PROCEEDINGS OF SCIENCE

Status of the Quantum Statistical Approach to the Parton Distributions

Franco Buccella[a,*]

[a] *INFN, Sezione di Napoli, Italy*

E-mail: buccella@roma1.infn.it

The role of Pauli principle advocated many years ago and the shapes of the valence and gluon parton distributions suggest to describe the parton distributios at the Q_0^2 separating the non perturbative and perturbative regimes of the evolution with Fermi-Dirac functions for the quarks and a Planck formula for the gluons. In such a way their shapes are well described, the unpolarized and polarized quark distributions are given at the same time and the isospin and spin asymmetries of the nucleon sea are related to the valence distributions.

Corfu Summer Institute 2021 "School and Workshops on Elementary Particle Physics and Gravity"
29 August - 9 October 2021
Corfu, Greece

*Speaker

PoS(CORFU2021)003

1. Introduction

Th sign of the β function for the renormalization group [1] [2]implies that QCD [3] accounts for the CONFINEMENT of the quarks ("INFRARED SLAVERY") and for the scale invariance of the structure functions, which describe DEEP INELASTIC SCATTERING ("ASYMPTOTIC FREEDHOM").
The proton and the other baryons, which at small Q^2 behave as states with three quarks combined into a color singlet, at high Q^2 appear as an incoherent set of quarks, gluons and antiquarks with distributions, which obey the sum rules of the parton model as the condition that at high p_z :

$$\int_0^1 \Sigma_i x p_i(x) dx = 1 \tag{1}$$

where x is the fraction of the proton momentum carried by the parton i .
QCD implies logarithmic violations of scale invariance described by DGLAP [4] [5] [6] equations, which allow to deduce the parton distributions at a Q^2 larger than a sufficiently high Q_0^2 from the ones at Q_0^2 . The standard parametrization at Q_0^2 is :

$$Ax^B(1-x)^C P(x) \tag{2}$$

with the parameter A, B e C and the polynome P(x) depending on the parton and such a form holds for the non polarized $q(x) = q^{\uparrow}(x) + q^{\downarrow}(x)$ and for the polarized $\Delta q(x) = q^{\uparrow}(x) - q^{\downarrow}(x)$ distributions .
Parton model and the consequent scale invariance hold for large values of Q^2 and $(p+q)^2 = M^2 + Q^2(\frac{1}{x} - 1)$ larger than M^2 and therefore the values $x = 0$ e $x = 1$ are exscluded as well as their neighboroods with amplitudes decreasing with Q^2 .
Therefore to fix the power behaviour around these points has not a strong motivation .
To fix the distributions at Q_0^2 one may be ispired by experiment, which suggests a role of quantum statistical mechanics .

2. The Parton Distributions Suggested by Quantum Statistical Mechanics

The quantum statistical mechanics approach to the parton distributions has been inspired by the idea that in the proton sea there is an isospin asymmetry, $\bar{d}(x)$ bigger than $\bar{u}(x)$, as a consequence of the Pauli principle [7] [8] .
This idea has been confirmed by the defect [9] in the Gottfried sum rule [10] and from the study of the production of Drell-Yan pais in proton proton and proton deuteron scattering [11] [12]. Also the correlation between the shapes and the first moments of the valence partons confirms the role of quantum statistical mechanics implying a broader distribution of u than d partons in the proton leading to the prediction of the decreasing of the ratio $\frac{F_2^n(x)}{F_2^p(x)}$ at increasing x confirmed by experiment [13].
It implies also that the ratio $\frac{\Delta u(x)}{u(x)}$ is an increasing function of x with negative curvature, while $\frac{\Delta d(x)}{d(x)}$ is a decreasing function with positive curvature .

