PROCEEDINGS OF SCIENCE

On the large-field equivalence between Starobinsky and Higgs inflation in gravity and supergravity

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Starobinsky inflation and Higgs inflation are reviewed with the focus on their equivalence in the large field limit due to the common inflaton scalar potential. This asymptotic equivalence is extended to supergravity in the minimal framework, where the Starobinsky and Higgs descriptions of inflation arise in two different gauges of a single model.

7th Symposium on Prospects in the Physics of Discrete Symmetries (DISCRETE 2020-2021)
29th November - 3rd December 2021
Bergen, Norway

[*] Speaker

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1. Introduction

The Starobinsky model of inflation [1] based on the modified $(R + R^2)$ gravity and the so-called Higgs inflation model [2] based on the non-minimal coupling of a Higgs-like field to gravity are apparently different but, nevertheless, lead to the same predictions for inflation. Their asymptotic equivalence in the large field limit is due to the fact that both models have the same inflaton potential during slow roll.

In this paper both inflationary models are introduced from scratch, without following chronological developments, and the underlying assumptions leading to their equivalence in describing inflation are made clear. The equivalence is extended to supergravity by proposing the supergravity model where both (Starobinsky and Higgs) descriptions of inflation arise in two different gauges.

The paper is organized as follows. In Sec. 2, the Starobinsky model of inflation is reviewed both in modified gravity and in scalar-tensor gravity. In Sec. 3, the Higgs inflation model with the non-minimal coupling to gravity is reviewed. The asymptotic equivalence between those models is confirmed in Sec. 4. In Sec. 5, the minimal embedding of the Starobinsky and Higgs inflation models into supergravity is proposed. In Sec. 6, the equivalence between the supergravity-based Starobinsky and Higgs inflation models is established during slow roll. Sec. 7 is our Conclusion.

2. Starobinsky inflation from scratch

The purpose of this Section is to review the Starobinsky (1980) model of inflation and argue about its privileged position amongst all inflation models, without following historical developments. The Starobinsky model is defined by the action [1]

$$S_{\text{Star.}} = \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g} \left(R + \frac{1}{6m^2} R^2 \right) , \tag{1}$$

where we have introduced the reduced Planck mass $M_{\text{Pl}} = 1/\sqrt{8\pi G_{\text{N}}} \approx 2.4 \times 10^{18}$ GeV, and the mass parameter m. We use the spacetime signature $(-,+,+,+,)$.

In the low curvature regime, the R^2 term can be ignored and the action (1) reduces to the standard Einstein-Hilbert action. In the high curvature regime relevant for inflation, the R term can be ignored and the action (1) reduces to the no-scale R^2 gravity with the dimensionless coupling constant in front of the action. The R^2-term has the positive coupling constant to avoid a ghost.

The $(R + R^2)$ gravity model (1) can be considered as a representative of the modified $F(R)$ gravity theories defined by

$$S_F = \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g}\, F(R) \tag{2}$$

with a function $F(R)$ of the scalar curvature R.

At first sight, the model (1) is rather *ad hoc*, being just one of many possible choices of the function $F(R)$. However, a closer theoretical inspection and the current observational data strongly favor Eq. (1) as the basic model of inflation. In order to demonstrate that, first, we recall the well known fact that the modified gravity theories can be reformulated as the scalar-tensor gravity theories [3], see also Refs. [4, 5] for some explicit examples.

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The $F(R)$ gravity action (2) is classically equivalent to

$$S[g_{\mu\nu},\chi] = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\,\left[F'(\chi)(R-\chi)+F(\chi)\right] \tag{3}$$

with the real scalar field χ, provided that $F''\neq 0$ as we always assume, and the primes denote the derivatives with respect to the argument. The χ-field equation of motion implies $\chi = R$ that brings back the action (2). Otherwise, the (positive) factor F' in front of the R in (3) can be eliminated by a Weyl transformation of metric $g_{\mu\nu}$, which transforms the action (3) into an action of the dynamical scalar field χ minimally coupled to Einstein gravity and having the scalar potential

$$V = \left(\frac{M_{\rm Pl}^2}{2}\right)\frac{\chi F'(\chi)-F(\chi)}{F'(\chi)^2}\quad . \tag{4}$$

The kinetic term of χ becomes canonically normalized after the field redefinition $\chi(\varphi)$ as

$$F'(\chi) = \exp\left(\sqrt{\frac{2}{3}}\varphi/M_{\rm Pl}\right)\ ,\qquad \varphi = \frac{\sqrt{3}M_{\rm Pl}}{\sqrt{2}}\ln F'(\chi)\ , \tag{5}$$

in terms of the canonical inflaton field φ. It results in the scalar-tensor gravity acton

$$S_{\rm quintessence}[g_{\mu\nu},\varphi] = \frac{M_{\rm Pl}^2}{2}\int {\rm d}^4x\sqrt{-g}R - \int {\rm d}^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi)\right]\ . \tag{6}$$

The classical and quantum stability conditions of $F(R)$ gravity theory (that we always assume) are given by (see e.g., Ref. [6])

$$F'(R) > 0 \quad \text{and} \quad F''(R) > 0\,, \tag{7}$$

while they are obviously satisfied by Eq. (1) for $R > 0$. The first condition in Eq. (7) means that graviton is not a ghost, whereas the second condition means that inflaton is not a tachyon.

