PROCEEDINGS OF SCIENCE

Gravi/dark photons in D-brane and holographic models, and the hypercharge portal

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This proceeding is based on [1], where we analyze and compare the mixing of graviphotons and dark brane photons to the Standard Model hypercharge. This analysis has been performed in full generality in two different frameworks. The first is a holography-inspired four-dimensional QFT where SM is coupled to a hidden large-N theory via heavy messengers. The second is a weakly-coupled string theory.

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1. Introduction

The Standard Model (SM) of particle physics is complete after discovering the Higgs particle. However, several theoretical and experimental puzzles signify that a greater Fundamental Theory lays beyond the SM.

There have been many attempts to identify this Fundamental Theory. Between them, string theory is an ultraviolet (UV) finite framework that includes a consistent description of quantum gravity. To do so, it introduces a natural cutoff, the string scale. The string's excited states become effective at this scale, resolving some gravitational and other interactions issues. At energies below the Planck scale, perturbative string theories are well-defined, but string perturbation theory breaks down at or above that scale. The AdS/CFT correspondence [1] and holography can now produce a UV-complete and non-perturbative theory.

In this proceeding, we use holographic ideas to construct a generic form of the Fundamental Theory and study its effective action at low energies [2]. We assume that *all interactions are described by four-dimensional (non-gravitational) quantum field theories*, defined on a flat Minkowski background metric, and gravity is an emerging interaction from a theory without gravity [2]. This Fundamental Theory is ultraviolet-complete, and it consists of three sectors,

a) The SM sector contains all known fields (quarks, leptons, gauge fields, Higgs). Initially, we will assume that any field beyond them comes from the hidden/messenger sector. Later on, we will study cases where some of the extra fields belong to the SM sector.

b) A hidden sector (HS) is disconnected from the SM sector and is UV-complete. Therefore, all couplings in the Hidden sector are either asymptotically free or conformal in the extreme UV region. The HS contains a gauge group with a large rank, and its structure is random. For simplicity, however, we will assume $SU(N_i)$ factors with N_i very large.

   At weak coupling (infrared regime (IR)), the spectrum of this HS contains the standard fields: vectors $\widehat{A}_\mu$, scalars $\widehat{\phi}$, and spin-1/2 particles $\widehat{\psi}$ (hatted fields denote field from the hidden sector), transforming under this $SU(N_i)$. The vectors will be in the adjoint; however, the rest can transform as fundamental or bifundamental representations, including adjoints, symmetric-, and antisymmetric-representations [2].

c) A messenger sector that communicates the SM and the HS. It consists of bifundamental fields charged under both the SM and hidden gauge groups. These messenger fields have large masses, M_{mess}, larger than all other scales in this framework.

Details of the hidden and messenger sectors will not be fixed since we want to investigate the general properties of this framework. The generic assumptions made above are based on string theory. Also, supersymmetry is not necessarily present in this setup.

At energies below the messenger scale, M_{mess}, the heavy messengers are integrated out. We obtain an effective action for the low-energy theory that involves operators of the SM and the hidden

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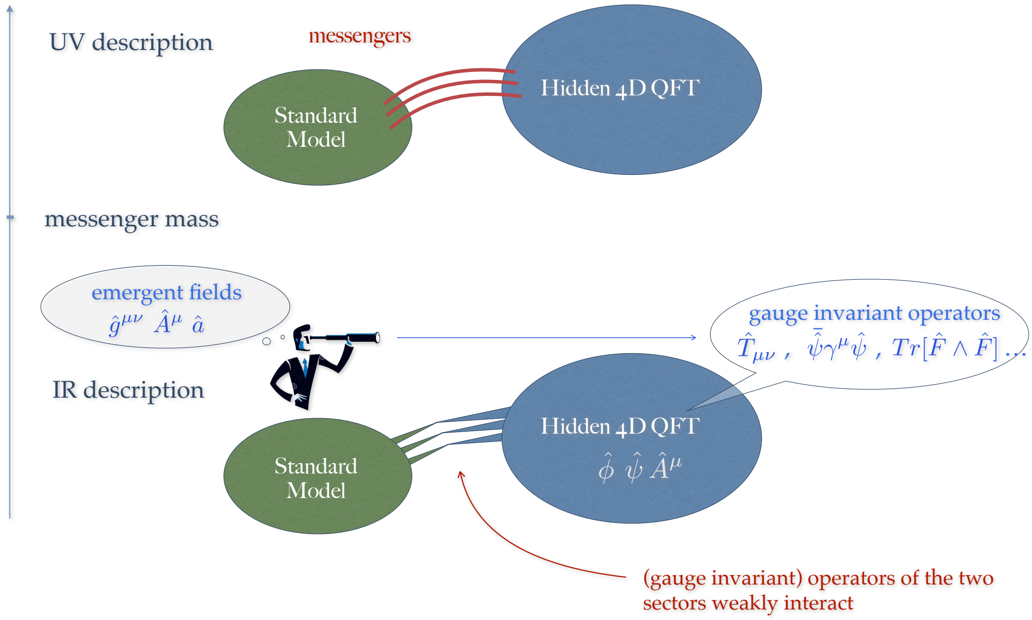
[1] According to the AdS/CFT correspondence, string theories at a given background are dual to large-N (and holographic) quantum field theories. Information for a strongly coupled QFT can be derived from a weakly coupled string theory and vice versa.

