appropriate regulator. To remove the UV-divergences, we could use the MS scheme with dimensional regularization (DR) and obtain a renormalized ZPE, but we prefer a more physical alternative. We start from the on-shell value of the vacuum EMT and perform a subtraction at an arbitrary mass scale M, which plays the role of renormalization point. The subtraction will remove divergences and the result will depend on purely geometric contributions, i.e. the higher derivative (HD) terms proportional to R, R^2, $R^{\mu\nu}R_{\mu\nu}$ etc., hence to H^2 and $\dot{H}$ (including higher powers of these quantities). There will be, however, some constant terms proportional to powers of the mass and of the subtraction scale M, in particular terms of order $\sim m^4$. These terms can be dangerous. However, let us emphasize that the VED is not just the 00th component of the renormalized vacuum part of the EMT but the sum of such component and the renormalized parameter ρ_Λ. In what follows we provide the main results, but refrain from entering technical issues and cumbersome computational details, which would lead us too far for this short presentation. For a more expanded exposition, see [16] as well as the technically comprehensive articles [19, 20].

Taking into account that the only adiabatic orders that are divergent in the case of the EMT in $n = 4$ spacetime dimensions are those up to order adiabatic 4th, the subtraction at the scale M is performed only up to this order. The terms beyond the 4th order are finite. The adiabatically renormalized EMT in this context therefore reads

$$\langle T_{\mu\nu}^{\delta\phi} \rangle_{\rm ren}(M) = \langle T_{\mu\nu}^{\delta\phi} \rangle(m) - \langle T_{\mu\nu}^{\delta\phi} \rangle^{(0-4)}(M) \,. \tag{3.1}$$

The on-shell value of the EMT can be computed of course at any order. Let us first consider Minkowskian space. Performing the subtraction (3.1) for the $00th$ component, we find [19, 20]

$$\langle T_{00}^{\delta\phi} \rangle_{\rm ren}^{\rm Mink}(M) = \frac{1}{128\pi^2}\left(-M^4 + 4m^2M^2 - 3m^4 + 2m^4 \ln\frac{m^2}{M^2}\right) . \tag{3.2}$$

Obviously, in the absence of the subtraction (3.1) the result would not be finite of its own accord. The renormalized and well-defined VED can now be constructed from the sum of the renormalized ρ_Λ term in the EH action and the renormalized ZPE given above:

$$\begin{aligned}\rho_{\rm vac}^{\rm Mink} = \rho_\Lambda(M) + \langle T_{00}^{\delta\phi} \rangle_{\rm ren}^{\rm Mink}(M) =& \rho_\Lambda(M) + \frac{1}{128\pi^2}\left(-M^4 + 4m^2M^2 - 3m^4 + 2m^4 \ln\frac{m^2}{M^2}\right) \\ =& \rho_\Lambda(M) + \frac{m^4}{64\pi^2}\left(\ln\frac{m^2}{M^2} - \frac{3}{2} - \frac{M^4}{2m^4} + \frac{2M^2}{m^2}\right) .\end{aligned} \tag{3.3}$$

The last equality in (3.3) is only to show in a more transparent way that such an expression boils down to the result obtained within the $\overline{\rm MS}$ scheme for $M^2 \ll m^2$, see [16]. In fact, in this limit only the logarithmic term and an additive constant survives, as it is characteristic of the MS. Now because the starting point is the sum of the bare parameter ρ_Λ and the unrenormalized ZPE, the expression (3.3) must be scale-invariant since it just corresponds to renormalizing a bare coupling[1]. Thereupon on acting with Md/M on both sides of (3.3) must yield zero. This tells us what is the β-function for ρ_Λ in the adiabatic renormalization scheme:

$$\beta_{\rho_\Lambda} = M\frac{d\rho_\Lambda(M)}{dM} = -\frac{1}{128\pi^2}\left(-4M^4+8m^2M^2-4m^4\right) = \frac{1}{2(4\pi)^2}(M^2-m^2)^2\,. \tag{3.4}$$

For $M^2 \ll m^2$ it reduces to the β-function for ρ_Λ in the $\overline{\rm MS}$ scheme [16]. As it is obvious, none of these formulas depend on the expansion rate H and therefore do not contain an inch of cosmological physics. The cosmological constant cannot be addressed in Minkowski spacetime, of course, although the above formula for β_{ρ_Λ} remains upright in the general case. Now the RGE for the VED in cosmological spacetime is not just the RGE for ρ_Λ. Once more we need the ZPE, but now in FLRW background.

