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Diagonal reflection symmetries and universal four-zero texture

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In this talk, we consider a set of new symmetries in the SM: *diagonal reflection* symmetries $R\, m_{u,\nu}^* \, R = m_{u,\nu}, \; m_{d,e}^* = m_{d,e}$ with $R = \text{diag}\,(-1,1,1)$. By combining the symmetries with the four-zero texture $(m_f)_{11} = (m_f)_{13} = 0$, the masses and mixing matrices of quarks and leptons are reproduced with precisions of 10^{-3}. Since this scheme has only eight parameters in the lepton sector, it has four predictions; the Dirac phase $\delta_{CP} \simeq 203°$, the Majorana phases $(\alpha_2, \alpha_3) \simeq (11.3°, 7.54°)$ up to 180°, and $m_1 \simeq 2.5$ or 6.2 meV with the normal hierarchy.

In this scheme, the type-I seesaw mechanism and a given neutrino Yukawa matrix Y_ν completely determine the structure of the right-handed neutrino mass matrix M_R. A $u-\nu$ unification predicts its masses to be $(M_{R1}, M_{R2}, M_{R3}) = (O(10^5), O(10^9), O(10^{14}))$ GeV with a strong hierarchy $M_R \sim Y_u^T Y_u$.

The symmetries are approximately stable under the renormalization of SM. This statement holds without the four-zero texture as long as couplings in the first row and column of the Yukawa matrices are sufficiently small. Then, they can possess information on a high energy scale.

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1. Diagonal reflection symmetries

To start, we show a new set of symmetries [1, 2]. The mass matrices of the SM fermions $f = u, d, e$, and neutrinos ν_L are defined by

$$\mathcal{L} \ni \sum_f -\bar f_{Li} m^{BM}_{f ij} f_{Rj} - \bar\nu_{Li} m^{BM}_{\nu ij} \nu^c_{Lj} + \text{h.c.}\,. \tag{1}$$

These matrices m_f^{BM} are assumed to satisfy $\mu - \tau$ reflection symmetries separately [3–5]:

$$T_u (m^{BM}_{u,\nu})^* T_u = m^{BM}_{u,\nu}\,, \quad T_d (m^{BM}_{d,e})^* T_d = m^{BM}_{d,e}\,, \tag{2}$$

where

$$T_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad T_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}. \tag{3}$$

A simultaneous redefinition of all fermion fields $f' = U_{BM} f$ and $\nu' = U_{BM}\nu$ by the following bi-maximal transformation U_{BM},

$$m_f \equiv U_{BM} m_f^{BM} U_{BM}^\dagger, \quad m_\nu \equiv U_{BM} m_\nu^{BM} U_{BM}^T, \quad U_{BM} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{i}{\sqrt2} & \frac{i}{\sqrt2} \\ 0 & -\frac{1}{\sqrt2} & \frac{1}{\sqrt2} \end{pmatrix}, \tag{4}$$

leads to the following mass matrices;

$$m_{u,\nu} = \begin{pmatrix} a_{u,\nu} & i b_{u,\nu} & i c_{u,\nu} \\ i d_{u,\nu} & e_{u,\nu} & f_{u,\nu} \\ i g_{u,\nu} & h_{u,\nu} & k_{u,\nu} \end{pmatrix}, \; m_{d,e} = \begin{pmatrix} a_{d,e} & b_{d,e} & c_{d,e} \\ d_{d,e} & e_{d,e} & f_{d,e} \\ g_{d,e} & h_{d,e} & k_{d,e} \end{pmatrix}, \tag{5}$$

with real parameters $a_f \sim k_f$.

In this basis, the $\mu - \tau$ reflection symmetries (2) are rewritten as

$$U_{BM} T_{u,d} U_{BM}^T m^*_{u,d} U^*_{BM} T_{u,d} U^\dagger_{BM} = m_{u,d}. \tag{6}$$

Surprisingly,

$$-U^*_{BM} T_u U^\dagger_{BM} = \text{diag}(-1,\ 1,\ 1) \equiv R, \quad U^*_{BM} T_d U^\dagger_{BM} = \text{diag}(1,\ 1,\ 1) = 1_3. \tag{7}$$

Then, the $\mu - \tau$ reflection symmetries in the basis are transformed into

$$R m^*_{u,\nu} R = m_{u,\nu}, \quad m^*_{d,e} = m_{d,e}. \tag{8}$$

The mass matrices (5) certainly satisfy these conditions. We call such a symmetry *diagonal reflection* because it is a diagonal remnant of $\mu - \tau$ reflection symmetry after deduction of $\mu - \tau$ symmetry [6]. Each of them is just a generalized CP symmetry [7–10] and no longer a $\mu - \tau$ reflection.