[2] Typically representations in string theory configurations.

theory coupled with double (or multiple) trace interactions, all irrelevant [3]. As a result, at low energies, the new interactions triggered between the two theories are weak.



A priori, all gauge-invariant operators of the hidden sector can couple to the SM gauge-invariant operators. Such operators are typically massive, with masses proportional to the messenger scale. However, there are operators which are **protected by symmetries**, and they do not acquire masses of the messenger scale, remaining“light” and hence are particularly interesting for phenomenology. *These light operators of the hidden sector appear as (emergent) weakly-coupled fields/particles from the SM perspective*, are listed bellow

– The stress-tensor $\widehat{T}_{\mu\nu}$ of the HS ∼ $g_{\mu\nu}$ (emergent) gravity coupled to the SM.

  The stress-energy tensor of the hidden sector appears as an external metric to the SM generating an emergent gravitational force [3]. It is worth mentioning that this emergent gravity scenario bypasses the Weinberg-Witten theorem since it predicts a non-vanishing cosmological constant that depends on the parameters of the hidden sector.

  Other low dimensional operators of the hidden sector can also couple to the stress-energy tensor of the SM. Nevertheless, being unprotected by symmetries will acquire very heavy masses, and they will be absent at low energies.

– The instanton density $Tr[F_{\widehat{A}} \wedge F_{\widehat{A}}]$ of the HS ∼ a emergent axions coupled to the SM.

  The instanton densities of the hidden sector are unique since approximate Peccei-Quinn-like symmetries protect them in perturbation theory. They couple to the instanton densities of the SM appearing as dynamical θ-angles. These operators generate axion-like particles [4].

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[3] There are two exceptions, however, couplings of the SM with the hidden sector which might be relevant.

– The Higgs mass term. This term is related to the hierarchy problem, and it will be avoided by assuming that all operators of the hidden theory have dimensions well above two.

– The field strength of the hypercharge. This mixing has been extensively studied in [1].

- Conserved global currents $Tr[\bar{\widehat{\psi}}\gamma^{\mu}\widehat{\psi}]$ of the HS $\sim A^{\mu}$ emergent gauge fields coupled to the SM [1, 5].

  These emergent gauge fields have very weak couplings with the SM, and none of the SM particles are directly charged under these gauge fields. This proceeding aims to parametrize the effective action and investigate the phenomenology of this “fifth-force," which can play the role of graviphotons/dark photons.

- Fermionic operators either from the messenger or the hidden sector ~ emergent right-handed neutrinos coupled to the SM. Contrary to the previous operators, these fermionic bound-states of messengers are not protected by symmetries, and they can become very massive, triggering the seesaw mechanism [6].

- Such a scenario can also provide a new portal to dark matter, as argued in [7].

These operators couple to the SM with specific properties. In this proceeding, we aim to investigate the effect of the conserved global currents of the HS (emergent graviphotons/dark photons) theoretically and explore their phenomenological consequences [8].

Holographic hidden sector.

AdS/CFT correspondence has shown us a *holographic* (gravitational), higher-dimensional picture of the framework discussed before. In that picture, the SM sector is localized on a four-dimensional brane, immersed in a five-dimensional space-time at an appropriate radial direction corresponding to the messenger mass cutoff M_{mess}. The hidden sector lives in all five dimensions.