So, let us next move to the renormalized result in curved spacetime and complete the job. The result will now depend on the Hubble rate and will be finite. Hence some physical considerations will be possible concerning the physical vacuum energy. Computing the renormalized ZPE for FLRW spacetime involves, however, a considerable amount of work which nevertheless can be expressed in a rather compact form. The final result is [19]

$$\begin{aligned}\langle T_{00}^{\delta\phi}\rangle_{\rm ren}(M) &= \frac{a^2}{128\pi^2}\left(-M^4+4m^2M^2-3m^4+2m^4\ln\frac{m^2}{M^2}\right)\\ &-\left(\xi-\frac{1}{6}\right)\frac{3\mathcal{H}^2}{16\pi^2}\left(m^2-M^2-m^2\ln\frac{m^2}{M^2}\right)+\left(\xi-\frac{1}{6}\right)^2\frac{9\left(2\mathcal{H}''\mathcal{H}-\mathcal{H}'^2-3\mathcal{H}^4\right)}{16\pi^2a^2}\ln\frac{m^2}{M^2}+\ldots\end{aligned} \tag{3.5}$$

where dots stand just for higher adiabatic orders. Notice the explicit dependence of this result on the Hubble rate. For $a=1$ ($\mathcal{H}=0$) the previous expression exactly reduces to the Minkowskian result (3.2), as it should be. The renormalized VED now follows from the sum of the renormalized ρ_Λ in the action and the renormalized ZPE, Eq. (3.5):

$$\begin{aligned}\rho_{\rm vac}(M,H) &= \rho_\Lambda(M)+\frac{\langle T_{00}^{\delta\phi}\rangle_{\rm ren}(M)}{a^2}\\ &= \rho_\Lambda(M)+\frac{1}{128\pi^2}\left(-M^4+4m^2M^2-3m^4+2m^4\ln\frac{m^2}{M^2}\right)\\ &-\left(\xi-\frac{1}{6}\right)\frac{3H^2}{16\pi^2}\left(m^2-M^2-m^2\ln\frac{m^2}{M^2}\right)\\ &-\left(\xi-\frac{1}{6}\right)^2\frac{9}{16\pi^2}\left(\dot H^2-2H\ddot H-6H^2\dot H\right)\ln\frac{m^2}{M^2}+\cdots\end{aligned} \tag{3.6}$$

Here we have expressed the result in terms of the ordinary Hubble rate in cosmic time, where we recall that $\mathcal{H}=aH$, and one can show from a simple calculation that $2\mathcal{H}''\mathcal{H}-\mathcal{H}'^2-3\mathcal{H}^4=$

[1]This is so because in Minkowski spacetime there is nothing else in the action apart from a constant term and the matter Lagrangian. In curved spacetime, however, we have in addition the curvature term in the EH action plus the geometric HD terms. The renormalization of the VED is then not just the renormalization of a bare term, as in fact the VED becomes explicitly dependent on H and its derivatives, as well as on M. The VED in FLRW spacetime indeed evolves independently with H and M, see Eq. (3.6) below. Only the full effective action (involving the classic part plus the vacuum effects) is scale-independent, i.e. RG-invariant [20].

PoS(CORFU2021)106

$-a^4\left(\dot{H}^2-2H\ddot{H}-6H^2\dot{H}\right)$. Moreover, in the arguments of the renormalized $\rho_{\rm vac}(M,H)$ we have remarked explicitly the dependence both on M and H. Obviously, it also depends on the derivatives of H, but we omit them in the arguments for brevity sake. The dependence of the VED on H is crucial and is inherited from that of the renormalized ZPE in curved spacetime. Of course, in the Minkowskian case such dependence was not possible since $H=0$.

We should now stress that Eq. (3.4) and hence the scale-independence of the sum of terms on the *r.h.s.* of (3.3) holds good also in FLRW spacetime since that equation is completely independent of H. This can be work out explicitly by rewriting Einstein's equations (2.2) using the renormalized parameters and including the higher derivative tensor $H_{\mu\nu}^{(1)}$, which is necessary for renormalization purposes [18]. The generalized Einstein's equations in the presence of higher derivative (HD) terms in the vacuum action read

$$\frac{G_{\mu\nu}}{8\pi G_N(M)}+\alpha(M)H_{\mu\nu}^{(1)}=\langle T_{\mu\nu}^{\rm vac}\rangle_{\rm ren}(M)+\cdots=-\rho_\Lambda(M)g_{\mu\nu}+\langle T_{\mu\nu}^{\delta\phi}\rangle_{\rm ren}(M)+\cdots \tag{3.7}$$

