PROCEEDINGS OF SCIENCE

Phenomenology of the superweak U(1) extension of the standard model

Zoltán TRÓCSÁNYI[*†]
Institute for Theoretical Physics, ELTE Eötvös Loránd University, Pázmány Péter sétány 1/A, H-1117 Budapest, Hungary
and ELKH-DE Particle Physics Research Group H-4010 Debrecen P.O.Box 105, Hungary
E-mail: zoltan.trocsanyi@cern.ch

The superweak force is a minimal, anomaly-free U(1) extension of the standard model, designed to explain the origin of (i) neutrino masses and mixing matrix elements, (ii) dark matter, (iii) cosmic inflation, (iv) stabilization of the electroweak vacuum and (v) leptogenesis. In this talk we discuss how the parameter space of the model is constrained by providing viable scenarios for the first four of this list.

Corfu Summer Institute 2021 "School and Workshops on Elementary Particle Physics and Gravity"
29 August - 9 October 2021
Corfu, Greece

[*]Speaker.

[†]This research was supported by grant K-125105 of the National Research, Development and Innovation Fund in Hungary.

PoS(CORFU2021)081

1. Introduction

The standard model (SM) of elementary particle interactions [1] has been studied at high-energy colliders extensively [2, 3]. The result of these experimental studies can be summarized simply: *the SM describes final states of particle collisions precisely*. Furthermore, extensive searches for new particles at the Large Hadron Collider have so far provided only exclusion limits, so *we do not have any proven sign of physics beyond the SM* (BSM) *from colliders* [4].[1]. Nevertheless, the precision of the SM parameters – in particular, that of the masses of the Higgs particle and the t quark together with the value of strong coupling [6] – provides strong evidence that the scalar potential of the SM becomes unbounded from below at energy scales around 10^{11} GeV [7, 8]. This means that the vacuum of the SM is metastable, and thus, *if new physics exist, it should not worsen the stability, but possibly push the vacuum to the stability region*.

There is a handful of established experimental facts that cannot be explained by the SM. The most outstanding ones are the following [6]: (i) the *measured abundance of dark matter in the universe*; (ii) the *non-vanishing neutrino masses*; (iii) the *observed matter–anti-matter asymmetry requiring lepto- and baryogenesis*[2]; (iv) the *accelerating expansion of the universe*, signaling the existence of dark energy. In addition to (i)–(iv), (v) inflation in the early universe is also considered fairly established, but without any direct proof for it. All these facts have to be explained by such an extension of the standard model that respects (a) the high precision confirmation of the standard model at collider experiments (b) and the lack of finding new particles beyond the Higgs boson by the collider experiments.

Neutrinos clearly must play a key role in the quest for the BSM theories. Neutrinos with non-zero masses must feel another force apart from the weak one. As all electrically charged fermions couple to the Higgs boson, it is at least not unnatural to assume that neutrinos do so as well, which requires the existence of right-handed neutrinos ν_{R}^f. We assume that these neutrinos come in three families ($f = 1$, 2 and 3) just like the SM fermions. Such neutrinos must be sterile under the SM gauge interactions, and might have observable effects only if they couple to the charged fermions through a new force.

The simplest and most economic extension of the SM gauge group $G_{\mathrm{SM}} = \mathrm{SU}(3)_{\mathrm{c}} \otimes \mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{U}(1)_Y$ is by a new U(1) to $G = G_{\mathrm{SM}} \otimes \mathrm{U}(1)_z$. Of course, such a new gauge interaction is not observed directly among the SM fermions, hence we expect it is broken. We assume the existence of a new scalar whose vacuum expectation value w breaks the new U(1) and simultaneously stabilizes the vacuum up to the Planck scale. The existence of the $\mathrm{U}(1)_z$ group calls for the fixing of the new z charges. The requirement of gauge and gravity anomaly cancellations and the inclusion of all possible gauge invariant Yukawa terms involving the right-handed neutrinos and the new scalar field allows for setting the z charges up to a normalization factor of the new gauge coupling g_z.

Such a model has the potential of explaining all the confirmed signs of new physics. Dirac and Majorana neutrino mass terms are generated by the spontaneous symmetry breaking (SSB) of the scalar fields, providing the origin of neutrino masses and oscillations. The lightest new particle

[1]There are notable deviations of experimental results from precision predictions in the flavor sector, but to date none has reached the significance of discovery [5]

[2]Baryogenesis can be explained in the standard model provided leptogenesis occurs, which is called lepto-baryogenesis

is a natural candidate for weakly or feebly interacting massive particle (WIMP/FIMP) matter if it is sufficiently stable. Diagonalization of neutrino mass terms leads to the emergence of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, which in turn may be the source of lepto-baryogenesis. The second scalar together with the established Brout-Englert-Higgs (BEH) field may be the source of accelerated expansion now and inflation in the early universe. Of course, extensive phenomenological studies are required to confront the predictions of the model with measurements, and decide whether or not these promises are fulfilled.

2. Particle model

We consider the usual three fermion families of the standard model extended with one right-handed Dirac neutrino in each family, using the notation

$$\psi^f_{q,1} = \begin{pmatrix} U^f \\ D^f \end{pmatrix}_{\rm L} \qquad \psi^f_{q,2} = U^f_{\rm R} \qquad \psi^f_{q,3} = D^f_{\rm R}; \qquad \psi^f_{l,1} = \begin{pmatrix} \nu^f \\ \ell^f \end{pmatrix}_{\rm L} \qquad \psi^f_{l,2} = \nu^f_{\rm R} \qquad \psi^f_{l,3} = \ell^f_{\rm R} \tag{2.1}$$

for the chiral quark fields ψ_q and chiral lepton fields ψ_l. The subscripts L and R denote the left and right-handed projections,

$$\psi_{\rm L/R} \equiv \psi_{\mp} = \frac{1}{2}(1 \mp \gamma_5)\,\psi \equiv P_{\rm L/R}\psi\,. \tag{2.2}$$

The field content in family f ($f = 1$, 2 or 3) consists of two quarks, U_f, D_f, a left-handed active neutrino ν_f, a charged lepton ℓ_f and a right-handed (SM) sterile neutrino $\nu^f_{\rm R}$. U_f is the generic notation for the u-type quarks while D_f is that for d-type quarks.