We will consider both possibilities of the hidden theory to be either Large-N or holographic in this proceeding. This will result in differences since the emerging fields in the first case do not exhibit five-dimensional properties, whereas in the second case, they do.

2. Emergent gauge fields

After integrating out heavy messengers, conserved global currents of the hidden sector couple to the SM as emergent gauge fields. They are weakly coupled to the SM, and they can play the role of graviphotons or/and dark photons [1, 5].

These emergent gauge fields have very light masses and a “compositeness" scale. Bellow the compositeness scale, these operators behave like standard (point-like) gauge fields. Above, they have non-local kinetic terms, with particular behavior that distinguishes them from similar fields studied in the past.

Next, we would like to classify our options for the SM and the global currents from the HS. The SM symmetries are

- Non-anomalous abelian gauge symmetries, like the hypercharge. The trace of such a symmetry under all fermionic fields of the SM satisfies $Tr[Y] = 0$.

- Anomalous abelian gauge symmetries. Such gauge fields appear in D-brane realizations of the SM (see [9–14]). Such symmetries have massive gauge bosons (whose effective interactions were analyzed in [12]), and B-L should be in that class. The structure of anomalies was analyzed in [13]. Typically the charge matrices of these symmetries satisfy $Tr[Q] \neq 0$.

- Non-abelian gauge fields, coming from the SM's SU(2) and SU(3).

On the other hand, we can have the following global symmetries.

- Symmetries that only messengers are charged under.

  Here we assume that the coupling is such that the masses of messenger-mesons are well above the SM scales. Such symmetries are therefore irrelevant for our discussion, as they will not be present at SM energies [1].

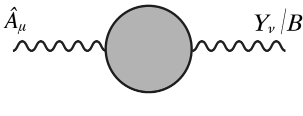
- R-like symmetries[4], which affect messengers and hidden sector fields. Such symmetries cannot be gauged under the SM, as in that case, the "bulk" states would have SM charges, a phenomenologically unappealing situation. However, as such symmetries act only on the hidden theory, they can be effectively treated in the low-energy theory as those of the following case after integrating out the messengers.

- Flavour symmetries of the large-N QFT. Messengers do not have a direct coupling to them, but couplings might occur by quantum corrections. All of the above can be, in principle, abelian or non-abelian. We will focus on this type of global currents.

Focusing on abelian (non-anomalous or anomalous) from the SM sector and Flavour symmetries from the HS side, we parametrize the effective action and these emergent gauge fields couple to all gauge-invariant antisymmetric tensors of the SM by using holographic principles and large-N expansions.

$$\begin{aligned} W_6 \sim\ & \frac{1}{NM^2} Tr[D_\mu H D_\nu H^\dagger] F^{\mu\nu}_{\widehat{A}} + \frac{1}{N^{\frac{3}{2}} M^2} F^{\mu\nu}_{\widehat{A}} [\bar{\psi}\gamma_{\mu\nu} H \psi + c.c.] \\ & + \frac{1}{N^{\frac{3}{2}} M^2} F^{\mu\nu}_{\widehat{A}} F_{Y,\mu\nu} H H^\dagger + \frac{1}{N^2 M^4} F^{\mu\nu}_{\widehat{A}} F_{Y,\mu\nu} [\bar{\psi} H \psi + c.c.] + \cdots \end{aligned} \tag{1}$$

where $\widehat{A}$ is the emergent gauge field from the hidden sector and H, ψ are the Higgs and fermions of the SM. The lowest dimensional operators of the SM are of the sixth order. It is worth mentioning that the kinetic mixing with the hypercharge is absent since messengers are neutral under these global symmetries of the hidden sector. However, quantum corrections involving SM fields generate such mixing.



$$\rightarrow \quad C_{\widehat{A}Y} \int d^4p\, F_{\widehat{A}}(p) \cdot F_Y(-p) \tag{2}$$

