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Relating the phases of CKM and PMNS matrices in 2HDM

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We propose a way of relating CP violation in the quark and in the lepton sectors, that as usual we parametrize by the CKM and PMNS phases. If the origin of the CP breaking is in the complex Yukawa couplings, both in the quark and lepton sectors, the previous relation will not be possible in general, since Yukawa couplings in the two sectors have independent flavour structures. We will show that both the δ_{CKM} and δ_{PMNS} phases can instead be generated by a vacuum phase in a class of two Higgs doublet models, and in this case a connection may be established. This scenario requires, both in the quark and lepton sectors, the presence of scalar FCNC at tree level. The appearance of these FCNC is an obstacle since one has to analyse which models are able to conform to the strict experimental limits on FCNC, both in the quark and lepton sectors. On the contrary, this class of models is falsifiable since FCNC arise at a level which can be probed experimentally in the very near future, especially in processes like $h \to e^{\pm}\tau^{\mp}$ and $t \to hc$. The connection between CP violations in δ_{CKM} and δ_{PMNS} is explicitly illustrated in models with Minimal Flavour Violation with very interesting predictions.

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1. Introduction

Our purpose in this work is to analyse the possibility of having a relation between CP violation in the quark and in the lepton sectors, parametrized by the corresponding complex phases: the Cabibbo-Kobayashi-Maskawa (CKM) phase δ_{CKM} in the quark sector [1,2] and the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) phase δ_{PMNS}, in the lepton sector [3,4].
We have a very solid experimental evidence that $\delta_{CKM} \neq 0$, corresponding to a complex CKM mixing matrix in the quark sector, even if one allows for the presence of New Physics contributing to CP violation [5,6]. Nevertheless, a complex CKM does not imply necessarily CP violation at the Lagrangian level through complex Yukawa couplings. Indeed, one may have a vacuum induced CP violation, from the Higgs potential, generating a complex CKM matrix in agreement with experiment [7,8].
In any extension of the Standard Model (SM) with non-vanishing neutrino masses and assuming that the origin of CP violation, in the lepton sector, is the presence of complex Yukawa couplings, then there is no relation between δ_{CKM} and δ_{PMNS}.
To obtain a relation between CKM and PMNS phases, an interesting possibility is to assume that the CP symmetry is spontaneously broken, with the irremovable vacuum phase generating both the phase in the quark and in the lepton sectors. We will follow this avenue

2. Spontaneous CP violation and the 2HDM

It was T.D. Lee, in 1973, that he proposed the first model of spontaneous CP violation (SCPV) in order to put the breaking of CP symmetry on the same footing as the breaking of the gauge symmetry [9]. It was achieved through the introduction of the two Higgs doublets model (2HDM), with vacuum expectation values having a relative phase which violates both, T and CP invariance. It is very important to realize that the general 2HDM [10,11] has Scalar Flavour Changing Neutral Couplings (SFCNC) at tree level which of course can be highly dangerous and therefore we need to put them under control.

One way of eliminating right away SFCNC is by imposing Natural Flavour Conservation (NFC) a la Glashow-Weinberg [12] through a Z_2 symmetry. A way of putting SFCNC – not eliminating completely- under control is by using Minimal Flavour Violation (MFV) models, obtained from a symmetry, like is the case of the Branco-Grimus-Lavoura (BGL) models [13-18]–in particular with a Z_4 symmetry-.

But it was proven long time ago by Branco that, quite generally, SCPV and NFC generates a real CKM [19,20]. By different reasons, BGL models cannot present SCPV because the Z4 symmetry is too constraining in the Higgs potential.

Therefore, in the framework of 2HDM we need to keep the possibility of having SCPV, avoid NFC and maintain SFCNC under control. All these ingredients appear in the so called generalized BGL models (gBGL) [21] with a softly broken Z_2 symmetry, realized in the Yukawa sector in a flavor dependent way, therefore not meeting NFC criteria and at the same time with SFCN with a limited intensity.

3. The gBGL model with SCPV

In general, the gBGL model is defined by

$$\begin{gathered}\Phi_1 \to \Phi_1 \; ; \; \Phi_2 \to -\Phi_2 \\ Q_{L_{1,2}} \to Q_{L_{1,2}} \; ; \; Q_{L_3} \to -Q_{L_3} \\ u_R \to u_R \; ; \; d_R \to d_R\end{gathered} \qquad (1)$$

With the introduction of right handed neutrino [22] –just considering the Dirac case for simplicity- we have for the Yukawa sector

$$\begin{gathered} L_Y = -\overline{Q_L}\left(\Gamma_1^{(d)}\Phi_1 + \Gamma_2^{(d)}\Phi_2\right)d_R - \overline{Q_L}\left(\Gamma_1^{(u)}\widetilde{\Phi}_1 + \Gamma_2^{(u)}\widetilde{\Phi}_2\right)u_R \\ -\overline{L_L}\left(\Gamma_1^{(e)}\Phi_1 + \Gamma_2^{(e)}\Phi_2\right)l_R - \overline{L_L}\left(\Gamma_1^{(\nu)}\Phi_1 + \Gamma_2^{(\nu)}\Phi_2\right)\nu_R + h.c. \end{gathered} \qquad (2)$$