Therefore one may assume Fermi-Dirac distributions for the quarks and Bose-Einstein for the gluons as a function of x, the variable appearing in the parton sum rules, as the boundary condition to the DGLAP equations [14]: therefore also the variables, which play the role of the "temperature", $\bar{x}$, and of the "potentials", X_q^h, depending on the flavor, q, and helicity, h, are dimensionless as x.
Let us remind that the choice of the energy as the variable , which appears in statistical mechanics, follows from its presence in the constraint on the total energy :

$$\Sigma n_i \epsilon_i = E \tag{3}$$

For the partons the constraint is Eq.(1), which implies that they carry the hadron momentum . The quark number and the Bjorken [15] sum rules imply :

$$u^{\uparrow} > d_d^{\downarrow} > u^{\downarrow} > d^{\uparrow} \tag{4}$$

which is obtained with :

$$X_u^{\uparrow} > X_d^{\downarrow} > X_u^{\downarrow} > X_d^{\uparrow} \tag{5}$$

At the Q_0^2, which separates the non perturbative and the perturbative regimes of the Q^2 evolution, the assumption of the equilibrium with respect to the processes, which determine the DGLAP equations, has two important consequences, a vanishing "potential" for the gluon implying a Planck formula for them and opposite signs for the "potentials" of the valence partons with given helicity and their antiparticles with oppposite helicity [16] [17] [18], which implies according to Eq.(4) in agreement with experiment [9] [19] [20] :

$$\Delta\bar{d}(x) < 0 < \Delta\bar{u}(x) < \bar{d}(x) - \bar{u}(x) < \Delta\bar{u}(x) - \Delta\bar{d}(x) \tag{6}$$

One needs also a diffractive term isospin invariant and unpolarized, which becomes dominant at small x and expected to have the same power behaviour in that region of the gluons .
Quantum statisticaL mechanics implies that the non diffractive fermion distributions are the products of Fermi-Dirac functions of the variables, which appear in the sum rules for the longitudinal component of the momentum and for the transverse energy . The following functions have been proposed for the parton distributions [21]:

$$xq(x) = \frac{AX_q^h x^b}{(\exp\frac{x-X_q^h}{\bar{x}}+1)} + \frac{\tilde{A}x^{\tilde{b}}}{(\exp\frac{x}{\bar{x}}+1)} \tag{7}$$

$$x\bar{q}^q(x) = \frac{\bar{A}x^{2b}}{X_q^{-h}(\exp\frac{x+X_q^{-h}}{\bar{x}}+1)} + \frac{\tilde{A}x^{\tilde{b}}}{(\exp\frac{x}{\bar{x}}+1)} \tag{8}$$

$$xG(x) = \frac{A_G x^{b_G}}{(\exp\frac{x}{\bar{x}}-1)} \tag{9}$$

where the second term for the quark distribution is the diffractive term, which originates from the gluons, which leads to relate their small x assuming $\tilde{b} = b_G - 1$. For the strange partons it has been assumed :

$$s(x) = \bar{s}(x) = \frac{\bar{u}(x) + \bar{d}(x)}{4} \tag{10}$$

The "ad hoc" factors X_q^h and $\frac{1}{X_q^h}$ have been introduced to comply with data, which have been successfully described in terms of the temperature $\bar{x}$, the four "potentials" for the valence quarks u and d with both helicities, the two exponents b and b_G and the four factors, A, $\bar{A}$, A_G and $\tilde{A}$, constrained by the moment sum rule and by the quark number sum rules :

$$\int_0^1 [u(x) - \bar{u}(x)]dx = 2 \tag{11}$$

$$\int_0^1 [d(x) - \bar{d}(x)] + dx = 1 \tag{12}$$

The values of the parameters have been $\bar{x} = 0.099$, $X_u^{\uparrow} = 0.461$, $X_d^{\downarrow} = 0.301$, $X_u^{\downarrow} = 0.299$, $X_{=}^{\uparrow}0.225$, $b = 0.41$, $b_G = 0.747$, $A = 1.75$, $\bar{A} = 1.91$, $A_G = 14.3$ and $\tilde{A} = 0.183$.
The statistical approach implies a common Boltzmann behaviour exp $\frac{-x}{\bar{x}}$ for x larger of the highest "potential" , $Xu^{\uparrow} = 0.46$. in good agreement with experiment.
The predictions for the polarized structure functions of the nucleons measured after [21] have been shown in agreement with successive measurement [22] . To account for the "ad hoc" actors previously mentioned one considered the transverse degrees of freedhom and their form coming from the sum rule proposed for the transverse energy , defined as the difference between the energy and the longitudinal component of the momentum [23] .
For the hadron of the target the transverse energy is given by $P_0 - P_z$, approximately equal at largeP_z to $\frac{M^2}{2P_z}$.
For a massless parton with the longitudinal component of the momentum xP_z and the transverse p_T the transverse energy is given by :

