PROCEEDINGS OF SCIENCE

Parity from $\mathrm{SO}(7,1)$ and $\mathrm{SO}(7,7)$ gauge symmetries

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Left-Right parity symmetry $\mathcal{P}$ can arise from a unified gauge symmetry, involving gravitational interactions. Parity can survive to the symmetry breaking of the gauge group at Planck scale and can be spontaneously broken at lower energies, as in Left-Right symmetric models with $\mathcal{P}$.

7th Symposium on Prospects in the Physics of Discrete Symmetries (DISCRETE 2020-2021)
29th November - 3rd December 2021
Bergen, Norway

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Weak interactions differ from the other known forces in Nature for their chiral asymmetry: parity is maximally broken. This special feature has led many authors to the hypothesis that at high energy scale one may recover a generalized parity $\mathcal{P}$, which combines standard parity and Left-Right gauge group exchange. This new discrete symmetry can be naturally embedded in the Left-Right symmetric models (LRSM) [1–5], based on the gauge group $\mathrm{SU}_L(2) \times \mathrm{SU}_R(2) \times \mathrm{U}_{B-L}(1)$. It must be broken at lower scale and, precisely, it can be spontaneously broken simultaneously to the $\mathrm{SU}_R(2)$ gauge symmetry. The generalized parity $\mathcal{P}$ has been known for a long time to have neither a UV completion nor a protection mechanism against higher energy physics (i.e. gravity.) This has motivated the proposal in [6].

The basic idea is to understand $\mathcal{P}$ as a discrete remnant of a continuous, gauge symmetry. The main obstacle to building this gauge symmetry is the non-commutation between the Lorentz symmetry and a continuous rotation changing the chiralities. An attractive solution to this issue is to mix chiral rotation and Lorenz group in a larger, internal gauge symmetry. Involving Lorentz symmetry necessary implies bringing in the game gravity, thus the internal gauge symmetry has to be broken at Planck scale. The underlying frameworks to model this proposal are usually known as gravi-weak and gravi-GUT scenarios [7–11]. The former unifies gravity with weak interactions, the latter represents a complete unification with gravity.

In particular, we showed in [6] that $\mathcal{P}$ can be seen as a discrete remnant of $SO(1,7)$ (gravi-weak unification) or $SO(7,7)$ (gravi-GUT) gauge symmetries. The framework requires to split $\mathcal{P}$ as the action on fields P and space inversion I_s:

$$\mathcal{P} = I_s \circ P\,, \tag{1}$$

where only P, which exchanges left and right fields, becomes part of a unifying gauge symmetry.

In this proceeding, we highlight our findings.

1. SO(1,7) case

Let us consider the Majorana real representation 16 of $\mathrm{SO}(1,7) \to \mathrm{SO}(1,3) \times \mathrm{SO}(0,4)$, where $\mathrm{SO}(1,3)$ is the Lorentz group. We denote with L, R (l, r) the left or right $SU(2)$ components of $\mathrm{SO}(0,4)$ $(\mathrm{SO}(1,3))$. The fermions transform as doublets under both $\mathrm{SU}(2)_{L,R}$ and Lorentz,

$$\mathbf{16}_{\mathbb{R}} \equiv \mathbf{8}_s \to (\mathbf{2}_L, \mathbf{1}_R, \mathbf{2}_l) \oplus (\mathbf{1}_L, \mathbf{2}_R, \mathbf{2}_r)\,. \tag{2}$$

The $\mathrm{SO}(1,7)$ generators acting on $\mathbf{8}_s$ are:

$$\Sigma_{M,N} = \frac{i}{2}\left(\begin{array}{cc|cc} 0 & i\sigma_i \otimes \mathbf{1} \otimes \sigma_3 & i\mathbf{1} \otimes \mathbf{1} \otimes \sigma_2 & i\mathbf{1} \otimes \sigma_b \otimes \sigma_1 \\ -i\sigma_j \otimes \mathbf{1} \otimes \sigma_3 & \epsilon_{ijk}\sigma_k \otimes \mathbf{1} \otimes \mathbf{1} & -\sigma_j \otimes \mathbf{1} \otimes \sigma_1 & \sigma_b \otimes \sigma_j \otimes \sigma_2 \\ \hline -i\mathbf{1} \otimes \mathbf{1} \otimes \sigma_2 & \boxed{\sigma_i \otimes \mathbf{1} \otimes \sigma_1} & 0 & -\mathbf{1} \otimes \sigma_b \otimes \sigma_3 \\ -i\sigma_b \otimes \mathbf{1} \otimes \sigma_1 & -\sigma_i \otimes \sigma_b \otimes \sigma_2 & \mathbf{1} \otimes \sigma_a \otimes \sigma_3 & \epsilon_{abc}\mathbf{1} \otimes \sigma_c \otimes \mathbf{1} \end{array}\right)$$

The upper-left block represents the $\mathrm{SO}(1,3)$ generators and the lower-right the $\mathrm{SO}(4)$ ones. In the respective spaces, we denote i, j or a, b as indices from 1 to 3, thus matching $M, N = 1,2,3 \to i, j = 1,2,3$ and $M, N = 5,6,7 \to a, b = 1,2,3$.