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By combining these symmetries with Hermitian four-zero texture (10) [11], the CKM matrix is reproduced with accuracies of $O(10^{-3})$;

$$V_{\rm CKM} \simeq \begin{pmatrix} 1 & \sqrt{m_u/m_c} & 0 \\ -\sqrt{m_u/m_c} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -i & 0 & 0 \\ 0 & c_q & s_q \\ 0 & s_q & c_q \end{pmatrix} \begin{pmatrix} 1 & -\sqrt{m_d/m_s} & 0 \\ \sqrt{m_d/m_s} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{9}$$

where $|s_q| \simeq 0.04$ comes from mixings of 23 generations in $M_{u,d}$. This scheme predicts $|V_{ub}| \simeq \sqrt{m_u/m_c}|V_{cb}| \simeq 0.0018$, and it does not match the current observation $|V_{ub}^{\rm obs}| \simeq 0.00361$. However, this mismatch can be solved by allowing large 23 elements with small complex phases [12] or by allowing finite $(M_u)_{11} \neq 0$ [2]. The Hermiticity of Yukawa matrices $Y_{u,d,e}$ is justified by the parity symmetry in the left-right symmetric models [13–15].

2. Universal four-zero texture

Here, we show the following universal four-zero texture

$$M_{u,\nu} = \begin{pmatrix} 0 & i\,C_{u,\nu} & 0 \\ \mp i\,C_{u,\nu} & \tilde B_{u,\nu} & B_{u,\nu} \\ 0 & B_{u,\nu} & A_{u,\nu} \end{pmatrix}, \quad M_{d,e} = \begin{pmatrix} 0 & C_{d,e} & 0 \\ C_{d,e} & \tilde B_{d,e} & B_{d,e} \\ 0 & B_{d,e} & A_{d,e} \end{pmatrix}, \tag{10}$$

is compatible with neutrino mixing parameters. The plus (minus) sign in $\mp$ corresponds to a symmetric matrix of neutrinos (a Hermitian matrix of up-type quarks). Since this system has only eight degrees of freedom, the following observables determine the mass matrices; three charged lepton masses at mass of Z boson m_Z [16],

$$m_e = 486.570\,{\rm keV}\,, \quad m_\mu = 102.718\,{\rm MeV}\,, \quad m_\tau = 1746.17\,{\rm MeV}\,, \tag{11}$$

the mixing angles and mass differences of the latest global fit [17]

$$\theta_{23}^{PDG} = 49.7^\circ, \quad \theta_{12}^{PDG} = 33.82^\circ, \quad \theta_{13}^{PDG} = 8.61^\circ, \tag{12}$$

$$\Delta m_{21}^2 = 73.9\,{\rm meV}^2\,, \quad \Delta m_{31}^2 = 2525\,{\rm meV}^2\,. \tag{13}$$

Thus, the remaining four parameters in the neutrino sector, namely the three CP phases δ, $\alpha_{2,3}$ and the lightest neutrino mass m_1 are predicted.

The Jarlskog invariant [18] determines the Dirac phase δ_{CP} as

$$\sin\delta_{CP} = -0.390, \quad \delta_{CP} \simeq 203^\circ. \tag{14}$$

This is very close to the best fit for the normal hierarchy (NH) $\delta_{CP}/^\circ = 217^{+40}_{-28}$ [17].

The Majorana phases are calculated from similar rephasing invariants [19]

$$I_1 = {\rm Im}\,[(U_{\rm MNS})_{12}^2 (U_{\rm MNS})_{11}^{*2}] = \frac{1}{4}\sin^2 2\theta_{12}^{PDG}\, \cos^4\theta_{13}^{PDG}\, \sin\alpha_2, \tag{15}$$

$$I_2 = {\rm Im}\,[(U_{\rm MNS})_{13}^2 (U_{\rm MNS})_{11}^{*2}] = \frac{1}{4}\sin^2 2\theta_{13}^{PDG}\, \cos^2\theta_{12}^{PDG}\, \sin\alpha_3', \tag{16}$$

where $\alpha_3' \equiv \alpha_3 - 2\delta_{CP}$. A reconstructed mixing matrix $U_{\rm MNS}$ yields the following results;