The couplings of the gBGL model are fixed by the Z_2 symmetry (1) in the following way

$$\Gamma_1^{(d)} \sim \Gamma_1^{(u)} \sim \Gamma_1^{(e)} \sim \Gamma_1^{(\nu)} \sim \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix} \qquad (3)$$

$$\Gamma_2^{(d)} \sim \Gamma_2^{(u)} \sim \Gamma_2^{(e)} \sim \Gamma_2^{(\nu)} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix} \qquad (4)$$

with real matrix elements. So in general we have the relations

$$\Gamma_2^{(f)} = P_3\Gamma_2^{(f)} \; ; \; \Gamma_1^{(f)} = (I - P_3)\Gamma_1^{(f)} \; ; P_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (5)$$

the Yukawa sector in the Higgs basis [23-25], with - $v^2 = v_1^2 + v_2^2, c_\beta = v_1/v_2, s_\beta = v_1/v_2$, -

$$\begin{pmatrix} e^{-i\theta_1}\Phi_1 \\ e^{-i\theta_2}\Phi_2 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ s_\beta & -c_\beta \end{pmatrix}\begin{pmatrix} \mathrm{H}_1 \\ \mathrm{H}_2 \end{pmatrix} \qquad (6)$$

can be written as

$$\begin{gathered} L_Y = -\frac{\sqrt{2}}{v}\overline{Q_L}\left(M_d^0 \mathrm{H}_1 + N_d^0 \mathrm{H}_2\right)d_R - \frac{\sqrt{2}}{v}\overline{Q_L}\left(M_u^0 \widetilde{\mathrm{H}}_1 + N_u^0 \widetilde{\mathrm{H}}_2\right)u_R \\ -\frac{\sqrt{2}}{v}\overline{L_L}\left(M_l^0 \mathrm{H}_1 + N_l^0 \mathrm{H}_2\right)l_R - \frac{\sqrt{2}}{v}\overline{L_L}\left(M_\nu^0 \widetilde{\mathrm{H}}_1 + N_\nu^0 \widetilde{\mathrm{H}}_2\right)\nu_R + h.c. \end{gathered} \qquad (7)$$

where

$$\begin{gathered} M_d^0 = \frac{v}{\sqrt{2}}\left(\Gamma_1^{(d)} c_\beta + \Gamma_1^{(d)} s_\beta e^{i\theta}\right) \; ; \; M_u^0 = \frac{v}{\sqrt{2}}\left(\Gamma_1^{(u)} c_\beta + \Gamma_1^{(u)} s_\beta e^{-i\theta}\right) \\ M_l^0 = \frac{v}{\sqrt{2}}\left(\Gamma_1^{(e)} c_\beta + \Gamma_1^{(e)} s_\beta e^{i\theta}\right) \; ; \; M_\nu^0 = \frac{v}{\sqrt{2}}\left(\Gamma_1^{(\nu)} c_\beta + \Gamma_1^{(\nu)} s_\beta e^{-i\theta}\right) \end{gathered} \qquad (8)$$

here $\theta = \theta_2 - \theta_1$ is the irremovable and CP violating relative phase among $\langle\Phi_1\rangle$ and $\langle\Phi_2\rangle$. For the N_f^0 couplings we get the simple and important result

$$N_f^0 = \left[t_\beta I - \left(t_\beta + t_\beta^{-1}\right)P_3\right]M_f^0 \qquad (9)$$

Note that even if N_f^0 are proportional to M_f^0 this proportionality involves a diagonal matrix different from the identity. This means that in general it will not be possible to bi-diagonalize both matrices simultaneously. The matrices N_f^0 control the scalar mediated flavour changing neutral couplings SFCNC that in general will be present in all sectors.

The Higgs potential is the standard for 2HDM with a Z_2 symmetry, including a soft breaking term. This way the possibility of having CP violation from the vacuum is open

$$\begin{gathered} V = \mu_{11}^2\Phi_1^\dagger\Phi_1 + \mu_{22}^2\Phi_2^\dagger\Phi_2 - \mu_{12}^2\left(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right) \\ + \left[\lambda_5\left(\Phi_1^\dagger\Phi_2\right)^2 + h.c.\right] + 2\lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) \\ + 2\lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 + \lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2 \end{gathered} \qquad (10)$$

the relative phase θ is fixed by the relation

$$\cos\theta = \frac{\mu_{12}^2}{2\lambda_5 \upsilon_1 \upsilon_2} \qquad (11)$$

and it signals CP violation from the vacuum provided $\theta \neq 0, \pm\frac{\pi}{2}, \pm\pi$.