The Lagrangian is

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \mathscr{L}_{\rm SW} \tag{2.3}$$

where $\mathscr{L}_{\rm SM}$ is the Lagrangian of the SM and $\mathscr{L}_{\rm SW}$ contains the terms due to the superweak (SW) extension. The U(1) part of the covariant derivative acting on the field ψ_F is extended by the term belonging to the new U(1) gauge field B'_μ:

$$\mathscr{D}^{(F),{\rm U}(1)}_\mu = -{\rm i}(y_F g_y B_\mu + z_F g_z B'_\mu)\,, \tag{2.4}$$

with y_F being the usual hypercharges and z_F to be specified below. The field strength tensor $F'_{\mu\nu}$ of B'_μ is gauge invariant itself, hence we allow for the presence of the kinetic mixing term

$$\mathscr{L}_{\rm SW} \supset -\frac{1}{4}F'^{\mu\nu}F'_{\mu\nu} - \frac{\varepsilon}{2}F^{\mu\nu}F'_{\mu\nu} \tag{2.5}$$

with mixing parameter ε.

The scalar sector is extended by a complex scalar χ that can mix with the usual SU$(2)_{\rm L}$-doublet Brout-Englert-Higgs (BEH) field ϕ, so

$$\mathscr{L}_{\rm SW} \supset \big(\mathscr{D}^{(F)}_\mu\chi\big)^*\mathscr{D}^{(F)\mu}\chi - \Big(-\mu_\chi|\chi|^2 + \lambda_\chi|\chi|^4 + \lambda|\phi|^2|\chi|^2\Big)\,. \tag{2.6}$$

After SSB we parametrize the fields as

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} -{\rm i}\sqrt{2}\sigma^+ \\ v + h' + {\rm i}\sigma_\phi \end{pmatrix}, \quad \chi = \frac{1}{\sqrt{2}}(w + s' + {\rm i}\sigma_\chi) \tag{2.7}$$

Table 1: Particle content and charge assignment of the SWSM, where ϕ and χ are complex scalars and the others are Weyl fermions. For $\mathrm{SU}(3)_\mathrm{c}\otimes\mathrm{SU}(2)_\mathrm{L}$ the representations, while for $\mathrm{U}(1)_Y\otimes\mathrm{U}(1)_z$ the charges (y and z) of the respective fields are given. Note that for $\mathrm{U}(1)_Y$, the eigenvalues of the half hypercharge operator Y are given.

	$\mathrm{SU}(3)_\mathrm{c}$	$\mathrm{SU}(2)_\mathrm{L}$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_z$
Q_L	**3**	**2**	$1/6$	$1/6$
U_R	**3**	**1**	$2/3$	$7/6$
D_R	**3**	**1**	$-1/3$	$-5/6$
L_L	**1**	**2**	$-1/2$	$-1/2$
N_R	**1**	**1**	0	$1/2$
e_R	**1**	**1**	-1	$-3/2$
ϕ	**1**	**2**	$1/2$	1
χ	**1**	**1**	0	-1

where v and w denotes the VEVs of the fields, whose values are

$$v=\sqrt{2}\sqrt{\frac{2\lambda_\chi\mu_\phi^2-\lambda\mu_\chi^2}{4\lambda_\phi\lambda_\chi-\lambda^2}}\,,\qquad w=\sqrt{2}\sqrt{\frac{2\lambda_\phi\mu_\chi^2-\lambda\mu_\phi^2}{4\lambda_\phi\lambda_\chi-\lambda^2}}\,,\tag{2.8}$$

with μ_ϕ^2 and λ_ϕ being the usual coefficients of the quadratic and quartic terms of the BEH potential. Assuming Yukawa interactions between the fermions and scalars, these VEVs provide masses to the fermions. In particular, the new Yukawa terms

$$-\mathscr{L}_\mathrm{SW}\supset\frac{1}{2}\overline{\nu_\mathrm{R}}\mathbf{Y}_N(\nu_\mathrm{R})^c\chi+\overline{\nu_\mathrm{R}}\mathbf{Y}_\nu\varepsilon_{ab}L_{\mathrm{L}a}\phi_b+\mathrm{h.c.}\,,\tag{2.9}$$

lead to both Dirac and Majorana mass terms for the neutrinos. In Eq. (2.9) L_L is the left-handed lepton doublet, ε_{ab} is the Levi-Civita symbol, a and b are SU(2) indices, and the superscript c denotes the charge conjugate of the field, $(\nu_\mathrm{R})^c=-\mathrm{i}\gamma_2\nu_\mathrm{R}^*$. The first term is gauge invariant provided the z-charge of the right-handed neutrinos and the new scalar satisfy the relation $z_\chi=-2z_{\nu_\mathrm{R}}$.

It is well known that the requirement of cancellation of gauge and gravity anomalies in U(1) extensions of the standard model lead to the parametrization of the z-charges in terms of two rational numbers Z_1 and Z_2 [9]. In the SWSM we assume that the left- and right-handed neutrinos have opposite z-charges, which fixes Z_2 with Z_1 given. With this choice and a suitable reparametrization of the U(1) couplings g_Y and g_z, we find that the model is equivalent to a U(1) extension when only the right-handed fermions are charged under the new U(1) interaction [10]. Had we chosen equal z-charges for the left- and right-handed neutrinos, we would end up with the well studied $\mathrm{U}(1)_{B-L}$ extension.

The remaining unkown Z_1 can be fixed freely, which sets the normalization of the coupling. For the sake of convenience, we choose the z-charge of the BEH scalar to be unity. The charge assignments are then obtained as given in Table 1.

3. Masses of neutrinos

After SSB the terms proportional to the VEVs provide the 3×3 Dirac and Majorana mass terms mass matrices

$$\mathbf{M}_{\mathrm{D}} = \frac{v}{\sqrt{2}}\mathbf{Y}_\nu\,, \quad \mathbf{M}_{\mathrm{R}} = \frac{w}{\sqrt{2}}\mathbf{Y}_N \tag{3.1}$$

where we chose a basis such that the Majorana mass matrix $\mathbf{M}_{\mathrm{R}}$ is real, positive and diagonal, while the Dirac mass matrix $\mathbf{M}_{\mathrm{D}}$ is complex. In flavour basis the full 6×6 mass matrix for the neutrinos can be written as

$$\mathbf{M}' = \begin{pmatrix} \mathbf{0}_3 & \mathbf{M}_{\mathrm{D}}^T \\ \mathbf{M}_{\mathrm{D}} & \mathbf{M}_{\mathrm{R}} \end{pmatrix} \tag{3.2}$$

so the *Dirac and Majorana mass terms appear already at tree level by SSB, i.e. not generated radiatively*.