The three generators in the box 123-4, namely $R_i \sim \sigma_i \otimes \mathbf{1} \otimes \sigma_1$, rotate among them one space direction with the timelike direction of $\mathrm{SO}(4)$. On the othert hand, one finds just above the standard

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rotations $L_i \sim \sigma_i \otimes \mathbf{1} \otimes \mathbf{1}$. The point now is that for any i, the combination $R_i - L_i$ generates a continuous rotation subgroup, $\mathrm{U}(1)_P$, and one can check that a rotation by π generates

$$P = \mathbf{1} \otimes \mathbf{1} \otimes \sigma_1 \,. \tag{3}$$

This rotation by π results in the inversion of all space directions, implementing parity, in addition to the timelike direction of SO(4). On the spinors, under the action of P, the swapping of left and right chirality is accompanied by the swapping of the Left and Right weak groups, as required.

2. Symmetric phase and breaking

A mechanism of symmetry breaking which preserves parity as a discrete remnant of the original continuous gauge group can be found within the extension of the first-order approach to gravity where the Lorentz symmetry is disentangled from spacetime transformations (diffeomorphisms) and treated as an internal gauge symmetry, further extended to include other interactions [8–10, 12].

For the group $G = \mathrm{SO}(1, 3 + N)$, preserving the metric $\eta_{MN} = \mathrm{diag}\,\{1, -1, -1, -1, -1 \ldots\}$ with $M = 0, \ldots N + 3$, a vierbein VEV, solution of the equations of motion, can be arranged in the first four directions,

$$\overline{e}_\mu^M = \begin{cases} M_{pl}\, \delta_\mu^M\,, & \text{for } 0 \leq M \leq 3 \\ 0\,, & \text{for } 4 \leq M \leq N+3 \end{cases} \,. \tag{4}$$

This breaks diffs and the 4D part of G down to global simultaneous Lorentz transformations of μ and the first four indices M, and it leaves unbroken a local subgroup $\mathrm{SO}(N)$, mixing the last N directions where the VEV vanishes.

This mechanism was used in [9], with SO(11, 3) broken in this single step to a SO(10) GUT. As analyzed there, the correct fermionic, gauge and gravitational lagrangians emerge after the symmetry breaking of the G-invariant unified theory. This VEV breaks $U(1)_P$ and accordingly, also P, under which we have $\overline{e}_\mu^M \to \overline{e}_\mu^M \eta^{MM}$ (no summation). Thus also P is broken, as it does not preserve $\overline{e}_\mu^M$.

On the other hand, the VEV is restored by adding a I_s spatial inversion, which completes the action of $\mathcal{P}$:

$$\mathcal{P}: \quad \overline{e}_\mu^M \;\to\; \eta_{\mu\mu}\overline{e}_\mu^M \eta^{MM} = \overline{e}_\mu^M \,, \tag{5}$$

i.e. the vierbein VEV is invariant under combined internal parity and spatial inversion, $\mathcal{P} = I_s \circ P$. This result shows that the breaking mechanism glues not only the gauge and diff Lorentz transformations but also glues internal parity with spatial inversion, to produce the standard behavior of parity in the low energy field theory.

Thus, if the Lagrangian is invariant under space inversion, then the low energy theory will be exactly $\mathcal{P}$ invariant.

3. Emergence of LRSM Yukawa terms

Accordingly, a SO(1, 7) Yukawa term is explicitly matched with the LRSM $\mathcal{P}$-invariant lagrangian at low energies,

$$\mathcal{L}_{Yuk} = Y_H \Psi^\dagger H \Psi + h.c. \;\to\; \overline{\psi}_L \left[Y\Phi + \tilde{Y}\tilde{\Phi} \right] \psi_R + h.c.\,. \tag{6}$$

p+q = 8	**spinor = $16_{\mathbb{R}}$ (Majorana)**	
$\mathrm{SO}(1,7)$	$\mathrm{SO}(1,3) \otimes \mathrm{SO}(0,4)$	$\mathcal{P}$
$\mathrm{SO}(5,3)$	$\mathrm{SO}(4,0) \otimes \mathrm{SO}(1,3)$	$\mathcal{T}$
p+q = 14	**spinor = $64_{\mathbb{R}}$ (Majorana-Weyl)**	
$\mathrm{SO}(7,7)$	$\mathrm{SO}(6,0) \otimes \mathrm{SO}(1,3) \otimes \mathrm{SO}(0,4)$	$\mathcal{T}_{col}$, $\mathcal{P}$
$\mathrm{SO}(11,3)$	$\mathrm{SO}(10,0) \otimes \mathrm{SO}(1,3)$	C $\mathcal{T}_{so10}$

Table 1: Breaking of unifying orthogonal groups and emerging discrete symmetries.