4. Generation of CP violating CKM and PMNS matrices

CP invariance of the Lagrangian imposes

$$\Gamma_i^{(f)} = \Gamma_i^{(f)*} \qquad (12)$$

From the structure of equations (5,8) it becomes evident that

$$M_f^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\sigma_f} \end{pmatrix} \widehat{M}_f^0 \equiv \Phi_3(\sigma_f)\widehat{M}_f^0 \qquad (13)$$

with $\widehat{M}_f^0$ an arbitrary real mass matrix and with σ_f taking the values $+\theta$ for $f = d, e$ and $-\theta$ for $f = u, \nu$.

Obviously

$$\begin{gathered} M_f^0 M_f^{0\dagger} = \Phi_3(\sigma_f)\widehat{M}_f^0 \widehat{M}_f^{0T} \Phi_3(-\sigma_f) \\ M_f^{0\dagger} M_l^0 = \widehat{M}_f^{0T}\, \widehat{M}_f^0 \end{gathered} \qquad (14)$$

therefore $M_f^{0\dagger} M_l^0$ will be diagonalized by a real orthogonal matrix O_{f_R}

$$O_{f_R}^T M_f^{0\dagger} M_f^0 O_{f_R} = \begin{pmatrix} m_{f_1}^2 & 0 & 0 \\ 0 & m_{f_2}^2 & 0 \\ 0 & 0 & m_{f_3}^2 \end{pmatrix} \qquad (15)$$

and in a similar way we get

$$U_{f_L}^\dagger M_f^0 M_f^{0\dagger} U_{f_L} = \begin{pmatrix} m_{f_1}^2 & 0 & 0 \\ 0 & m_{f_2}^2 & 0 \\ 0 & 0 & m_{f_3}^2 \end{pmatrix};\ U_{f_L} = \Phi_3(\sigma_f) O_{f_L} \qquad (16)$$

where O_{f_L} will be a real orthogonal matrix, in such a way that the bi-diagonalization becomes

$$M_f = U_{f_L}^\dagger M_f^0 O_{f_R} = \begin{pmatrix} m_{f_1} & 0 & 0 \\ 0 & m_{f_2} & 0 \\ 0 & 0 & m_{f_3} \end{pmatrix} \qquad (17)$$

Therefore, because de CKM and PMNS matrices are defined by $V_{CKM} = U_{u_L}^\dagger U_{d_L}$ and $U_{PMNS} = U_{e_L}^\dagger U_{\nu_L}$ respectively we will have

$$\begin{gathered} V \equiv V_{CKM} = O_{u_L}^T \Phi_3(2\theta) O_{d_L} \\ U \equiv U_{PMNS} = O_{e_L}^T \Phi_3(-2\theta) O_{\nu_L} \end{gathered} \qquad (18)$$

Since O_{f_L} are arbitrary real rotations, it is evident that there is enough freedom to obtain arbitrary V and U, except for the fact that any CP violating observable in the quark sector and any CP violating observable in the lepton sector, must vanish with $\theta \to 0$.

It is thus interesting to scrutinize in detail the relation that must exist among the CP violating phases in V and U, δ_{CKM} and δ_{PMNS} respectively. Where δ_{CKM} and δ_{PMNS} will simply correspond to the CP phases in a standard parametrization.

5. CP violation in the Charged Currents and the presence of SFCNC

To present the relation among δ_{CKM} and δ_{PMNS} the simplest approach would be to impose that SFCNC are absent, since there is no evidence yet of the existence of SFCNC beyond the SM. But this, as we will see, leads to a real CKM, contrary to evidence, and thus SFCNC are necessary.

The appearance of SFCNC is encoded in the N_f^0 matrices which control the Yukawa couplings to H_2. In the fermion mass basis, where one can fully appreciate SFCNC, N_f^0 becomes N_f:

$$N_f = U_{f_L}^{\dagger} M_f^0 O_{f_R} = \left[t_\beta I - \left(t_\beta + t_\beta^{-1}\right) P_3^f\right] M_f$$
$$= \left[t_\beta I - \left(t_\beta + t_\beta^{-1}\right) P_3^f\right] \begin{pmatrix} m_{f_1} & 0 & 0 \\ 0 & m_{f_2} & 0 \\ 0 & 0 & m_{f_3} \end{pmatrix} \qquad (19)$$

where we have introduced the projector operators

$$P_3^f \equiv U_{f_L}^{\dagger} P_3\, U_{f_L} = O_{f_L}^T P_3 O_{f_L} \qquad (20)$$

therefore SFCNC are controlled by the real projectors P_3^f, in particular by the off-diagonal entries of P_3^f, which are controlled by the O_{f_L} matrices, which also give the CKM and PMNS mixing matrices.