The flavour eigenstates $(\nu_e,\, \nu_\mu,\, \nu_\tau,\, \nu_{R,1}^c,\, \nu_{R,2}^c,\, \nu_{R,3}^c)$ can be transformed into the ν_i $(i = 1-6)$ mass eigenstates with a 6×6 unitary matrix $\mathbf{U}$ where the mass matrix is diagonal,

$$\mathbf{U}^T\mathbf{M}'\mathbf{U} = \mathbf{M} = \mathrm{diag}(m_1, m_2, m_3, m_4, m_5, m_6)\,. \tag{3.3}$$

It is helpful to decompose the matrix $\mathbf{U}$ into two 3×6 blocks $\mathbf{U}_{\mathrm{L}}$ and $\mathbf{U}_{\mathrm{R}}^*$,

$$\mathbf{U} = \begin{pmatrix} \mathbf{U}_{\mathrm{L}} \\ \mathbf{U}_{\mathrm{R}}^* \end{pmatrix}, \tag{3.4}$$

so $\mathbf{U}^T = (\mathbf{U}_{\mathrm{L}}^T, \mathbf{U}_{\mathrm{R}}^\dagger)$ where both blocks are 6×3 matrices. It may be worth emphasizing that in spite of what might be implied by the notation, the matrices $\mathbf{U}_{\mathrm{L}}$ and $\mathbf{U}_{\mathrm{R}}^*$ are only semi-unitary. Useful relations of these matrices are collected in the Appendix of Ref. [11].

While the full diagonalization in Eq. (3.3) is cumbersome, one can first block-diagonalize the block mass matrix up to small corrections:

$$\begin{pmatrix} \mathbf{M}_\nu & 0 \\ 0 & \mathbf{M}_N \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{U}_{\mathrm{as}} \\ -\mathbf{U}_{\mathrm{as}}^\dagger & \mathbf{1} \end{pmatrix}^T \begin{pmatrix} 0 & \mathbf{M}_{\mathrm{D}}^T \\ \mathbf{M}_{\mathrm{D}} & \mathbf{M}_{\mathrm{R}} \end{pmatrix} \begin{pmatrix} \mathbf{1} & \mathbf{U}_{\mathrm{as}} \\ -\mathbf{U}_{\mathrm{as}}^\dagger & \mathbf{1} \end{pmatrix} \approx \begin{pmatrix} -\mathbf{M}_{\mathrm{D}}^T\mathbf{M}_{\mathrm{R}}^{-1}\mathbf{M}_{\mathrm{D}} & 0 \\ 0 & \mathbf{M}_{\mathrm{R}} \end{pmatrix} \tag{3.5}$$

where the matrix $\mathbf{U}_{\mathrm{as}} = \mathbf{M}_{\mathrm{D}}^\dagger\mathbf{M}_{\mathrm{R}}^{-1}$ is the active-sterile mixing matrix having elements U_{ai} $(a = e,\, \mu,\, \tau,\, i = 4,\, 5,\, 6)$. In the last step of (3.5) we neglected blocks suppressed in the see-saw limit, such as the off-diagonal blocks $\mathbf{U}_{\mathrm{as}}^*\mathbf{M}_{\mathrm{D}}\mathbf{U}_{\mathrm{as}}$ and its transpose, as well as $\mathbf{M}_{\mathrm{D}}\mathbf{U}_{\mathrm{as}}$ and its transpose as compared to $\mathbf{M}_{\mathrm{R}}$. Whether or not such terms are indeed negligible at the physical points can be justified numerically a-posteriori, once we have numerical estimates for the Yukawa matrices (see below). For now we assume that $-\mathbf{M}_{\mathrm{D}}^T\mathbf{M}_{\mathrm{R}}^{-1}\mathbf{M}_{\mathrm{D}}$ and $\mathbf{M}_{\mathrm{R}}$ are the approximate mass matrices for active and sterile neutrinos, i.e. the see-saw limit can be applied. At this point $\mathbf{M}_N$ is already diagonal, but $\mathbf{M}_\nu$ is not so. Hence, next we diagonalize the light neutrino mass matrix $\mathbf{M}_\nu$:

$$\mathbf{U}_2^T\mathbf{M}_\nu\mathbf{U}_2 = \mathbf{M}_\nu^{\mathrm{diag}} \tag{3.6}$$

where $\mathbf{U}_2$ is a 3×3 unitary matrix.

We have experimental constraints on the upper limits the elements of $\mathbf{M}_\nu^{\mathrm{diag}}$ [13, 14]. Even if the tree-level matrix $\mathbf{M}_\nu^{\mathrm{diag}}$ satisfies those limits, one has to check that the inclusion of loop corrections to the mass matrix

$$\delta\mathbf{M}' = \begin{pmatrix} \delta\mathbf{M}_{\mathrm{L}} & \delta\mathbf{M}_{\mathrm{D}}^T \\ \delta\mathbf{M}_{\mathrm{D}} & \delta\mathbf{M}_{\mathrm{R}} \end{pmatrix} = \mathbf{U}^*\delta\mathbf{M}\mathbf{U}^\dagger \tag{3.7}$$

do not upset them (after diagonalization of the corrected matrix). Here the 3×3 blocks can be computed as

$$\delta\mathbf{M}_\mathrm{L} = \mathbf{U}_\mathrm{L}^*\delta\mathbf{M}\mathbf{U}_\mathrm{L}^\dagger,\quad \delta\mathbf{M}_\mathrm{D} = \mathbf{U}_\mathrm{R}\delta\mathbf{M}\mathbf{U}_\mathrm{L}^\dagger,\quad \delta\mathbf{M}_\mathrm{R} = \mathbf{U}_\mathrm{R}\delta\mathbf{M}\mathbf{U}_\mathrm{R}^T\,. \tag{3.8}$$

We are especially interested in the correction $\delta\mathbf{M}_L$. Its computation is straightforward, but involves highly non-trivial cancellation of the gauge-dependent terms and divergent contributions (see Ref. [11] for details). The final form of the one-loop correction can be compactly given as

$$\delta\mathbf{M}_L = \frac{1}{16\pi^2}\sum_{k=1,2}\left[3(\mathbf{Z}_\mathrm{G})^2_{k1}\frac{M^2_{V_k}}{v^2}\mathbf{F}(M^2_{V_k}) + (\mathbf{Z}_\mathrm{S})^2_{k1}\frac{M^2_{S_k}}{v^2}\mathbf{F}(M^2_{S_k})\right], \tag{3.9}$$

and can easily be generalized to arbitrary number of $\mathrm{U}(1)$ gauge bosons V_k and complex scalars S_k (see [11]). In Eq. (3.9) the finite matrix valued function

$$\mathbf{F}_{ij}(M^2) = \sum_{a=1}^{6}(\mathbf{U}_L^*)_{ia}(\mathbf{U}_L^\dagger)_{aj}\frac{m_a^3}{M^2}\frac{\ln\frac{m_a^2}{M^2}}{\frac{m_a^2}{M^2}-1} \tag{3.10}$$