$$\alpha_2^0 \simeq 11.3^\circ, \quad \alpha_3^0 \simeq 7.54^\circ. \tag{17}$$

Meanwhile, the original $\mu - \tau$ reflection symmetry restrict the Majorana phases to be $\alpha_{2,3}/2 = n\pi/2$ $(n = 0, 1)$ [20]. The nontrivial phase $\pi/2$ comes from negative masses (after a real rotation). We parameterize these effects as

$$m_2 = e^{i\beta_2}|m_2|, \quad m_3 = e^{i\beta_3}|m_3|, \quad \beta_{2,3} = 0 \text{ or } \pi. \tag{18}$$

The whole Majorana phases are found to be

$$(\alpha_2, \, \alpha_3) = (\alpha_2^0 + \beta_2, \, \alpha_3^0 + \beta_3) = (11.3^\circ \text{ or } 191.3^\circ, \; 7.54^\circ \text{ or } 187.54^\circ). \tag{19}$$

Finally, The numerical values of the lightest mass m_1 are found to be

$$m_1 = 6.20\,\text{meV} \;\; \text{for} \;\; (\beta_2, \beta_3) = (0,0) \text{ or } (\pi, \pi), \tag{20}$$

$$= 2.54\,\text{meV} \;\; \text{for} \;\; (\beta_2, \beta_3) = (0,\pi) \text{ or } (\pi, 0), \tag{21}$$

for the NH case. For the inverted mass hierarchy, the solutions do not have real values and thus contradict the diagonal reflection.

2.1 Right-handed neutrino mass M_R

The right-handed neutrino mass matrix M_R can be reconstructed from the type-I seesaw mechanism [21–24] with some GUT relations. For example, a simple $u - \nu$ unification as realized in Pati–Salam GUT [13] determines Y_ν;

$$Y_\nu = Y_u \simeq \frac{0.9 m_t \sqrt{2}}{v} \begin{pmatrix} 0 & 0.0002\,i & 0 \\ -0.0002\,i & 0.10 & 0.31 \\ 0 & 0.31 & 1 \end{pmatrix}. \tag{22}$$

Here, the value of Y_u is taken from one of the recent best fits [12].

From the type-I seesaw mechanism and Eq. (22), M_R also displays a four-zero texture because the four-zero texture is seesaw invariant [25, 26],

$$M_R = \frac{v^2}{2} Y_\nu M_\nu^{-1} Y_\nu^T = \begin{pmatrix} 0 & -1.08\,i \times 10^8 & 0 \\ -1.08\,i \times 10^8 & 1.26 \times 10^{14} & 4.07 \times 10^{14} \\ 0 & 4.07 \times 10^{14} & 1.32 \times 10^{15} \end{pmatrix} \text{GeV}. \tag{23}$$

Evidently, M_R also satisfies diagonal reflection symmetry (8),

$$R M_R^* R = M_R. \tag{24}$$

Therefore, all the fermion masses respect the diagonal reflection symmetry with the four-zero textures.

The masses of M_R are found to be

$$(M_{R1} \, , M_{R2} \, , M_{R3}) = (2.86 \times 10^6 \, , 3.73 \times 10^9 \, , 1.44 \times 10^{15}) \, \text{GeV}. \tag{25}$$

This is just an example calculation because it depends on unification schemes. Also, the Yukawa matrix Y_ν (22) is evaluated at m_Z scale. Renormalized values of quark masses at a GUT scale will lead to $O(10)$ smaller masses of M_R.

Moreover, these symmetries are almost renormalization invariant and realized by vevs of scalar fields $\langle \theta_u \rangle = i v_u$ and $\langle \theta_d \rangle = v_d$ that only couple to the first generations of SM fermions. Detailed discussions are found in the original papers [1, 2].

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References

[1] M.J.S. Yang, *Diagonal reflection symmetries and universal four-zero texture*, *Chin. Phys. C* **45** (2021) 043103 [2003.11701].

[2] M.J.S. Yang, *Almost exact diagonal reflection symmetries and three-zero texture*, *Nucl. Phys. B* **972** (2021) 115549 [2103.12289].

[3] P.F. Harrison and W.G. Scott, *mu - tau reflection symmetry in lepton mixing and neutrino oscillations*, *Phys. Lett.* **B547** (2002) 219 [hep-ph/0210197].

[4] W. Grimus and L. Lavoura, *A Nonstandard CP transformation leading to maximal atmospheric neutrino mixing*, *Phys. Lett.* **B579** (2004) 113 [hep-ph/0305309].