where $Y_H = Y + i\tilde{Y}$ is a generic complex Yukawa matrix, $H \in 8_c 8_c^\dagger$, Φ is the LRSM bidoublet, and where under the breaking $\mathrm{SO}(1,7) \to \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SO}(1,3)$, the field H decomposes as

$$H = L_\mu(1_L, 1_R, 4_l) + L^a_\mu(3_L, 1_R, 4_l) + L_i\big(2^*_L, 2_R, 3_{\bar{l}r}\big) + \Phi_{\overline{L}R}\big(2^*_L, 2_R, 1_{\bar{l}r}\big) + (L \leftrightarrow R)\,.$$

Thus H contains Lorentz 4-vector, 3-vector, and singlet representations transforming under the weak groups. The field Φ is a suitable combination of $\Phi_{\overline{L}R}$ and $\Phi^\dagger_{\overline{R}L}$, which has to remain light to make contact with the LRSM.

The other bidoublet has a natural mass at the Planck breaking scale, disappearing from the low energy spectrum. Incidentally, the same fate has to be assumed for all the other components transforming nontrivially under Lorentz, also avoiding possible issues with the signature of their nonstandard kinetic terms.

Complete modeling should pay attention to the norm positivity of states surviving below the Planck scale. In the unbroken phase above the Planck scale instead, there is no background metric and thus no standard quadratic kinetic terms exist. This phase thus belongs arguably to a topological nonperturbative regime of quantum gravity.

4. Complete unifications and other symmetries

Larger groups can be proposed to include both weak and strong interactions, see Table 1 for the realistic pseudo-orthogonal cases.

The various components can be arranged with spatial and/or timelike signature, and the rotations exploited above to generate $\mathcal{P}$ give rise to possible new symmetries.

For instance for $\mathrm{SO}(5,3)$, one can rotate one of the $\mathrm{SO}(4)$ directions with internal direction 0 to obtain its inversion, and the VEV may be preserved by adding a time inversion I_t. We indicate the symmetry as $\mathcal{T}$ in the table, amounting to time-reversal plus exchange of the Left and Right weak groups.

In the $\mathrm{SO}(7,7)$ case, $\mathrm{SO}(4)$ is spatial and leads to $\mathcal{P}$, while $\mathrm{SO}(6)$ is time-like. Thus, one finds a time-reversal plus SU(4) color conjugation, $\mathcal{T}_{col}$ in the table. This additional discrete symmetry may or may not survive the lower stages of symmetry breaking.

For $\mathrm{SO}(11,3)$ we find an analogous symmetry, $\mathcal{T}_{so10}$, while $\mathcal{P}$ is absent. In the table, it is shown also the standard C LR-symmetry, charge conjugation plus exchange of Left and Right weak groups, which is part of $\mathrm{SO}(10)$.

5. Discussion

We established that $\mathcal{P}$ arises from the gluing of internal parity P and spatial inversion I_s. While P is gauged, spatial inversion need not be assumed to be an invariance of the theory. This is still compatible with diffeomorphisms and the weak equivalence principle in General Relativity.

If I_s is assumed, the theory would have no $\mathcal{P}$ violating terms. In particular, $\theta F\tilde{F}$ is forbidden, so this choice can be viewed as a solution to the Strong CP problem, as in [1, 5, 13].

In case I_s is not assumed to be exact, $\theta F^M_N \wedge F^N_M$ is admitted (the two-form F^M_N being the curvature of ω^M_N) This term violates space inversion but respects internal parity P being gauge-invariant. In the low-energy theory, it generates the term $\theta F\tilde{F}$, as well as the gravitational $\theta\tilde{R}R$, both $\mathcal{P}$ violating. Another example breaking spatial parity but not the gauge symmetry is the Immirzi term $\alpha R^{MN} \wedge e_M \wedge e_N$.

The two scenarios shall be disentangled. One of the most stringent tests is the experimental bound from the electric dipole moments (EDM) (e.g. of the neutron [14]). The relative bounds of the order $\overline{\theta} < 10^{-10}$ directly translate for us into limits on the gravitational analogous, $\theta\tilde{R}R$. This is argued to be physical [15], and the question of how it could be measured is the subject of some recent studies, e.g. [16–18].

In the LRSM, exact parity $\mathcal{P}$, along with flavor constraints [19–23], imposes that the QCD θ is strictly zero, so that the neutron EDM is computable and pushes the Left-Right scale M_{W_R} beyond ~ 28 TeV [23, 24]. The present framework instead motivates also the situation as in Ref. [19, 23], namely, only $\overline{\theta}$ is free. In this case, $\mathcal{P}$ symmetry is valid in the Yukawa sector, but strong CP poses no additional constraints, in complete analogy with the case of C symmetry [19]. In this scenario, the W_R scale can be lowered to ~ 6 TeV, at the reach of future LHC runs and next-generation colliders [25], helping to clarify the underlying mechanism behind $\mathcal{P}$.

Acknowledgements. The work of F.N. was partially supported by the research grant No. 2017X7X85K under the program PRIN 2017 funded by the Ministero dell'Istruzione, Università e della Ricerca.

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