is of dimension mass and the summation runs over all neutrinos. As the correction is finite, it is also independent of the renormalization scale. The 2×2 rotation matrices $\mathbf{Z}_\mathrm{S}$ and $\mathbf{Z}_\mathrm{G}$ connect the mass eigenstates of the scalars (h, s) and Goldstone bosons (σ_Z, $\sigma_{Z'}$) to their flavour eigenstates h', s' and σ_ϕ, σ_χ:

$$\begin{pmatrix} h \\ s \end{pmatrix} = \mathbf{Z}_\mathrm{S}(\theta_S)\begin{pmatrix} h' \\ s' \end{pmatrix} \quad\text{and}\quad \begin{pmatrix} \sigma_Z \\ \sigma_{Z'} \end{pmatrix} = \mathbf{Z}_\mathrm{G}(\theta_\mathrm{G})\begin{pmatrix} \sigma_\phi \\ \sigma_\chi \end{pmatrix} \tag{3.11}$$

where θ_S and θ_G are the scalar and Goldstone mixing angles obtained by diagonalizing the mass matrix of the real scalars and that of the neutral Goldstone bosons. Explicitly, θ_S can be expressed with the parameters of the scalar sector, while the Goldstone mixing angle is related simply to the mixing angle θ_Z between the massive neutral gauge bosons:

$$\tan(2\theta_S) = -\frac{\lambda v w}{\lambda_\phi v^2 - \lambda_\chi w^2}\,,\quad \tan(\theta_\mathrm{G}) = \tan(\theta_Z)\frac{M_{Z'}}{M_Z}\,. \tag{3.12}$$

Fig. 1 shows the range that the matrix elements $\mathbf{F}_{ij}$ can take as a function of the mass m_loop of the boson in the loop (blue band), assuming normal neutrino mass hierarchy. We have highlighted with vertical bands the relevant mass regions where the masses of the bosons in the loop lie. Following [12], we require the scalar s to have mass between 144 and 558 GeV needed for the stability of the vacuum.

The eigenvalues of the $\mathbf{F}$ matrices can be large, even larger than 1 eV for tree-level masses in the allowed range for the active neutrinos, depending on the mass of the boson in the loop and the tree-level neutrino masses. However, the coupling factors suppress those significantly. For instance, assuming the active neutrino masses to be $\mathrm{O}(10^{-3})$ eV, the corrections to the matrix elements can be estimated as

$$(\delta\mathbf{M}_\mathrm{L})_{ij} < \mathrm{O}(10^{-7})\,\mathrm{eV} + \mathrm{O}(10^{-21})\times\left(\frac{M_{Z'}}{100\ \mathrm{MeV}}\right)^2\mathbf{F}_{ij}(M^2_{Z'})\,. \tag{3.13}$$

Hence, a rough estimate for the relative correction to active neutrino masses in this region of the parameter space is $\mathrm{O}(10^{-4})$.

PoS(CORFU2021)081

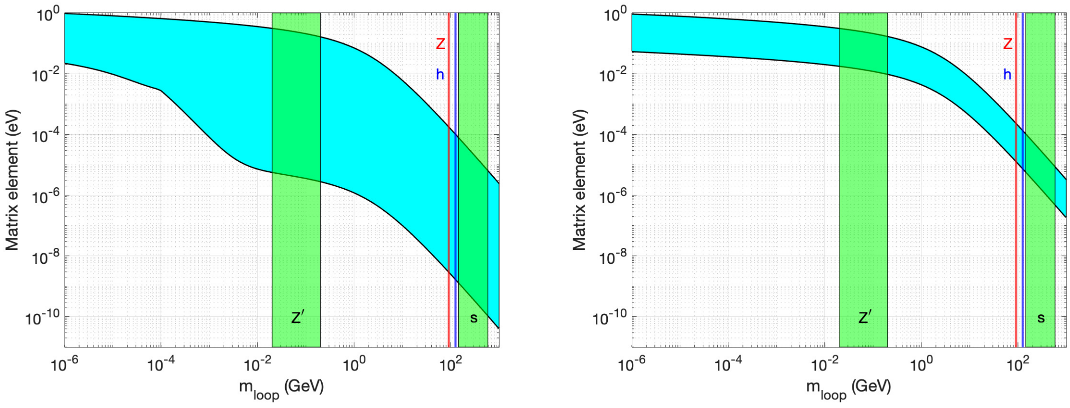

Figure 1: Range of the matrix elements $\mathbf{F}_{ij}$represented by the blue band as a function of the mass $m_{\rm loop}$ of the boson in the loop. Left plot: $m_1^{\rm tree} = 0.01\,{\rm eV}$, $m_4^{\rm tree} = 30\,{\rm keV}$, $m_5^{\rm tree} \approx m_6^{\rm tree} = 2.5\,{\rm GeV}$. Right plot: $m_1^{\rm tree} = 0.001\,{\rm eV}$, $m_4^{\rm tree} = 7.1\,{\rm keV}$, $m_5^{\rm tree} \approx m_6^{\rm tree} = 3.0\,{\rm GeV}$. Taken from Ref. [11].

4. Dark matter candidate

We have firm evidence that dark matter (DM) exists in the Universe [13]. However, so far all known evidence is based solely on the gravitational effect of the dark matter on the luminous astronomical objects and on the Hubble-expansion of the Universe, which allows for various sources of DM. Nevertheless, we know that the Universe is filled with different types of stable SM particles or bound states of those. Hence, it might seem natural to assume that the DM has particle origin. The only chance to observe such a particle in the laboratory or in Nature if it interacts with the SM particles. Such an interaction must be mediated by a field, which is called *portal*. There are three portals studied extensively in the literature: (i) vector boson portal when a new gauge boson is coupled to the SM fermions, for instance through kinetic mixing; (ii) Higgs portal when the BEH field couples to the DM particles; (iii) neutrino portal when the DM is a fermion (of dimension 3/2) coupled to the HL operator (of dimension 5/2). With the Z' boson coupled to the SM particles and also to the right-handed neutrinos, in the SWSM the vector boson portal Z' with the lightest sterile neutrino ν_4 as dark matter candidate is a natural scenario.