It is important to notice, that by construction

$$P_3^u = V P_3^d V^{\dagger}\ ;\ \ P_3^e = U P_3^{\nu} V U^{\dagger}$$
$$\left[O_{u_L}^T P_3 O_{u_L}\right] = \left[O_{u_L}^T \Phi_3(2\theta) O_{d_L}\right]\left[O_{d_L}^T P_3 O_{d_L}\right]\left[O_{d_L}^T \Phi_3(-2\theta) O_{u_L}\right] \qquad (21)$$

that means that SFCNC in the up and down quark sectors are not independent, they are related through the CKM matrix. This fact will be particularly relevant in order to address appropriately the counting and the election of the independent parameters.

The elements of the matrices P_3^f are

$$\left(P_3^f\right)_{ij} \equiv \left(O_{f_L}^T P_3 O_{f_L}\right)_{ij} = \left(O_{f_L}^T\right)_{i3}\left(O_{f_L}\right)_{3j} \equiv \hat{r}_{[f]i}\hat{r}_{[f]j} \qquad (22)$$

where $\hat{r}_{[f]i} \equiv \left(O_{f_L}^T\right)_{i3}$ are the components of real, unit vectors in three dimensions $\hat{r}_{[f]}$: the third rows of the orthogonal matrices O_{f_L}

$$O_{f_L} = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \hat{r}_{[f]1} & \hat{r}_{[f]2} & \hat{r}_{[f]3} \end{pmatrix} \qquad (23)$$

In principle each $\hat{r}_{[f]}$ would require two independent parameters but because the relations in equation (21) we only need two parameters to control SFCNC in the quark sector in addition to VCKM: these make a total of six parameters. Similar should be in the lepton sector.

Coming back to our objective of relating SFCNC and CP violation in the mixing matrix we can ask for the consequences of eliminating SFCNC in some sector [26]. It is clear from equations (19) and (22) that to have N_f diagonal we need to have P_3^f diagonal and this can only be achieved by imposing one of its components equal to 1 and therefore vanishing the others:

$$\left(P_3^f\right)_{ij} = \delta_{ik}\delta_{jk} \equiv (P_k)_{ij} \qquad (24)$$

Under this condition we will show that the corresponding mixing matrix does not violate CP. For definiteness we work in the lepton sector:

$$U = O_{e_L}^T \Phi_3(-2\theta) O_{\nu_L} = O_{e_L}^T\left[I + \left(e^{-2i\theta}\right)P_3\right]O_{\nu_L}$$
$$= O_{e_L}^T O_{\nu_L} O_{\nu_L}^T\left[I + \left(e^{-2i\theta}\right)P_3\right]O_{\nu_L} = O_{e_L}^T O_{\nu_L}\left[I + \left(e^{-2i\theta}\right)P_3^{\nu}\right] \qquad (25)$$

and from this result it is clear that if $P_3^{\nu} = P_k$ it turns out that the matrix $\left[I + \left(e^{-2i\theta}\right)P_3^{\nu}\right]$ is a diagonal of phases and therefore in this case equation 25 represents an orthogonal matrix times a real of phases that can be reabsorbed in the neutrino field phases and therefore U_{PMNS} can be chosen as a real orthogonal matrix conserving CP. The argument in equation (25) can be extended to the charged lepton sector or to both quark sectors concluding that in the 2HDM with SCPV of the type gBGL:

1) To have CP violation in the CKM matrix, there must be tree level SFCNC both in the up and in the down quark sectors.

2) To have a non-vanishing CP violating phase in the PMNS matrix, there must be tree level SFCNC both in the neutrino and in the charged lepton sectors.

6. The general relation between δ_{CKM} and δ_{PMNS}

Before arriving to the relation among δ_{CKM} and δ_{PMNS} we need to clarify the parameter counting in these models. Let us introduce the usual rotations ($s_x \equiv \sin x$, $c_x \equiv \cos x$)

$$R_{12}(x) = \begin{pmatrix} c_x & s_x & 0 \\ -s_x & c_x & 0 \\ 0 & 0 & 1 \end{pmatrix} ; \; R_{13}(x) = \begin{pmatrix} c_x & 0 & s_x \\ 0 & 1 & 0 \\ -s_x & 0 & c_x \end{pmatrix} \qquad (26)$$
$$R_{13}(x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_x & s_x \\ 0 & -s_x & c_x \end{pmatrix}$$

If for definiteness we analyse the quark sector, we can choose as a completely general parametrization

$$V = O_{u_L}^T \Phi_3(2\theta) O_{d_L}$$
$$O_{u_L} = R_{12}(p_1^u) R_{23}(p_2^u) R_{13}(p_3^u) \qquad (27)$$
$$O_{d_L} = R_{12}(p_1^d = 0) R_{23}(p_2^d) R_{13}(p_3^d)$$