[5] M.J.S. Yang, *Interplay between μ-τ reflection symmetry, four-zero texture and universal texture*, *Phys. Lett. B* **806** (2020) 135483 [2002.09152].

[6] T. Fukuyama and H. Nishiura, *Mass matrix of Majorana neutrinos*, hep-ph/9702253.

[7] G. Ecker, W. Grimus and H. Neufeld, *Spontaneous CP Violation in Left-right Symmetric Gauge Theories*, *Nucl. Phys. B* **247** (1984) 70.

[8] M. Gronau and R.N. Mohapatra, *Maximal Spontaneous CP Violation and Left-right Symmetry*, *Phys. Lett. B* **168** (1986) 248.

[9] F. Feruglio, C. Hagedorn and R. Ziegler, *Lepton Mixing Parameters from Discrete and CP Symmetries*, *JHEP* **07** (2013) 027 [1211.5560].

[10] M. Holthausen, M. Lindner and M.A. Schmidt, *CP and Discrete Flavour Symmetries*, *JHEP* **04** (2013) 122 [1211.6953].

[11] H. Fritzsch and Z.-z. Xing, *A Symmetry pattern of maximal CP violation and a determination of the unitarity triangle*, *Phys. Lett.* **B353** (1995) 114 [hep-ph/9502297].

[12] Z.-z. Xing and Z.-h. Zhao, *On the four-zero texture of quark mass matrices and its stability*, *Nucl. Phys. B* **897** (2015) 302 [1501.06346].

[13] J.C. Pati and A. Salam, *Lepton Number as the Fourth Color*, *Phys. Rev.* **D10** (1974) 275.

[14] G. Senjanovic and R.N. Mohapatra, *Exact Left-Right Symmetry and Spontaneous Violation of Parity*, *Phys. Rev. D* **12** (1975) 1502.

[15] R.N. Mohapatra and J.C. Pati, *Left-Right Gauge Symmetry and an Isoconjugate Model of CP Violation*, *Phys.Rev.* **D11** (1975) 566.

[16] Z.-z. Xing, H. Zhang and S. Zhou, *Impacts of the Higgs mass on vacuum stability, running fermion masses and two-body Higgs decays*, *Phys. Rev. D* **86** (2012) 013013 [1112.3112].

[17] I. Esteban, M.C. Gonzalez-Garcia, A. Hernandez-Cabezudo, M. Maltoni and T. Schwetz, *Global analysis of three-flavour neutrino oscillations: synergies and tensions in the determination of* θ_{23}, δ_{CP}, *and the mass ordering*, *JHEP* **01** (2019) 106 [1811.05487].

[18] C. Jarlskog, *Commutator of the Quark Mass Matrices in the Standard Electroweak Model and a Measure of Maximal CP Violation*, *Phys. Rev. Lett.* **55** (1985) 1039.

[19] J.F. Nieves and P.B. Pal, *Minimal Rephasing Invariant CP Violating Parameters With Dirac and Majorana Fermions*, *Phys. Rev. D* **36** (1987) 315.

[20] Z.-z. Xing and J.-y. Zhu, *Neutrino mass ordering and* $\mu - \tau$ *reflection symmetry breaking*, *Chin. Phys.* **C41** (2017) 123103 [1707.03676].

[21] P. Minkowski, $\mu \to e\gamma$ *at a Rate of One Out of* 10^9 *Muon Decays?*, *Phys. Lett.* **67B** (1977) 421.

[22] T. Yanagida, *Horizontal gauge symmetry and masses of neutrinos*, *Conf. Proc.* **C7902131** (1979) 95.

[23] R.N. Mohapatra and G. Senjanovic, *Neutrino Mass and Spontaneous Parity Nonconservation*, *Phys. Rev. Lett.* **44** (1980) 912.

[24] M. Gell-Mann, P. Ramond and R. Slansky, *Complex spinors and unified theories*, *Conf. Proc.* **C790927** (1979) 315.

[25] H. Nishiura, K. Matsuda and T. Fukuyama, *Lepton and quark mass matrices*, *Phys. Rev.* **D60** (1999) 013006 [hep-ph/9902385].

[26] H. Fritzsch and Z.-z. Xing, *Mass and flavor mixing schemes of quarks and leptons*, *Prog. Part. Nucl. Phys.* **45** (2000) 1 [hep-ph/9912358].