In order to check if such a scenario is feasible, we have to estimate the abundance of ν_4 in the Universe today, for which our starting point is Boltzmann's equation. It is convenient to define the comoving number density $\mathscr{Y}_i$ of particle species i. Starting from the Boltzmann equation, we can derive the differential equation for the dark matter candidate a. Schematically we have

$$\frac{{\rm d}\mathscr{Y}_a}{{\rm d}z} \propto \sum_{\rm particles} \Big[(\text{rate of creation processes of particle } a) \\ - (\text{rate of processes annihilating particle } a)\Big] \tag{4.1}$$

where z denotes the dimensionless inverse temperature Λ/T, with Λ being an arbitrary mass scale. The rate of a particular process can be estimated by the schematic formula

$$\text{rate} = (\text{cross section or decay rate}) \times (\text{available initial particle abundance})\,. \tag{4.2}$$

The first factor depends on the particle physics model. The initial abundance can be either in or out of equilibrium. Both factors depend on the temperature. In Eq. (4.2) the cross section refers to the thermally averaged cross section

$$\langle \sigma v_{\text{Møl}} \rangle \propto \int_{4\mu^2}^{\infty} \mathrm{d}s \, \sigma(s)(s - 4m_{\text{in}}^2) \sqrt{s} K_1 \left(\frac{\sqrt{s}}{T} \right) \tag{4.3}$$

where $\mu = \max(m_{\text{in}}, m_{\text{out}})$ with the masses of incoming and outgoing particles. For a decaying particle of mass m the natural choice for the mass scale Λ is m itself and the decay rate is simply

$$\langle \Gamma \rangle = \Gamma \frac{K_1(z)}{K_2(z)}, \tag{4.4}$$

with K_i denoting the modified Bessel-function of the second kind. The ratio $K_1(z)/K_2(z)$ is a monotone increasing function of z, reaching unity at infinity.

There are two possible ways to reach the current abundance: (i) by freeze-out or (ii) freeze-in mechanism. As the latter requires very small – so called feeble – couplings (smaller than 10^{-10}), it is very difficult test that scenario experimentally. While it is possible in the SWSM [15], here we focus only on the freeze-out case, which will be testable in the near and medium term future.

The freeze-out mechanism for a DM species of mass m requires that (i) it decouples from the other particles in the cosmic soup at some temperature T_{dec} that is typically $T_{\text{dec}} \approx 0.1m$, and (ii) it had been in equilibrium with the other species before decoupling. The way in which the equilibrium distribution had been achieved is unimportant. The only necessary condition is that it was the case before decoupling. Decoupling is a result of the Hubble-expansion, and occurs when the rate of scattering processes becomes smaller than the rate of expansion.

The dark matter particles are produced dominantly by the decay of the Z'. Current exclusion limits on this vector boson portal leave room for $M_{Z'} \gtrsim 20\,\text{MeV}$. However, a sufficiently heavy Z' can change Big-Bang Nucleosynthesis (BBN) dramatically through the production of SM particles. Hence we focus on the mass window with upper end below the muon pair production threshold. With the choice of these Z' masses it is assured that their abundance has mostly diminished by the onset of BBN, and thus their effect will be negligible. Nevertheless, for $M_{Z'} > m_\pi$ pion production is kinematically allowed, which would still affect the proton-neutron conversion rate [16]. In our analysis we neglected pion production, as Z' with mass above $\simeq 130\,\text{MeV}$ will turn out to be already excluded by laboratory experiments. As a result, we consider the decays of the Z' into electrons, active neutrinos, and ν_4.

The dark matter candidate species is produced by the decay of the Z', hence $m_4 < M_{Z'}/2$. Specifically, we consider $m_4 \in [10, 50]\,\text{MeV}$, so the decoupling temperature is $T_{\text{dec}} = \mathrm{O}(1)\,\text{MeV}$. At this temperature *electrons* and *active neutrinos* are abundant in the cosmic soup, while the presence of heavier fermions are negligible.

An example solution of Eq. (4.1) is shown in Fig. 2 (solid line). The initial condition was given by the equilibrium comoving number density for ν_4, and the starting temperature can be chosen around $T_0 \simeq m_4/10$. At high temperatures the solution follows the equilibrium comoving number density (dashed black), while at low temperatures the dark matter decouples, and a non-zero relic density is frozen out. To obtain the correct relic density, one needs a $\mathrm{U}(1)_z$ coupling that is too large, and is excluded by SM precision measurements. In the freeze-out mechanism decreasing the

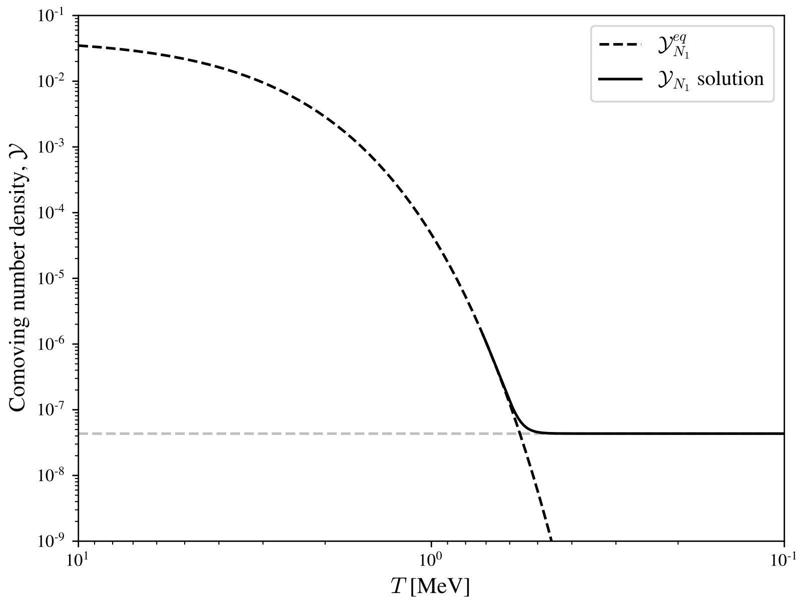

Figure 2: Example solution to the Boltzmann equation in the freeze-out case. The horizontal line indicates the relic density corresponding to $\Omega_{\text{DM}} = 0.265$, $M_{Z'} = 30\,\text{MeV}$, $M_1 = 10\,\text{MeV}$, $g_z = 1.06 \cdot 10^{-3}$.

coupling, hence the interaction rate increases the relic density. It is essential for the SWSM DM candidate that the resonance can dominate the integral in the rate (4.3). Exploiting this condition we can decrease the value of the coupling while keeping the relic density unchanged.