$$\frac{p_T^2}{p_z + \sqrt{p_z^2 + p_T^2}} = \frac{p_T^2}{P_z(x + \sqrt{x^2 + \frac{p_T^2}{P_z^2}})} \tag{13}$$

where P_z is the momentum of the initial hadron in the reference system of the final hadrons and is given, neglecting terms in $(xM)^2$, by :

$$P_z^2 = \frac{Q^2}{4x(1-x)} \tag{14}$$

Multiplying $\times 2P_z$ we obtain a sum rule with M^2 in the right hand side .
The sum rule for the transverse energy fixes the dependence on P_T of the transverse distribution, which is given by :

$$\frac{2}{[\exp(\frac{(p_T)^2}{x + \sqrt{x^2 + (\frac{p_T}{P_z})^2}(\mu)^2} - Y_q^h) + 1]}. \tag{15}$$

where Y_q is the "transverse potential and μ has the dimension of a mass and it is fixed by the sum rule for the transverse energy .
The transformation :

$$p_T^2 = \frac{\mu^2 \eta (x + \sqrt{x^2 + \frac{p_T^2}{P_z^2}})}{2} \tag{16}$$

gives rise to the integral in the variable η, which has the value, neglecting terms proportional to the ratio $(\frac{\mu}{Q})^2$:

$$\ln\left[(1 + \exp\left(Y_q^h - \frac{k}{x}\right)\right] \tag{17}$$

where $k = (\frac{m_q}{\mu})^2$ and may be negliged for the lightest partons u and d, but not for the strange partons . For the non diffractive contribution of the non valence fermion partons $Y_q^h = 0$, which implies for the lightest partons in the mesons the value $\ln 2$, while the non diffractive term for the strange partons is reduced at low x and for the strange valence partons the factor in Eq.(17) takes the value $\ln 2$, when $x = \frac{k}{Y_s^h}$. Therefore for the light valence partons one should have instead of the factors AX_q^h the factors $A' \ln\left(1 + \exp Y_q^h\right)$ and one could recover the form proposed in [21] for the valence quarks simply assuming the proportionality between X_q^h and $\ln\left(1 + \exp Y_q^h\right)$. Indeed in [24], where both X_q^h and Y_q^h are fixed by comparing with the fermion distributions proposed in [25] the proportionality holds with a good approximation . For the non diffractive part of their antiparticles one has a slight change, since $\ln\left(1 + \exp Y_q^h\right) \ln\left[1 + \exp\left(-Y_q^h\right)\right]$ is not costant, but the product gets its maximun, $(\ln 2)^2$, at $Y_q^h = 0$ and the more relevant change concerns $\bar{u}^{\downarrow}$, which has a small non diffractive contribution .
There is an important difference with respect to the standard form $Ax^B(1 - x)^C P(x)$ at high x :in fact the different parton distributions are fixed by the exponent C, which comes out different for the different valence quarks with the conseguence that the limit $\frac{d(x)}{u(x)}$ for $x \to 1$ is 0 or infinity. In the fit by Hera [25] the parameter C is larger for u than for d, while for the sea is still smaller with the consequence to be dominant in that limit. To agree with the experimental behaviour of the ratio $\frac{d(x)}{u(x)}$ the "ad hoc" factor $(1 + 9.7x^2)$ is introduced for the parton u . Instead for the statistical approach above the highest "potential", $X_u^{\uparrow} = 0.461$ all the distributions approach the universal Boltzmann behaviour proportional to $\exp\left(\frac{-x}{\bar{x}}\right)$, which describes for a larger range the gluon and diffractive distributions as a consequence of their vanishing potential . For the valence partons the costant, which multiples $\exp\left(\frac{-x}{\bar{x}}\right)$, depends on their "potential" .
An important feature of the statistical approach is to describe at the same time the unpolarized and the polarized parton distributions .