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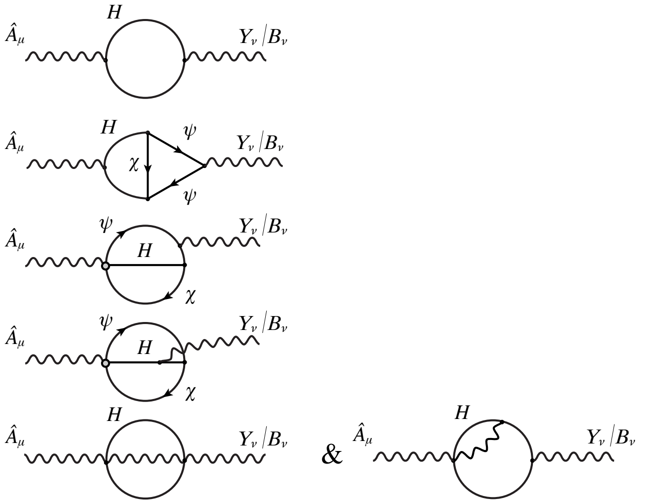
[4] R-like symmetries are global symmetries that resemble R-symmetries in supersymmetric theories. For example, one can break supersymmetry without breaking an R-symmetry. Alternatively, as in N=1 sQCD, the R-symmetry can mix with another U(1) global symmetry to make a non-anomalous R-like symmetry. In string theory, R-symmetries are never exact. They can be symmetries of the massless sector and are subgroups of the orthogonal symmetry of the internal space. They may be present even in the absence of supersymmetry in the massless sector. The compactification breaks them of α' corrections.

In [1], we have evaluated the contribution to the kinetic mixing, up to the second loop order in the unbroken and the broken phase of the SM. The results are presented bellow

- In the unbroken phase, the mixings are generated by loop diagrams from the action (1).



$$\begin{aligned} C_{\widehat{A}Y} &\sim \frac{\Omega_3}{8}\frac{Q_Y^H}{N}\frac{\Lambda^2}{M^2} \\ C_{\widehat{A}Y} &\sim Q_Y^\psi |g_{H\psi\chi}|^2 \frac{\Lambda^2}{NM^2} \\ C_{\widehat{A}Y} &\sim \frac{Q_Y^H g_{H\psi\chi}}{N^{\frac{3}{2}}} \\ C_{\widehat{A}Y} &\sim \frac{g_{H\psi\chi} m^2}{M^2 N^{\frac{3}{2}}} \log\frac{\Lambda^2}{m^2} \\ C_{\widehat{A}Y} &\sim \frac{(Q_Y^H)^2}{N} \end{aligned} \tag{3}$$

Therefore, the leading contribution comes from the 1-loop diagram with the Higgs circulating in the loop.

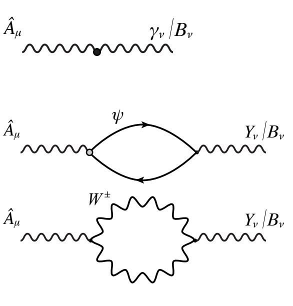
- In the broken phase, the action becomes

$$\begin{aligned} W_{BROKEN} \quad \sim \quad & \frac{4g_w^2}{NM^2}(h+v)^2 F^{\widehat{A}}_{\mu\nu} W^\mu_+ W^\nu_- + \frac{4ie}{NM^2}(h+v) F^{\widehat{A}}_{\mu\nu} A^\mu_\gamma \partial^\nu h \\ & + \frac{4e}{NM^2}\sqrt{g_w^2+g_Y^2}(h+v)^2 F^{\widehat{A}}_{\mu\nu} A^\mu_\gamma Z^\nu + \frac{1}{N^{\frac{3}{2}}M^2} F^{\mu\nu}_{\widehat{A}} \left[(h+v)\bar{\psi}\gamma_{\mu\nu}\psi + c.c.\right] \\ & + \frac{1}{NM^2} F^{\widehat{A}}_{\mu\nu}(\cos\theta_w F^{\gamma,\mu\nu} - \sin\theta_w F^{Z,\mu\nu})(h+v)^2 \\ & + \frac{1}{N^2M^4} F^{\widehat{A}}_{\mu\nu}(\cos\theta_w F^{\gamma,\mu\nu} - \sin\theta_w F^{Z,\mu\nu}) \left[\bar{\psi}\psi(h+v) + c.c.\right] \end{aligned}$$

and the kinetic mixing involve the photon γ and the Z boson.

$$C_{\widehat{A}\gamma Z}\int d^4p\, F^{\widehat{A}}_{\mu\nu}(p)\Big(\cos\theta_w F^{\gamma,\mu\nu} - \sin\theta_w F^{Z,\mu\nu}\Big)(-p) \tag{4}$$

The loop diagrams now contribute with



$$\begin{aligned} C_{\widehat{A}\gamma Z} &\sim \frac{v^2}{NM^2} \\ C_{\widehat{A}\gamma Z} &\sim Q_Y^\psi |g_{H\psi\chi}|^2 \frac{\Lambda^2}{NM^2} \\ C_{\widehat{A}\gamma Z} &\sim \frac{Q_Y^H g_{H\psi\chi}}{N^{\frac{3}{2}}} \end{aligned} \tag{5}$$

Results.