Note that V will depend on $R_{12}^T(p_1^u) R_{12}(p_1^d) = R_{12}^T(p_1^u - p_1^d)$ therefore, without loss of generality, we can chose $p_1^d = 0$ in such a way that the number of independent parameters in the quark sector are six $\{p_1^u, p_2^u, p_3^u, p_2^d, p_3^d, \theta\}$ matching the four standard CKM $\{\theta_{12}^q, \theta_{13}^q, \theta_{23}^q, \delta_q\}$ and two from SFCNC $\{\hat{r}_{[u]1}, \hat{r}_{[u]2}\}$ or equivalently, two independent $\hat{r}_{[u]j}, \hat{r}_{[d]k}$.

The same happens in the lepton sector, six independent parameters in our PMNS matrix U, $\{p_1^e, p_2^e, p_3^e, p_2^\nu, p_3^\nu, \theta\}$ should match the four standard PMNS parameters $\{\theta_{12}^l, \theta_{13}^l, \theta_{23}^l, \delta_l\}$ and two from the SFCNC $\{\hat{r}_{[e]1}, \hat{r}_{[e]2}\}$ for example.

In summary, the experimental information constrains $\{\theta_{12}^q, \theta_{13}^q, \theta_{23}^q, \delta_q, \hat{r}_{[u]1}, \hat{r}_{[u]2}\}$ and could fix the model parameters $\{p_1^u, p_2^u, p_3^u, p_2^d, p_3^d, \theta\}$. A full analysis along these lines was presented in reference [7]. The most important aspect to be emphasized here is that, ideally, one can fix θ with this procedure, since CP violation is well established in the quark sector. Finally with the well-known PMNS mixing angles $\{\theta_{12}^l, \theta_{13}^l, \theta_{23}^l\}$ together with the knowledge of $\{\hat{r}_{[e]1}, \hat{r}_{[e]2}\}$ (from for example, $h \to l_i l_j$ or other processes changing flavor lepton number), and incorporating θ we should be able to predict δ_l. This is the precise way the connection between δ_{CKM} and δ_{PMNS} operates.

7. Quark sector analysis results

In the Higgs basis the components of the scalar doublets are

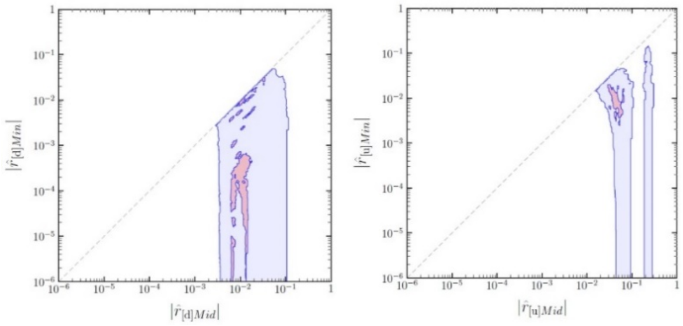
$$H_1 = \begin{pmatrix} G^+ \\ (v + H_0 + iG^0)/\sqrt{2} \end{pmatrix} ; \; H_2 = \begin{pmatrix} H^+ \\ (R_0 + iI_0)/\sqrt{2} \end{pmatrix} \qquad (28)$$

where G^+, G^0 are the would-be Goldstone bosons, H^+correspond to the charged scalars and the neutral one mass eigenstates h, H and A, are in general combinations of H_0, R_0 and I_0, fixed by the Higgs potential in equation (10) and determined by the orthogonal matrix R:

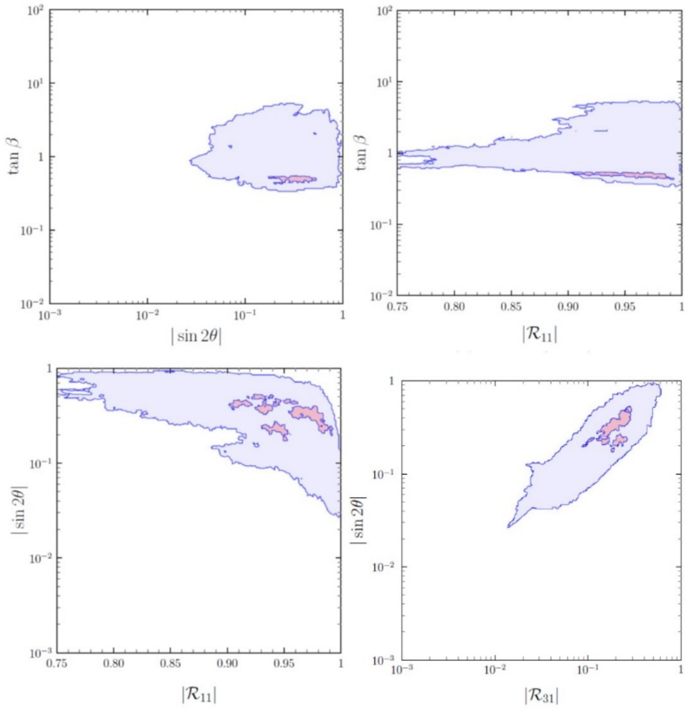
$$\begin{pmatrix} h \\ H \\ A \end{pmatrix} = R^T \begin{pmatrix} H_0 \\ R_0 \\ I_0 \end{pmatrix} \qquad (29)$$