We have written only the vacuum part of the EMT since we want to perform a subtraction of Eq. (3.7) at the two scale values M and M_0 and hence the background contribution of the field ϕ (and any other one, indicated above by $\ldots$) will cancel. Using on its *r.h.s.* the explicit form for $\langle T_{00}^{\delta\phi}\rangle_{\rm ren}(M)$ as given by Eq, (3.5) as well as the explicit forms of G_{00} and $H_{00}^{(1)}$ in the FLRW metric (cf. Appendix A1 of [20], for example), we may perform such subtraction and project the 00th component of this expression. Among other relations involving the couplings α and G_N, we find that the following quantity involving ρ_Λ is scale invariant [16]:

$$\begin{aligned}\rho_\Lambda(M)+&\frac{1}{128\pi^2}\left(-M^4+4m^2M^2-3m^4+2m^4\ln\frac{m^2}{M^2}\right)\\ =&\rho_\Lambda(M_0)+\frac{1}{128\pi^2}\left(-M_0^4+4m^2M_0^2-3m^4+2m^4\ln\frac{m^2}{M_0^2}\right).\end{aligned}\tag{3.8}$$

For $H=0$ Eq. (3.6) reduces to the Minkowskian result (3.3). However, the latter must necessarily vanish when gravity is turned on, i.e. when we impose the generalized Einstein's equations (3.7). In fact, the *l.h.s.* of these equations exactly vanishes, in Minkowski space and so must vanish its *r.h.s.*, that is: $-\rho_\Lambda(M)\eta_{\mu\nu}+\langle T_{\mu\nu}^{\delta\phi}\rangle_{\rm ren}(M)=0$. The 00$th$ component reads $\rho_\Lambda(M)+\langle T_{00}^{\delta\phi}\rangle_{\rm ren}(M)=0$, which expresses vanishing VED in Minkowski vacuum, as it should be.

From the foregoing, it is clear that Eq.(3.4) is still the correct β-function for ρ_Λ also for curved spacetime. Despite the quantity (3.8) is identically zero in Minkowski spacetime, it is no longer zero in curved spacetime. Such expression remains scale-invariant and hence acting on it with Md/M on both sides leads again to Eq.(3.4). Finally, once the β-function for the renormalized parameter ρ_Λ has been correctly identified, the β-function for the VED, $\rho_{\rm vac}(M)$, immediately follows from Eq. (3.6):

$$\beta_{\rho_{\rm vac}}=M\frac{\partial\rho_{\rm vac}(M)}{\partial M}=\left(\xi-\frac{1}{6}\right)\frac{3H^2}{8\pi^2}\left(M^2-m^2\right)+\left(\xi-\frac{1}{6}\right)^2\frac{9\left(\dot{H}^2-2H\ddot{H}-6H^2\dot{H}\right)}{8\pi^2}. \tag{3.9}$$

This result is significant, it says that the running associated to the renormalization group equation (RGE) for $\rho_{\rm vac}(M)$ is completely free from the troublesome quartic mass terms $\sim m^4$ and rests only on the presence of quadratic mass scales in the final result, which are highly smoothed by the

PoS(CORFU2021)106

accompanying factor of the Hubble rate, namely we are left with just soft terms of the form $\sim m^2H^2$ plus, of course, the higher order contributions $\mathcal{O}(H^4)$ which are all carrying time derivatives, i.e. they are of the form $\sim \dot{H}^2, H\ddot{H}, H^2\dot{H}$. All these higher order contributions are irrelevant for the current universe. The important result (3.9) for the β-function of the VED was first derived in the recent paper [20]. It shows that a perfectly sensible RGE for the VED (and hence for the physical cosmological term) exists, in contrast to existing claims in the literature. The upshot is that the physical cosmological term in Einstein equations is not a rigid cosmological constant in the presence of quantized matter fields, but a running quantity with the cosmic expansion.