Emergent (composite) dark photons arising from large-N strongly-coupled or hidden holographic theories provide new physics associated with the hypercharge portal of the SM. We have estimated that the strength of the hypercharge mixing is naturally tiny and can be of order $O(N^{-3/2})$ and in tuned cases $O(N^{-2})$. Therefore, one can evade today's experimental constraints with relative ease. Moreover, such physics may have significant effects in the dark matter arena. There is also the exceptional case of an extension of the SM by an adjoint scalar, whose vev can enhance the mixing to $O(N^{-1})$. An analysis of this case may also be interesting.

In case there are anomalous $U(1)$s in the SM sector, we expect no difference on the mixings with the emergent $U(1)$ [10–14].

3. Comparison with string theory.

The above scenario often appears in string theory. In addition, AdS/CFT correspondence indicates that strongly coupled large-N QFTs are related to weakly-coupled string theories and vice-versa. In this framework, emergent vectors are related to gravi/dark photons in string theory. Our goal is to compare couplings between the abelian gauge fields and SM fields in the two scenarios.

In string theory, there are two different sources for abelian gauge fields.

- Closed sector: These $U(1)$s come from the RR/NSNS closed string sector. Since this is the same sector where the graviton is, we call them $graviphotons$.
- Open sector: $U(1)$ factors appear from open strings with both ends on the same stack of D-branes. In our case, we will assume that this stack of D-branes lives at a distance from the SM stack, and therefore it lives on a hidden "dark" stack of D-branes. These $U(1)$s will be called "dark-brane-photons".

Our goal here is to reproduce the couplings in (1) by string scattering amplitudes that involve the gravi/dark $U(1)$'s and SM fields in D-brane realizations of the SM. We perform our computations in flat four-dimensional space-time with internal tori, orbifolds, or CYs at points in their moduli space where a CFT is available. Also, we include closed-string fluxes in the NSNS and RR sector, representing the first step towards a computation in a warped space such as AdS or alike.

The stringy amplitudes that we will evaluate have a different structure, depending on the closed/open string type that describes the abelian field. In addition, SM fields are described by open strings, and they have to be inserted on a boundary [15]. Therefore, we have the following cases

- Graviphotons (closed strings): The closed string is inserted in the bulk of the surface with at least one boundary. The lowest such surface is the disk. On the disk boundary, we insert the vertex operators for the SM fields. As we have already mentioned, the presence of RR/NSNS fluxes are also considered.
- Dark-brane photons (open strings): In this case, the vertex operator should be inserted in a different boundary than the vertex operators of the SM fields. The lowest order diagram, in this case, is the cylinder.

Bellow, we present the EFT couplings (first column) that appear in (1) and the lowest string amplitudes (2nd, 3rd, 4th columns), which can provide them. By A^μ, ϕ, λ, we denote the vertex operators (VO) of gauge, scalar, and fermionic fields from the SM sector. As usual, in D-brane realizations of the SM, these fields belong to the open sector, and they live on D-branes. The $\widehat{A}^\mu$ in the 2nd and 3rd column is a graviphoton, aka a $U(1)$ from the closed sector, inserted in the bulk of the amplitudes. In the 3rd column, we additionally evaluate the couplings in the presence of RR/NSNS fluxes, denoted by $W_{RR/NSNS}$. In the 4th column, the $\widehat{A}^\mu$ is a dark-photon, living on a stack far from the SM stack of D-branes.