The first important result in the quark sector is that the model is viable after surmounting flavour constraints, Higgs constraints, electroweak constraints and overall that, as we have shown, in spite of the fact that SFCNC cannot be eliminated to produce a correct δ_{CKM} [7].

If we plot the allowed regions for the minimal and medium components of $\hat{r}_{[q]}$ we get



In the figure to the left it is plotted the minimum and medium components of $\hat{r}_{[d]}$, named as $|\hat{r}_{[d]Min}|$ and $|\hat{r}_{[d]Mid}|$ and the same for $\hat{r}_{[u]}$ to the right of the figure. The main existence of a minimum for $|\hat{r}_{[d]Mid}|$ and for $|\hat{r}_{[u]Mid}|$ is a consequence of the necessity of having SFCNC in each sector in order to reproduce δ_{CKM}.
A much more surprising result is the one represented in the following figures



Here one can read out the ranges of $\tan\beta$, $|R_{11}|$, $|R_{31}|$ and $|\sin 2\theta|$. A minimum value for $|R_{31}|$ is a consequence of the CP violation in the Higgs potential after spontaneous CP breaking. One also sees that we cannot have neither very large nor very small $\tan\beta$. The most surprising result is the large range allowed for $|\sin 2\theta|$ between a few per cent up to almost one. This result is telling us that contrary to δ_{CKM} the determination of θ is much more poor.
Therefore, generalizing the full analysis to include the leptonic sector does not look the more promising way to begin with, especially if we are trying to show how it works the connection among δ_{CKM} and δ_{PMNS} in this kind of models, and as we have seen we have a very poor determination of θ implying that we will get a very poor determination of δ_{PMNS} or δ_l.
We need more simplified models were θ get fixed in a similar way how δ_{CKM} is fixed by experimental data in the Standard Model.

8. Simplified model to show the δ_{CKM} δ_{PMNS} connection

Therefore, the idea to show how can work the connection among δ_{CKM} and δ_{PMNS} will be to go to more restricted scenarios where, instead of using just the bare experimental constraints, we will make simplifying assumptions about the SFCNC sector. It is well-known that there is not any confirmed experimental evidence of the existence of SFCNC, therefore the idea will be to assumes as much as possible constraints in all the SFCNC sectors.

8.1 Simplified model: the quark sector

Once we have shown that we cannot assume the absence of SFCNC, the next level of simplicity will be to eliminate as much as possible SFCNC in the up and in the down quarks. If we remember that we have 6 parameters in the quark flavour sector, the idea will be to assume two experimentally guided constraints in the SFCNC sector in order to work with a parametrization consistent with the 4 parameters of the CKM matrix. But note that the way of eliminating as much as possible SFCNC –keeping a complex CKM- is to impose a zero in one of the components of the vector $\hat{r}_{[u]}$ and a zero in another of the components of the vector $\hat{r}_{[d]}$:

$\hat{r}_{[u]}$	$(0,\times,\times)$	$(\times,0,\times)$	$(\times,\times,0)$
$\hat{r}_{[d]}$	$(0,\times,\times)$	$(\times,0,\times)$	$(\times,\times,0)$

(30)

in this way there will be just SFCNC in the $d_i \leftrightarrow d_j$ transitions if the zero is in the $\hat{r}_{[d]k}$ component with $k \neq i,j$ and also just in the $u_l \leftrightarrow u_m$ if the zero is in the $\hat{r}_{[u]n}$ component with $n \neq l,m$. Note that these models should incorporate automatically the Minimal Flavour Violation (MFV) ansatz, because we will have as many parameters in the flavour sector as the four CKM one. In fact the still allowed SFCNC, in each sector, will be fixed by one of the 3 mixing angles of the V_{CKM} matrix, what certainly will be a very strong prediction.
Because the simplified models we considered are fixed by the shape we chose for the vectors $\hat{r}_{[u]}$ and $\hat{r}_{[d]}$ following the previous table, we have 9 models in the quark sector times 9 models in the lepton sector. 81 simplified models in total that has been analysed. A very important result [8] is that only one of these models survives all experimental data in the quark and in the lepton sectors. From now one we will present this model in the quark sector.
The surviving model in the quark sector is defined by

$$\hat{r}_{[u]} = (0, -\sin p_2^u, \cos p_2^u)\,; \quad \hat{r}_{[d]} = \left(-\sin p_2^d, 0, \cos p_2^d\right) \tag{31}$$

and since $\hat{r}_{[f]i} \equiv \left(O_{f_L}^T\right)_{i3}$, that is the third row of O_{f_L}, it is clear that

$$O_{u_L} = R_{12}(p_1^u) R_{23}(p_2^u) \sim \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix} \tag{32}$$

and also

$$O_{d_L} = R_{13}(p_2^d) \sim \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & 0 & \times \end{pmatrix} \tag{33}$$