In Fig. 3 we present the parameter space for the freeze-out scenario of dark matter production. The dark matter particle is assumed to be the lightest right-handed neutrino with mass M_1 in this plot. The required dark couplings g_z reproducing $\Omega_{\text{DM}} = 0.265$ are plotted against the mass of the new gauge boson Z' for various values of the dark matter mass. The shaded region of the parameter space is excluded by the a_e bound (dashed gray) obtained from the $\text{U}(1)_z$ contribution to the electron anomalous magnetic moment [17], and the NA64 bound (dashed light-blue) obtained from missing energy searches [18]. The steep parts of the lines correspond to the resonant amplification, not yet excluded by the NA64 constraint. The lightly shaded region $M_{Z'} > m_\pi$ is not excluded, but it may be in conflict with the observed proton-to-neutron ratio [16], which we have not taken into account because the relevant couplings for $M_{Z'} \gtrsim 130$ MeV are already ruled out by NA64.

5. Neutrino benchmarks

While the resonant dark matter production in the freeze-out mechanism is an exciting explanation to the DM puzzle, one might ask whether such neutrinos are at all allowed by known experimental constraints, or will they be testable within the not too far future. In fact, there are stringent bounds on the elements of the active-sterile mixing matrix in the mass range 1–80 GeV of the sterile neutrinos, $|U_{ai}|^2 \lesssim 10^{-5}$ $(a = e, \mu, \tau, i = 4, 5, 6)$ [19, 20]. As mentioned before, this matrix emerges in the diagonalization of the neutrino mass matrix in the see-saw limit. In order to utilize those constraints, we have to parametrize the active neutrino mass matrix.

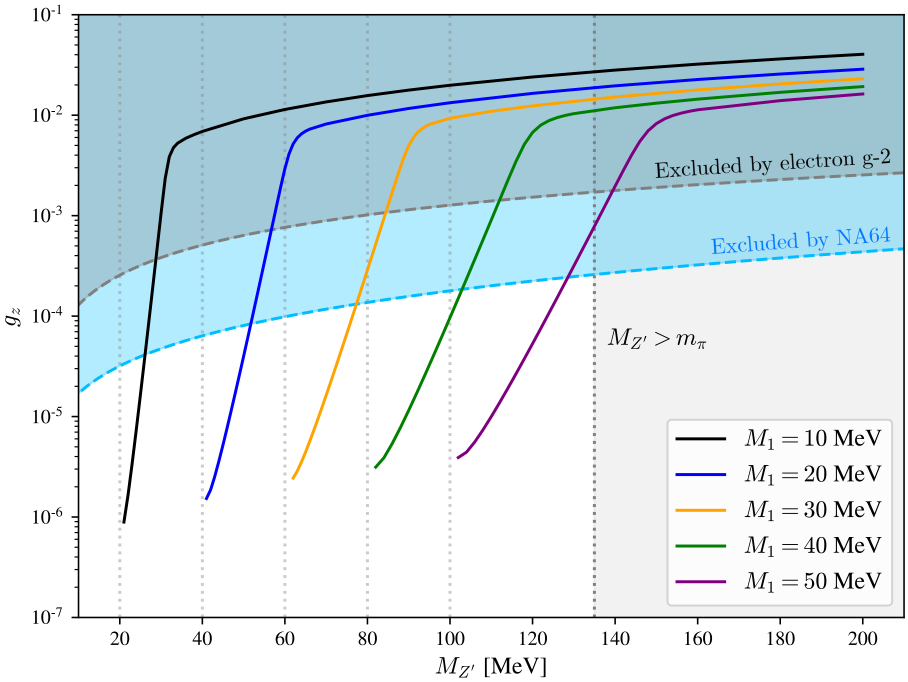

Figure 3: Parameter space for the freeze-out scenario of dark matter production in the supeweak model. Taken from Ref. [15].

According to Eqs. (3.5) and (3.6) we can write the diagonalized light neutrino mass matrix as

$$\mathbf{M}_\nu^{\text{diag}} = \mathbf{U}_2^T \mathbf{M}_\nu \mathbf{U}_2 = -\mathbf{U}_2^T \mathbf{M}_\text{D}^T \mathbf{M}_\text{R}^{-1} \mathbf{M}_\text{D} \mathbf{U}_2 = -\frac{v^2}{2} \mathbf{U}_2^T \mathbf{Y}_\nu^T \mathbf{M}_\text{R}^{-1} \mathbf{Y}_\nu \mathbf{U}_2 . \tag{5.1}$$

To find connection to the neutrino Yukawa matrix, we use the Casas-Ibarra parameterization [21] by introducing the matrix

$$\mathbf{R} = \mathrm{i} \frac{v}{\sqrt{2}} \mathbf{M}_\text{R}^{-1/2} \mathbf{Y}_\nu \mathbf{U}_2 (\mathbf{M}_\nu^{\text{diag}})^{-1/2} . \tag{5.2}$$

Utilizing Eq. (5.1), we obtain $\mathbf{R}^T \mathbf{R} = \mathbf{1}$, i.e. $\mathbf{R}$ is an orthogonal matrix. In general it can be complex, but for the sake of simplicity here we focus only on real $\mathbf{R}$, hence it can be parametrized in terms of three real numbers s_{ij} (sines of the Euler angles) over the unit cube.

We can solve Eq. (5.2) for the neutrino Yukawa matrix, and obtain its adjoint as

$$\mathbf{Y}_\nu^\dagger = \frac{\sqrt{2}}{v} \mathbf{U}_2 (\mathbf{M}_\nu^{\text{diag}})^{1/2} (\mathrm{i}\mathbf{R}^\dagger) \mathbf{M}_\text{R}^{1/2} . \tag{5.3}$$

The inclusion of sterile neutrinos results in non-unitary active-light neutrino mixing matrix [22, 23], but the violation of unitarity is proportional to the active-sterile mixing squared, which is tiny, so we neglect it in this study. Thus the active-light mixing is described by the unitary matrix $\mathbf{U}_\text{PMNS} = \mathbf{U}_{\ell\text{L}}^\dagger \mathbf{U}_2$ (usual PMNS matrix). We may choose to set $\mathbf{U}_{\ell\text{L}}$ to unit matrix, leading to $\mathbf{U}_\text{PMNS} = \mathbf{U}_2$, which ensures that for the charged leptons the flavour and mass eigenstates coincide. For the active neutrinos the same choice is not possible. The $\mathbf{U}_2$ PMNS matrix may also include the CP violating and the unknown, complex Majorana phases. We set those to zero, as we do not expect that such phases will change our conclusions significantly.