EFT coupling string theory amplitudes

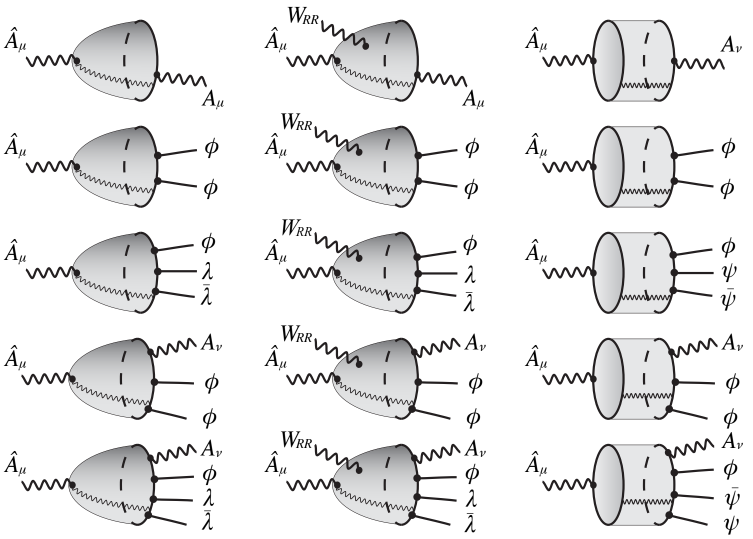
$$\frac{\Lambda^2}{NM^2} F^{\widehat{A}}_{\mu\nu} F^{Y,\mu\nu}$$

$$\frac{1}{NM^2} D^\mu H^\dagger D^\nu H F^{\widehat{A}}_{\mu\nu}$$

$$\frac{1}{N^{\frac{3}{2}}M^2} \bar{\psi}\gamma^{\mu\nu} H\psi F^{\widehat{A}}_{\mu\nu}$$

$$\frac{1}{N^{\frac{3}{2}}M^2} F^{\widehat{A}}_{\mu\nu} F^{Y,\mu\nu}\ H^\dagger H$$

$$\frac{1}{N^2M^4} F^{\widehat{A}}_{\mu\nu} F^{Y,\mu\nu} \bar{\psi} H\psi$$



Results.

The results are encoded in the following table, where the relation between the large-N and the string scale g_s is given by $g_s = 1/N$.

EFT coupling	EFT estimate	graviphoton	graviphoton +bulk fluxes	dark photon
$F\widehat{F}$	$O\left(\frac{1}{N}\right)$	$O\left(g_s^2\right)$	$O(g_s^{3/2})$	$O\left(g_s\right)$
$\phi\ F\widehat{F}$	$O\left(\frac{1}{N}\right)$	$O\left(g_s\right)$		
$DHDH^\dagger\widehat{F}$	$O\left(\frac{1}{N}\right)$	$O\left(g_s^2\right)$	$O\left(g_s^2\right)$	$O(g_s^{3/2})$
$HH^\dagger F\widehat{F}$	$O\left(\frac{1}{N^{3/2}}\right)$	$O(g_s^{5/2})$	$O(g_s^{5/2})$	$O\left(g_s^2\right)$
$\bar{\psi}H\gamma^{\mu\nu}\psi\widehat{F}_{\mu\nu}$	$O\left(\frac{1}{N^{3/2}}\right)$	$O(g_s^{3/2})$	$O(g_s^{5/2})$	$O\left(g_s^2\right)$
$\bar{\psi}H\psi F\widehat{F}_{\mu\nu}$	$O\left(\frac{1}{N^2}\right)$	$O\left(g_s^2\right)$	$O\left(g_s^3\right)$	$O(g_s^{5/2})$

Notice that only in the $\phi\ F\widehat{F}$, $\bar{\psi}H\gamma^{\mu\nu}\psi\widehat{F}_{\mu\nu}$ and $\bar{\psi}H\psi F\widehat{F}_{\mu\nu}$ we have an agreement at leading order between the EFT and the graviphoton case. The rest (graviphotons+fluxes and dark photons) are subleading to the EFT estimation.

Directions.

The case of emergent abelian factors is not very much studied, and we plan to fill this gap [16, 17].

Studying the emergent gauge fields above the compositeness scale is interesting. It is also worth mentioning that the actions obtained in [5] are similar to those postulated earlier by Kraus and Tomboulis [18]. Therefore, it is interesting to study cases where conserved global currents of the hidden sector acquire non-vanishing vevs. In this case, Lorentz invariance breaks at the vev's scale. However, the low energy theory remains approximately Lorentz invariant.

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