We have therefore for the CKM matrix:

$$V = R_{23}^T(p_2^u)\Phi_3(2\theta)R_{12}^T(p_1^u)R_{13}\left(p_2^d\right) \tag{34}$$

And the result of the fit of V to the experimental data (to the known VCKM) gives

2θ	p_1^u	p_2^u	p_2^d
$1.077^{+0.039}_{-0.031}$	0.2269 ± 0.0005	$(4.235 \pm 0.059) \times 10^{-2}$	$(3.77 \pm 0.10) \times 10^{-3}$

(35)

In order to relate δ_{CKM} and δ_{PMNS} it is especially relevant that the quark sector fixes θ with an important precision at the few per cent level. Obviously $\hat{r}_{[u]}$ and $\hat{r}_{[d]}$ get fixed by

$$\begin{aligned} \hat{r}_{[u]} &\equiv (0, r_c, r_t) = (0, -0.0423, 0.9991) \\ \hat{r}_{[d]} &\equiv (r_d, 0, r_b) = (-0.0038, 0, 0.9999) \end{aligned} \tag{36}$$

A non-trivial result is that these values are within the allowed regions of the previous figures. Even if we have fixed the intensity of the SFCNC the precise effects in specific processes depend on other parameters like t_β and R_{ij} -the Higgs mixing matrix-. From the previous figures and taking, from the obtained θ value, $|sin\, 2\theta| = 0.88$ we get the ranges

$$R_{11} \in (0.82,0.90) \quad ; \quad t_\beta \in (0.5,1.8) \qquad (37)$$

The most relevant prediction of this model in the SFCNC sector concerns the transition $t \to ch$

$$B_r(t \to ch) = 0.1306(1 - R_{11}^2)\left(t_\beta + t_\beta^{-1}\right)^2 r_c^2 r_t^2$$
$$2.7 \times 10^{-4} \le B_r(t \to ch) \le 4.3 \times 10^{-4} \qquad (38)$$

In the down sector we get a much more less interesting prediction: $B_r\left(h \to b\bar{d} + d\bar{b}\right) \sim 10^{-6}$.

8.2 Simplified model: the lepton sector

In this sector the most stringent constraint comes from $\mu \to e + \gamma$. If we allowed just SFCNC in this sector the intensity of the couplings will be controlled by $\left|U_{\mu i} U_{ei}\right|^2$ and being nonhierarchical the PMNS matrix, we estimate [18] that to avoid the actual upper bound of $B_r(\mu \to e + \gamma) \le 4.2 \times 10^{-13}$[27], we need a cancellation or fine tuning at the level of $10^{-4} - 10^{-5}$ among the neutral scalar and pseudoscalar contributions in the 2 loop Barr-Zee contribution [28-29]. Therefore, it is mandatory to eliminate these kind of transitions. This in turn means that we have to put a zero in $\hat{r}_{[e]1}$ or in $\hat{r}_{[e]2}$. Still in the neutrino sector we have three possibilities of putting the zero in each one of the components of $\hat{r}_{[\nu]}$. Therefore, we are left with 6 different models in the lepton sector.

Out of these six cases the only one allowed experimentally is:

$$\hat{r}_{[e]} = (-sin\, p_2^e, 0, cos\, p_2^e)\ ; \quad \hat{r}_{[\nu]} = (-sin\, p_2^\nu, cos\, p_2^\nu, 0\,) \qquad (39)$$

and again, because $\hat{r}_{[f]i} \equiv \left(O_{f_L}^T\right)_{i3}$, we must have:

$$O_{e_L} = R_{12}(p_1^e) R_{13}(p_2^e) \sim \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & 0 & \times \end{pmatrix} \qquad (40)$$

together with

$$O_{\nu_L} = P_{23} R_{12}(p_2^\nu) \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (41)$$

therefore, the PMNS matrix will be

$$U = R_{13}^T\left(p_2^l\right) \Phi_3(-2\theta) R_{12}^T\left(p_1^l\right) P_{23} R_{12}(p_2^\nu) \qquad (42)$$