3. Comparison with HERA and NNQCD

When HERA presented the parton distributions derived by the combined fit to H1 and ZEUS data, Jacques Soffer immediately realized their similarity with the ones found in [21] .
Therefore in [24] the free parameters introduced in [21] were fixed by minimizing the difference from the unpolarized distributions proposed by HERA and from the polarized in [21], which were

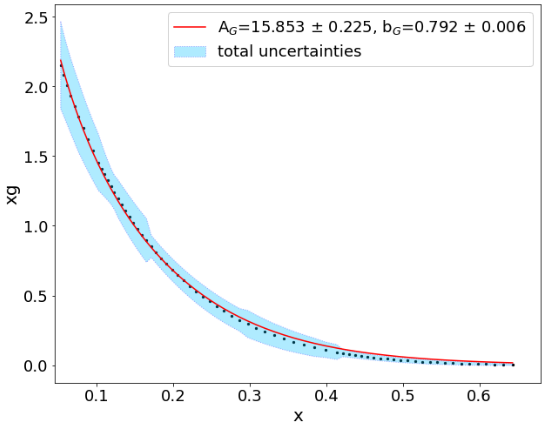

Figure 1: The red curve represents the best fit of the gluon momentum distribution $xg(x)$ obtained in the ATLAS experiment, performed using the functional form in Eq.(9), with A_G and b_G as free parameters, and $\bar{x} = 0.099$. The dots correspond to the experimental points, and the (cyan) shaded area to their uncertainty.

found in very good agreement with the experiments performed after . Instead of the "ad hoc" factors X_q^h and $\frac{1}{X_q^h}$ the factors $\ln(1 + \exp Y_q^h)$ and $\ln(1 + \exp -Y_q^h)$ coming from the extension to the transverse momenta were considered .
In Table 1 we compare the "temperature" and the "potentials" found in [21] with the ones obtained in [26] and in [24] from the comparison with HERA.

Parameter	[21]	[26]	[24]
$\bar{x}$	0.099	0.090	0.102
X_u^+	0.461	0.475	0.446
X_u^-	0.298	0.307	0.297
X_d^+	0.228	0.245	0.222
X_d^-	0.302	0.309	0.320

Table 1: Values of the statistical model parameters found in previous works. The temperature $\bar{x}$ is involved in both the fermion and gluon distributions. The "potentials" X_u^+, X_u^-, X_d^+ and X_d^- determine the non-diffractive parts of the fermion distributions.

Also the proportionality between X_q^h and $\ln(1 + \exp Y_q^h)$ is very well respected .

Instead for the gluons the agreement holds up to about $x = 0.2$, while above the different parametrization lead to a faster decrease for the distribution proposed by HERA . The comparison was repeated [14] with the distributions found by [27] better in agreement with [24] than with [25] . More recently the Planck formula for the gluons was compared [28] with the distribution found by ATLAS [29] and the very good agreement shown in Fig. 1 is obtained with the same $\bar{x}$ and very similar values for A_G and b_G found in [21].

4. Conclusion

The proposal that the boundary conditions at $Q_0^2 = 4\frac{GeV}{c^2}$ for DGLAP equations are Fermi-Dirac functions for the quarks and a PlancK formula for the gluons allows to make many predictions in agreement with experiment and to write both the unpolarized and polarized distributions in terms of few parameters, which are rather stable with respect to the comparison with new data .
The degeneracy of the gas of the valence partons realizes the idea proposed in [7] and [8] that Pauli principle accounts for the isospin asymmetry in the proton sea.
In the phase transition from $Q^2 = 0$ to the deep inelastic regime the DGLAP equations may be applied, when the narrowing of the distributions implied by them is consistent with the increase of the available phase space and this happens at a certain Q_0^2, which a posteriori is around the value chosen in [21] . As it happened for the transformation between constituent and current quarks [30] [31] the transverse degrees of freedom for the constituents of a hadron with large P_z play an important role [32] [23] .