Using that $\mathbf{M}_\mathrm{R}$ is real and diagonal, we can write the active-sterile mixing matrix as

$$\mathbf{U}_\mathrm{as} = \frac{v}{\sqrt{2}}\mathbf{Y}_\nu^\dagger \mathbf{M}_\mathrm{R}^{-1}\,. \tag{5.4}$$

We substitute the matrix $\mathbf{Y}_\nu$ as given in Eq. (5.3) to obtain

$$\mathbf{U}_\mathrm{as} = \mathbf{U}_\mathrm{PMNS}\sqrt{\mathbf{M}_\nu^\mathrm{diag}}(\mathrm{i}\mathbf{R}^\dagger)\mathbf{M}_\mathrm{R}^{-1/2}\,. \tag{5.5}$$

We see that even though the light and heavy neutrino masses and PMNS matrix are independent of the choice of $\mathbf{R}$ matrix, the mixing between active and sterile neutrinos is not so. Hence, knowing the PMNS matrix experimentally and assuming values for the masses of the neutrinos, we have to scan over the full parameter space of the $\mathbf{R}$ matrix to find the possible $\mathbf{U}_\mathrm{as}$ matrix elements.

The various accelerator, beam dump and decay search experiments constrain the combinations

$$U_X^2 = \sum_{i=4}^{6}|U_{Xi}|^2\,, \quad (X = e \text{ or } \mu) \tag{5.6}$$

of the elements of the active-sterile mixing matrix. We can use these sums to investigate the dependence of the neutrino sector of SWSM on the $\mathbf{R}$ matrix, the mass m_1 of the lightest neutrino and the sterile neutrino masses m_4, m_5 and m_6. The sum U_X^2 in Eq. (5.6) represents the weight of sterile components in ν_X ($X = e$ or μ).

We scanned the parameters of the $\mathbf{R}$ matrix over the whole parameter space $(s_{12}, s_{13}, s_{23}) \in [0,1]^3$ to enhance the active-sterile mixings U_e^2 and U_μ^2 enough, so that those will be testable at different upcoming experiments. We performed systematic iterative searches by locating the optimal region in the unit cube, followed by a search again in the optimal sub-volume with a denser sampling until we reached the desired accuracy of the s_{ij} values.

We searched for benchmark points giving valid physics scenarios, and being sensitive to different combinations of the experiments. We chose the benchmark points BP1–BP5 in such a way that they all evade the present experimental bounds, but can be tested at future experiments. These points are exhibited in Fig. 4. While at points BP1–BP4 the lightest sterile neutrino has mass in the keV range, relevant to the freeze-in mechanism of SWSM DM production, the at the point BP5 $m_4 = 25\,\mathrm{MeV}$, showing that the freeze-out mechanism is also possible. We have also checked that active-ν_4 mixing satisfied the β decay electron energy spectrum kink bounds given in [24].

It turns out that the (2,2), (2,3), (3,2) and (3,3) elements dominate $\mathbf{Y}_\nu$, as they correspond to the heavy right-handed neutrinos ν_5 and ν_6. Similarly, the first column in the active-sterile mixing matrix corresponds to mixing of the active neutrinos to ν_4. As $m_4 \ll m_5$ and m_6, active-ν_4 mixing is stronger than active-ν_5 and -ν_6 mixing,

$$|U_{a4}| \gg |U_{a5}|, |U_{a6}|, \quad a = e, \mu, \tau\,. \tag{5.7}$$

6. Conclusions and outlook

In this contribution we have studied the allowed parameter space of the superweak extension of the standard model of particle interactions via focusing on three facets of the model: (i) neutrino mass generation, (ii) possible source for dark matter candidate and (iii) and searching for neutrino

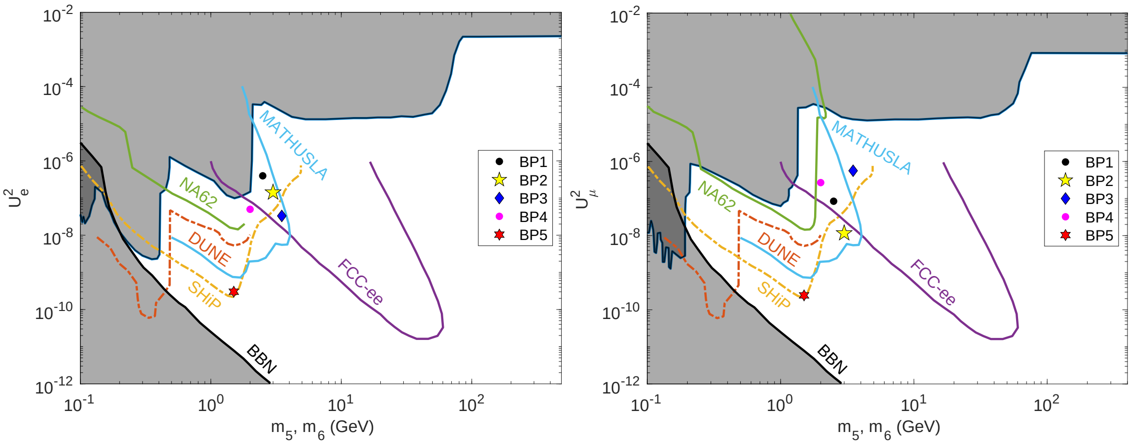

Figure 4: Constraints in logarithmic (U_X^2, m_i) plane $i = 5, 6$ from above are given by several experiments (shaded area), collected from [19, 25, 26]. Experimental sensitivities of future experiments are given by colored lines. Left plot: $X = e$. Right plot: $X = \mu$.

benchmark points that can be tested experimentally in the near future. We have found that the model provides viable phenomenology to solve these puzzles. Of course, many more studies are needed in other sectors of the model in order to establish whether or not the SWSM can provide explanation to all the outstanding observations at the intensity and cosmological frontiers. Even if the model provided explanation to all such questions, it would not necessarily mean that it is the correct BSM extension of the SM. Nevertheless, it is an interesting question whether or not a single model can provide explanation to all puzzles and check to what extent the parameter space can be tested in near future experiments.

References

[1] S. Weinberg, *A Model of Leptons*, Phys. Rev. Lett. **19** (1967), 1264-1266 doi:10.1103/PhysRevLett.19.1264

[2] S. Schael *et al.* [ALEPH, DELPHI, L3, OPAL, SLD, LEP Electroweak Working Group, SLD Electroweak Group and SLD Heavy Flavour Group], *Precision electroweak measurements on the Z resonance*, Phys. Rept. **427** (2006), 257-454 doi:10.1016/j.physrep.2005.12.006 [arXiv:hep-ex/0509008 [hep-ex]].