4. The RVM: vacuum evolution from QFT in curved spacetime

Let us now show that the QFT-driven universe remains close to the ΛCDM paradigm, although it being fundamentally different in conceptual terms. We start from the general form of the renormalized VED, Eq, (3.6). For simplicity, we focus on the current universe, so we neglect the $\sim H^4$ terms in that expression. We explore the VED evolution between epochs characterized by the values H and H_0 of the Hubble rate (in particular, H_0 could be the current epoch). Thus, from Eq, (3.6) we find

$$\begin{aligned}\rho_{\rm vac}(M,H)-\rho_{\rm vac}(M_0,H_0)=\frac{3\left(\xi-\frac{1}{6}\right)}{16\pi^2}&\left[H^2\left(M^2-m^2+m^2\ln\frac{m^2}{M^2}\right)\right.\\ &\left.-H_0^2\left(M_0^2-m^2+m^2\ln\frac{m^2}{M_0^2}\right)\right]+\cdots\end{aligned}\tag{4.1}$$

where the ... stand for the neglected higher order terms. Notice that we have used the important Eq. (3.8), which insures the cancellation of the quartic mass terms $\sim m^4$. What about the scale M? As a matter of fact, $M=M(t)$ becomes a dynamical variable in FLRW spacetime since we must choose M and M_0 at particular epochs of the cosmological evolution in order to evaluate the physical difference of VED values in these epochs. Thus, denoting respectively by $\rho_{\rm vac}(H)$ and $\rho_{\rm vac}(H_0)$ the values of $\rho_{\rm vac}(M=H,H)$ and $\rho_{\rm vac}(M_0=H_0,H_0)$ and recalling that the higher order powers are negligible for the late universe, the final result may be cast as follows [20]:

$$\rho_{\rm vac}(H)\simeq\rho^0_{\rm vac}+\frac{3\nu_{\rm eff}(H)}{8\pi}\,(H^2-H_0^2)\,m_{\rm Pl}^2=\rho^0_{\rm vac}+\frac{3\nu_{\rm eff}(H)}{\kappa^2}\,(H^2-H_0^2)\,,\tag{4.2}$$

with $\nu_{\rm eff}(H)$ a very slow evolving function of H which can be approximated by the constant value

$$\nu_{\rm eff}\equiv\nu_{\rm eff}(H_0)\simeq\frac{1}{2\pi}\left(\xi-\frac{1}{6}\right)\frac{m^2}{m_{\rm Pl}^2}\ln\frac{m^2}{H_0^2}\,.\tag{4.3}$$

Formula (4.2) with constant $\nu_{\rm eff}$ constitutes the canonical form of the RVM [14], but here it has been derived from our QFT framework. As noted, $\rho^0_{\rm vac}\equiv\rho_{\rm vac}(H_0,H_0)$ in it can be identified with today's VED value and $\rho_{\rm vac}(H)\equiv\rho_{\rm vac}(H,H)$ is the VED value at some expansion history time $H(t)$ around the current time. We naturally expect $|\nu_{\rm eff}|\ll 1$ owing to the ratio $M_X^2/m_{\rm Pl}^2\ll 1$. Thus, the RVM formula (4.2) predicts a smooth evolution of the VED between different expansion history epochs without the disturbance of $\sim m^4$ contributions, which have been washed out.

5. RVM: a possible cure for the σ_8 and H_0 tensions

The RVM has been tested since long in a variety of papers, where the basic parameter $\nu_{\rm eff}$ has been fitted to the overall cosmological data [36–42]. Despite of the fact that $|\nu_{\rm eff}| \ll 1$ can be connected to QFT quantum effects (this fact being crucial for a solid theoretical underpinning of the RVM), such parameter can only be picked out from observations since we ignore at this point the details of the underlying Grand Unified Theory (GUT) which ultimately accounts for such a vacuum dynamics. To test the RVM, we are going to use a generalized expression which goes beyond the canonical form (4.2). It includes both a term in H^2 and in $\dot{H}$, each one with an independent coefficient, which we call ν and $\tilde{\nu}$:

$$\rho_{\rm vac}(H) = \frac{3}{\kappa^2}\left(c_0 + \nu H^2 + \tilde{\nu}\dot{H}\right) + \mathscr{O}(H^4)\,. \tag{5.1}$$

The higher powers of H are of course irrelevant around our time, so we neglect them from now on. With the choice $\tilde{\nu} = \nu/2$, the VED reads $\rho_{\rm vac}(H) = \frac{3}{\kappa^2}\left(c_0 + \frac{\nu}{12}R\right)$ since $R = 12H^2 + 6\dot{H}$ is the curvature scalar. For this reason we may call this particular implementation of the VED the 'RRVM'. This scenario was analyzed in [43] and will be summarized here. It is particularly well-behaved in the radiation dominated era, as in it $R \simeq 0$ and hence the BBN physics is not altered [36]. The following RRVM situations will be considered: 1) type-I, which assumes interaction of vacuum with matter, and 2) type-II, which assumes matter conservation and a slowly evolving gravitational coupling $G(H)$. In case 1), the vacuum exchanges energy only with cold dark matter (CDM), but not with baryons:

$$\dot{\rho}_{dm} + 3H\rho_{dm} = -\dot{\rho}_{\rm vac}\,. \tag{5.2}$$

On solving explicitly the model leads to the following evolution of the matter densities [43]:

$$\rho_m(a) = \rho_m^0 a^{-3\xi}\,, \quad \rho_{dm}(a) = \rho_m^0 a^{-3\xi} - \rho_b^0 a^{-3}\,, \tag{5.3}$$

in wich $\xi \equiv \frac{1-\nu}{1-\frac{3}{4}\nu} \simeq 1 - \nu/4 + \mathscr{O}\left(\nu^2\right)$. Here $\rho_m = \rho_{dm} + \rho_b$ is the total matter density (CDM plus baryons). The ΛCDM behavior is obtained for $\nu = 0$ ($\xi = 1$). The presence of $\nu \neq 0$ permits a soft dynamical evolution of the VED:

$$\rho_{\rm vac}(a) = \rho_{\rm vac}^0 + \left(\frac{1}{\xi} - 1\right)\rho_m^0\left(a^{-3\xi} - 1\right) \simeq \rho_{\rm vac}^0 + \frac{1}{4}\,\nu\,\rho_m^0\left(a^{-3\xi} - 1\right) + \mathscr{O}\left(\nu^2\right)\,. \tag{5.4}$$

Once more, for $\nu = 0$ we have $\rho_{\rm vac} = \rho_{\rm vac}^0$, i.e. the ΛCDM . In the case of type-I models we test the possibility that the vacuum dynamics might have started only very recently (as also contemplated e.g. in [44]). Thus, we introduce a threshold redshift z_* and assume that for $z \leq z_*$ the vacuum is dynamical in the way indicated above, whereas for $z > z_*$ we assume it is constant. Specifically, we take the threshold point of this dynamical vacuum transition at $z_* \simeq 1$, as in [43]. It turns out that the possibility of such a transition can actually be a fundamental consequence of the vacuum behavior in QFT, see [20]. For type II models, case 2), matter is conserved and in this case the vacuum can still evolve at the expense of a very mild (logarithmic) running of the gravitational coupling: $G_{\rm eff} = G_{\rm eff}(\ln H)$. For this second type of model we do not assume any threshold effect. The approximate behavior of the VED for type-II RRVM models at around the present time reads

$$\rho_{\rm vac}(a) = \frac{3c_0}{\kappa^2}(1+4\nu) + \nu\rho_m^0 a^{-3} + \mathscr{O}(\nu^2)\,. \tag{5.5}$$

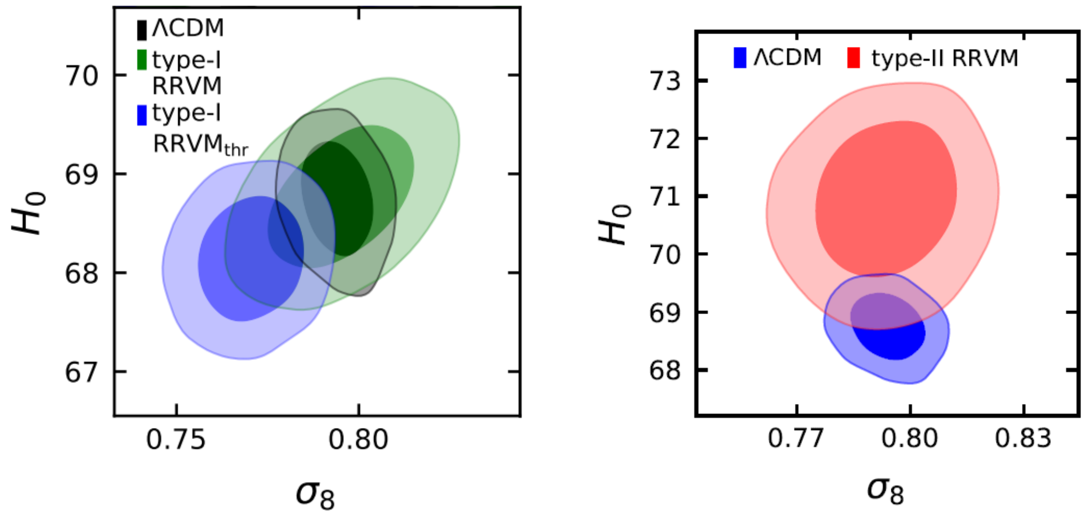

Figure 1. *Left:* Regions at 1σ and 2σ c.l. in the (σ_8-H_0)-plane corresponding to type I RRVM. In the presence of a threshold redshift $z_* \simeq 1$ it reveals efficient for alleviating the σ_8 tension, but not the H_0 one; *Right:* As previously, but for the type-II RRVM. The latter, in contrast, is effective to mitigate the H_0 tension and improves also the σ_8 one. H_0 in the plots is expressed in km/s/Mpc.