References

[1] H. D. Politzer, *Phys. Rev. Lett.* **30** (1973) 1346

[2] D. J. Gross and F. Wilczek, *Phys. Rev. Lett.* **30** (1973) 1343

[3] H. Fritzsch, M. Gell-Man and H. Leutwyler *Phys. Lett.* **B47** (1973) 365

[4] V. N. Gribov and L. N. Lipatov, *Sov. Nucl.Phys.* **15** (1972) 438,
L. N. Lipatov, *Sov. J. Nucl.Phys.*, **20** (1975) 94

[5] G. Altarelli and G. Parisi, *Nucl.Phys.* **B126** (1977) 298

[6] Y. Dockshitzer, *J. Exp. Theor. Phys.*, **46** (1977) 1946

[7] A. Niegawa and K. Sasaki, *Progr. Theor. Phys.* **54** (1975) 192

[8] R. P. Feynman and J. D. Field, *Phys. Rev.* **D15** (1977) 2590

[9] M. Arneodo et al. (New Muon Collaboration), *Phys. Rev.* **D50** (1994) R1

[10] K. Gottfried *Phys. Rev. Lett.* **18** (1967) 1174

[11] A. Baldit et al. (NMC Collaboration) *Phys. Lett.* **B332** (1994) 244

[12] J. Dove (Sea Quest Experiment), *PoS.* **DIS2019** (2019) 011

[13] T. Sloan, G. Smadja and R. Voss, *Phys. Rev.* **162** (1988) R45

[14] F.Buccella, S. Sohaily and F.Tramontano *J. Stat. Mech : Theory Exp.* **2019** (2019) 073302

[15] J. Bjorken, *Phys. Rev.* **D1** (1970) 376

[16] R. S. Bhalerao, *Phys. Lett.* **B380** (1996) 1 *Eur. Phys. J.* **C41** (2005) 327 *Eur. Phys. J.* **C41** (2005) 327

[17] R. S. Bhalerao, N. G. Keipar and G. Ram, *Phys. Lett.* **B476** (2000) 285

[18] R. S. Bhalerao, *Phys. Rev.* **C63** (2001) 025208

[19] L. Adamczyk et al., *Phys. Rev. Lett.* **C113** (2014) 072301

[20] J. Adam et al., *Phys. Rev.* **CD09** (2019) 051102)

[21] , C. Bourrely, F. Buccella and J. Soffer, *Eur. Phys. J.* **C23** (2002) 187

[22] , C. Bourrely, F. Buccella and J. Soffer, *Eur. Phys. J.* **C41** (2005) 327

[23] , C. Bourrely, F. Buccella and J. Soffer, *Mod. Phys. Lett.* **A18** (2006) 143 and *Int. Jour. of Mod. Phys.* **28** (2013) 13500

[24] F.Buccella and S. Sohaily *Mod. Phys. Lett.* **A30** (2015) 1550203 .

[25] F. D. Aaron (HERA Collaboration), *JHEP.* **109** (2010) 1001

[26] , C. Bourrely and J. Soffer, *Nucl. Phys.* **A941** (2015) 307

[27] R. D. Ball et al. (NNQCD Collaboration) *Eur. Phys. J.* **C77** (2017) 663

[28] L. Bellantuono, R. Bellotti and F. Buccella, arXiv : 2201.07640 [hep-ph]

[29] G. Aad et al. (ATLAS Collaboration), *JHEP.* **07** (2021) 223

[30] C. Boldrighini, F. Buccella, E. Celeghini, E. Sorace and L. Triolo, *Nucl.Phys.* **B122** (1970) 651,

[31] F. Buccella, E. Celeghini, H. Kleinert, C. A. Savoy and E. Sorace *Nuovo Cimento* **A69** (1970) 133 .

[32] H. J. Melosh, *Phys. Rev.* **D9** (1975) 1995