At this point we must stress that the PMNS matrix is fully fixed by three mixing angles and the CP violating phase θ already fixed by the quark sector. Now we can fit U to the experimental information on PMNS encoded in $\left\{\theta_{12}^l, \theta_{13}^l, \theta_{23}^l\right\}$. In this fit we fix the quark fit result $2\theta = 1.077^{+0.039}_{-0.031}$. Although different PMNS analyses show some sensitivity to the phase δ_l, we do not include that information in the fit since we are precisely interested in its prediction. The fit gives the following two solutions:

Solution 1	$p_1^e = 0.7496$	$p_2^e = 1.3541$	$p_2^\nu = 0.8974$
Solution 2	$p_1^e = 2.3889$	$p_2^e = 1.3541$	$p_2^\nu = 1.0542$

(43)

The relevant SFCNC are controlled by

$$\hat{r}_{[e]} = (r_e, 0, r_\tau) = (-0.9765, 0, 0.2156) \qquad (44)$$

and also very important is the fact that the two solutions differ in the values of the (unique) CP violating imaginary part of an invariant quartet:

$$J_{PMNS} = Im\left(U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^*\right) \qquad (45)$$

and the phase $\delta_{PMNS} = \delta_l$.

Case	J_{PMNS}	$\delta_{PMNS} = \delta_l$	$\Delta\chi^2_{NO}$	$\Delta\chi^2_{IO}$
Solution 1	-0.0316	$1.629\pi\ (293^\circ)$	5	0
Solution 2	0.0282	$0.679\pi\ (126^\circ)$	13	> 20

(46)

The values of $\Delta\chi^2_{NO}$ and $\Delta\chi^2_{IO}$ show the contribution that corresponds to δ_{PMNS} attending to the $\Delta\chi^2$ profiles for δ_l obtained for normal and inverted neutrino mass ordering.
We stress that using the information on CP violation in the quark sector, we have been able to predict the phase in PMNS using the connection that SCPV provides in this model; in particular, Solution 1 has $\delta_{PMNS} = 1.629\pi$, which is in good agreement with the most likely values in PMNS analyses.
We have also the parameters that control the SFCNC in the $\tau \leftrightarrow e$ sector, $r_e = 0.9765$ and $r_\tau = 0.2156$: These figures give rise again to a definite prediction for$B_r(h \to \tau\bar{e} + e\bar{\tau})$, through the equation

$$B_r(h \to e\tau) = (1 - R_{11}^2)\left(t_\beta + t_\beta^{-1}\right)^2 r_e^2 r_\tau^2 \left(\frac{\Gamma_{SM}(h)}{\Gamma(h)}\right) \qquad (47)$$

If we take into account the allowed variation of R_{11} and t_β; we end up with the sharp prediction

$$3 \times 10^{-3} \leq \left(\frac{\Gamma(h)}{\Gamma_{SM}(h)}\right) B_r(h \to e\tau) \leq 5 \times 10^{-3} \qquad (48)$$

Note that this result should be seen or disproved soon because the actual experimental bound [30-33] is $B_r(h \to e\tau) \leq 4.7 \times 10^{-3}$ (CMS has recently [34] announced $B_r(h \to e\tau) \leq 2.2 \times 10^{-3}$).

9. Conclusions

We have discussed the possibility of having a framework where there is a connection between the CP violations in the quark and in the lepton sectors. The natural place is in models with spontaneous breaking of CP symmetry, and in particular we have worked with two Higgs doublet models with a softly broken flavour dependent Z_2 symmetry realized in the way of the so-called generalized BGL models (gBGL).
In this framework it has been shown that in order to generate a complex CKM matrix, one has to have scalar flavour changing neutral couplings both in the up and down quark sectors. In the lepton sector it is also needed to have SFCNC both in the charged lepton sector and in the neutrino one.
We have shown that within those gBGL models, there is a connection between δ_{CKM} and δ_{PMNS}.The interplay among CPV and the existence of SFCNC makes these relations quite involved implying connections or predictions for processes mediated by SFCNC in all the sectors: up, down quarks and charged leptons and even neutrinos. To clarify all these relations, we have worked out different models that have the minimal amount of SFCNC needed to keep SCPV generating complex mixing matrices. These simplified models verify the Minimal Flavour Violation ansatz. Because they are controlled by the four unit vectors $\hat{r}_{[u]}, \hat{r}_{[d]}, \hat{r}_{[e]}$ and $\hat{r}_{[\nu]}$ having a zero in some of the three entries, there are $3^4 = 81$ possible models of this type. There is only one of this models surviving experimental constraints. There are two possible solutions for the PDG CP violating phase in the lepton sector δ_l: either 293°or 126°. Those solutions goes together with the following predictions in the SFCNC sector $2.7 \times 10^{-4} \leq B_r(t \to ch) \leq 4.3 \times 10^{-4}$ and $3 \times 10^{-3} \leq \left(\Gamma(h)/\Gamma_{SM}(h)\right)B_r(h \to e\tau) \leq 5 \times 10^{-3}$.

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