For $\nu = 0$ the VED is constant and $\rho_{\rm vac} = 3c_0/\kappa^2$ (i.e. we recover again the ΛCDM behavior), but for nonvanishing ν it has a moderate dynamics as for type-I models.

In a summarized way the main results of the phenomenological analysis of the two RRVM types can be seen in Fig. 1, including the ΛCDM [43]. We emphasize that the effect of the threshold $z_* \simeq 1$ is significant to help lessening the value of σ_8 and hence to improve the status of this tension. Without threshold, however, the effect is only moderate. Unfortunately, type-I models with fixed $G_{\rm eff} = G_N$ do not alleviate the H_0 tension, as H_0 stays around the CMB value [43]. On the other hand, type-II models may alleviate the two tensions at a time, which is remarkable (cf. Fig. 1 right). A more detailed analysis will be considered elsewhere.

6. Conclusions

In this work, we have reviewed in a very summarized way the recent investigations on the problem of the cosmological vacuum energy in a QFT context [19, 20]. For an extended presentation, see [16]. These studies confirm that the running vacuum model (RVM) proposal can be formally derived within QFT in curved spacetime. The intrinsic dependency that renormalized quantities have on the renormalization point M makes the VED a function of M, and this turns into a cosmic evolution with H. What we call the 'cosmological constant' Λ is only the nearly sustained value of $\rho_{\rm vac}(H)$ around any given cosmic epoch. There is no cosmological constant problem (CCP) in this approach since, as we have shown, the evolution of $\rho_{\rm vac}(H)$ is perfectly smooth, it does not depend on the quartic powers of the masses $\sim m^4$. The dark energy that we observe is just the (non-constant) vacuum energy which, according to QFT in FLRW spacetime, remains naturally of order H^2 at any (post-inflationary) observational time without fine tuning. In point of fact, there

PoS(CORFU2021)106

cannot be any cosmological constant in the context of QFT in curved spacetime since renormalization theory enforces that the vacuum energy density, $\rho_{\rm vac}(H)$, must always be evolving with the expansion. Such evolution is very soft and proportional to H^2 through a small coefficient $\nu_{\rm eff}$ which is calculable in QFT and plays the role of β-function coefficient of the running. At any cosmic time t characterized by $H(t)$ there is a (different) 'CC' term $\Lambda(H) = 8\pi G_N \rho_{\rm vac}(H)$ acting (approximately) as a cosmological constant for a long period around that time, but there is no true CC valid at all times!

We have illustrated these facts by considering the renormalization of the energy-momentum tensor (EMT) of a real quantum scalar field non-minimally coupled to classical gravity in the cosmological context. The method is based on an off-shell extension of the adiabatic regularization and renormalization procedure [19, 20] and where for the first time we provided a calculation of the zero point energy (ZPE) of a quantum scalar field that is free from the need of extreme fine tuning. The calculational procedure in our approach is based on the WKB expansion of the field modes in the FLRW spacetime and the use of an appropriately renormalized EMT. The resulting EMT becomes finite because we subtract the first four adiabatic orders (the only ones that can be divergent). Since the off-shell renormalized EMT is a function of the arbitrary renormalization point M, we can compare the renormalized result at different epochs of the cosmic history by setting M to the value o the expansion rate H at each epoch. This is how we find that the evolution of the VED throughout the cosmic history is tiny (but nonvanishing) as indicated above.

These properties are confirmed by recent phenomenological analyses based on a large set of updated cosmological data on SnIa+H(z)+BAO+LSS+CMB [43,45]. One finds that a slow dynamics of the cosmic vacuum is helpful to describe the cosmological observations, and in particular to reduce the persisting σ_8 and H_0 tensions that are afflicting since long the standard ΛCDM. Needles to say, more work will be required to confirm the phenomenological status of the RVM proposal, but in the meantime the theoretical and phenomenological advantages of this approach are made evident.

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PoS(CORFU2021)106