[3] Kostas Kordas on behalf of ATLAS and CMS collaborations, *SM (EW+QCD) measurements in ATLAS and CMS*, https://indico.cern.ch/event/1066112/timetable/#20210831

[4] Ann-Kathrin Perrevoort on behalf of ATLAS and CMS collaborations, *Exotics and BSM in ATLAS and CMS*, https://indico.cern.ch/event/1066112/timetable/#20210901

[5] Monica Pepe-Altarelli on behalf of LHCb collaboration, *Review of rare decays and tests of Lepton Flavour Universality at LHCb*, https://indico.cern.ch/event/1066112/timetable/#20210902

[6] P. A. Zyla *et al.* [Particle Data Group], *Review of Particle Physics*, PTEP **2020** (2020) no.8, 083C01 doi:10.1093/ptep/ptaa104

[7] F. Bezrukov and M. Shaposhnikov, *Standard Model Higgs boson mass from inflation: Two loop analysis*, JHEP **07** (2009), 089 doi:10.1088/1126-6708/2009/07/089 [arXiv:0904.1537 [hep-ph]].

[8] G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori and A. Strumia, *Higgs mass and vacuum stability in the Standard Model at NNLO*, JHEP **08** (2012), 098 doi:10.1007/JHEP08(2012)098 [arXiv:1205.6497 [hep-ph]].

[9] T. Appelquist, B. A. Dobrescu and A. R. Hopper, *Nonexotic neutral gauge bosons*, *Phys. Rev.* **D68** (2003) 035012 [hep-ph/0212073].

[10] Z. Trócsányi, *Super-weak force and neutrino masses*, Symmetry **12** (2020) no.1, 107 doi:10.3390/sym12010107 [arXiv:1812.11189 [hep-ph]].

[11] S. Iwamoto, T. J. Kärkkäinen, Z. Péli and Z. Trócsányi, *One-loop corrections to light neutrino masses in gauged U(1) extensions of the standard model*, Phys. Rev. D **104**, no.5, 055042 (2021) doi:10.1103/PhysRevD.104.055042 [arXiv:2104.14571 [hep-ph]].

[12] Z. Péli and Z. Trócsányi, *Stability of the vacuum as constraint on U(1) extensions of the standard model*, [arXiv:1902.02791 [hep-ph]].

[13] N. Aghanim *et al.* [Planck], *Planck 2018 results. VI. Cosmological parameters*, Astron. Astrophys. **641** (2020), A6 [erratum: Astron. Astrophys. **652** (2021), C4] doi:10.1051/0004-6361/201833910 [arXiv:1807.06209 [astro-ph.CO]].

[14] M. Aker *et al.* [KATRIN], *Improved Upper Limit on the Neutrino Mass from a Direct Kinematic Method by KATRIN*, Phys. Rev. Lett. **123** (2019) no.22, 221802 doi:10.1103/PhysRevLett.123.221802 [arXiv:1909.06048 [hep-ex]].

[15] S. Iwamoto, K. Seller and Z. Trócsányi, *Sterile neutrino dark matter in a U(1) extension of the standard model*, JCAP **01** (2022) no.01, 035 doi:10.1088/1475-7516/2022/01/035 [arXiv:2104.11248 [hep-ph]].

[16] A. Boyarsky, M. Ovchynnikov, O. Ruchayskiy and V. Syvolap, *Improved big bang nucleosynthesis constraints on heavy neutral leptons*, Phys. Rev. D **104** (2021) no.2, 023517 doi:10.1103/PhysRevD.104.023517 [arXiv:2008.00749 [hep-ph]].

[17] L. Morel, Z. Yao, P. Cladé and S. Guellati-Khélifa, *Determination of the fine-structure constant with an accuracy of 81 parts per trillion*, Nature **588** (2020) no.7836, 61-65 doi:10.1038/s41586-020-2964-7

[18] D. Banerjee, V. E. Burtsev, A. G. Chumakov, D. Cooke, P. Crivelli, E. Depero, A. V. Dermenev, S. V. Donskov, R. R. Dusaev and T. Enik, *et al. Dark matter search in missing energy events with NA64*, Phys. Rev. Lett. **123**, no.12, 121801 (2019) doi:10.1103/PhysRevLett.123.121801 [arXiv:1906.00176 [hep-ex]].

[19] S. Alekhin, W. Altmannshofer, T. Asaka, B. Batell, F. Bezrukov, K. Bondarenko, A. Boyarsky, K. Y. Choi, C. Corral and N. Craig, *et al. A facility to Search for Hidden Particles at the CERN SPS: the SHiP physics case*, Rept. Prog. Phys. **79**, no.12, 124201 (2016) doi:10.1088/0034-4885/79/12/124201 [arXiv:1504.04855 [hep-ph]].

[20] K. Bondarenko, A. Boyarsky, D. Gorbunov and O. Ruchayskiy, *Phenomenology of GeV-scale Heavy Neutral Leptons*, JHEP **11**, 032 (2018) doi:10.1007/JHEP11(2018)032 [arXiv:1805.08567 [hep-ph]].

[21] J. A. Casas and A. Ibarra, *Oscillating neutrinos and* $\mu \to e, \gamma$, Nucl. Phys. B **618**, 171-204 (2001) doi:10.1016/S0550-3213(01)00475-8 [arXiv:hep-ph/0103065 [hep-ph]].

[22] E. Fernandez-Martinez, M. B. Gavela, J. Lopez-Pavon and O. Yasuda, *CP-violation from non-unitary leptonic mixing*, Phys. Lett. B **649**, 427-435 (2007) doi:10.1016/j.physletb.2007.03.069 [arXiv:hep-ph/0703098 [hep-ph]].

[23] M. Blennow and E. Fernandez-Martinez, *Parametrization of Seesaw Models and Light Sterile Neutrinos*, Phys. Lett. B **704**, 223-229 (2011) doi:10.1016/j.physletb.2011.09.028 [arXiv:1107.3992 [hep-ph]].

[24] A. Atre, T. Han, S. Pascoli and B. Zhang, *The Search for Heavy Majorana Neutrinos*, JHEP **05**, 030 (2009) doi:10.1088/1126-6708/2009/05/030 [arXiv:0901.3589 [hep-ph]].

[25] M. Drewes, J. Hajer, J. Klaric and G. Lanfranchi, *NA62 sensitivity to heavy neutral leptons in the low scale seesaw model*, JHEP **07**, 105 (2018) doi:10.1007/JHEP07(2018)105 [arXiv:1801.04207 [hep-ph]].

[26] D. Curtin, M. Drewes, M. McCullough, P. Meade, R. N. Mohapatra, J. Shelton, B. Shuve, E. Accomando, C. Alpigiani and S. Antusch, *et al. Long-Lived Particles at the Energy Frontier: The MATHUSLA Physics Case*, Rept. Prog. Phys. **82**, no.11, 116201 (2019) doi:10.1088/1361-6633/ab28d6 [arXiv:1806.07396 [hep-ph]].