Actually, the *inverse* transformation is more illuminating, and it reads [5, 7–9]

$$R = \left[\frac{\sqrt{6}}{M_{\rm Pl}}\frac{dV}{d\varphi}+\frac{4V}{M_{\rm Pl}^2}\right]\exp\left(\sqrt{\frac{2}{3}}\varphi/M_{\rm Pl}\right)\ ,\quad F = \left[\frac{\sqrt{6}}{M_{\rm Pl}}\frac{dV}{d\varphi}+\frac{2V}{M_{\rm Pl}^2}\right]\exp\left(2\sqrt{\frac{2}{3}}\varphi/M_{\rm Pl}\right)\ , \tag{8}$$

defining the function $F(R)$ in the parametric form for a (given) inflaton scalar potential $V(\varphi)$.

The key physical requirement to the inflaton potential is its *flatness* enabling slow roll of inflaton (see the next Section for precise definitions). It exactly corresponds to the smallness of the first term against the second one in the square brackets of Eq. (8). Ignoring the first term immediately gives rise to the F-function as the R^2 term driving inflation.

As regards the $(R+R^2)$ gravity of Eq. (1), the exact inflaton potential is given by

$$V(\varphi) = \frac{3}{4}M_{\rm Pl}^2m^2\left[1-\exp\left(-\sqrt{\frac{2}{3}}\varphi/M_{\rm Pl}\right)\right]^2\ . \tag{9}$$

It has a *plateau* of the positive height (related to the inflationary scale), which gives rise to the slow roll of inflaton along the plateau.

The duration of inflation is measured by the e-foldings number

$$N_e \approx \frac{1}{M_{\rm Pl}^2}\int_{\varphi_{\rm end}}^{\varphi_*} \frac{V}{V'} d\varphi \ , \tag{10}$$

where φ_* is the inflaton value at the reference scale (horizon crossing), and $\varphi_{\rm end}$ is the inflaton value at the end of inflation when one of the slow roll parameters,

$$\varepsilon_V(\varphi) = \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \quad \text{and} \quad \eta_V(\varphi) = M_{\rm Pl}^2 \left|\frac{V''}{V}\right| \ , \tag{11}$$

is no longer small (close to 1).

The amplitude of scalar perturbations at the horizon crossing is given by [10]

$$A = \frac{V_*^3}{12\pi^2 M_{\rm Pl}^6 (V_*{}')^2} = \frac{3m^2}{8\pi^2 M_{\rm Pl}^2}\sinh^4\left(\frac{\varphi_*}{\sqrt{6}M_{\rm Pl}}\right) \ . \tag{12}$$

As regards its phenomenological applications, the Starobinsky model (1) is the excellent model of cosmological inflation. The Planck satellite mission measurements of the Cosmic Microwave Background (CMB) radiation [11] give the scalar perturbations tilt as $n_s \approx 1 + 2\eta_V - 6\varepsilon_V \approx 0.9649 \pm 0.0042$ (with 68% CL) and restrict the tensor-to-scalar ratio as $r \approx 16\varepsilon_V < 0.064$ (with 95% CL). The Starobinsky inflation yields $r \approx 12/N_e^2 \approx 0.004$ and $n_s \approx 1 - 2/N_e$, where N_e is the e-foldings number between 50 and 60, with the best fit at $N_e \approx 55$ [12, 13].

Being based on only gravitational interactions, the Starobinsky model (1) is geometrical, while its (inflaton mass) parameter m is fixed by the observed CMB amplitude (COBE, WMAP) as

$$m \approx 3 \cdot 10^{13}\ \text{GeV} \quad \text{or} \quad \frac{m}{M_{\rm Pl}} \approx 1.3 \cdot 10^{-5} \ . \tag{13}$$

A numerical analysis of (10) with the potential (9) and $N_e \approx 55$ yields [10]

$$\sqrt{\frac{2}{3}}\varphi_*/M_{\rm Pl} \approx \ln\left(\frac{4}{3}N_e\right) \approx 5.5 \quad \text{and} \quad \sqrt{\frac{2}{3}}\varphi_{\rm end}/M_{\rm Pl} \approx \ln\left[\frac{2}{11}(4+3\sqrt{3})\right] \approx 0.5 \ . \tag{14}$$

The scalar potential (9) is *nonrenormalizable*. Expanding it in powers of $\varphi/M_{\rm Pl}$ clearly shows that the nonrenormalizable (marginal) terms beyond the fourth power of φ are suppressed by the powers of $M_{\rm Pl}$ so that the *UV-cutoff* of the Starobinsky model is given by $\Lambda_{\rm Star.UV} = M_{\rm Pl}$, in agreement with [14]. The classical equivalence of $F(R)$ gravity and scalar-tensor gravity can be extended to the (on-shell) *quantum* equivalence in the one-loop approximation [15].

The Starobinsky solution to Hubble function in the R^2 gravity reads $H(t) \approx \left(\frac{m}{6}\right)^2 (t_{\rm end} - t)$, and it is an *attractor*, with the inflaton being the Nambu-Goldstone boson associated with spontaneous breaking of the *scale* invariance [16].

3. Higgs inflation

The basic idea of Higgs inflation is to identify inflaton field with a Higgs field. Strictly speaking, it does not have to be the Higgs field of the Standard Model. It is phenomenologically possible when such inflaton is non-minimally coupled to gravity [2].

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The Lagrangian of Higgs inflation in the Jordan frame reads (we take $M_{\rm Pl} = 1$ fo simplicity)

$$\mathcal{L}_J = \sqrt{-g}\left[\frac{1}{2}(1+\xi\phi^2)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V_H(\phi)\right] , \tag{15}$$

where the scalar potential has the Higgs form

$$V_H(\phi) = \frac{\lambda}{4}\left(\phi^2 - v^2\right)^2 , \tag{16}$$

with the coupling constants v and λ, while the non-minimal coupling to gravity (in front of the R) is measured by the new coupling constant $\xi > 0$.

The transfer from the Jordan frame to the Einstein frame is achieved by a Weyl transformation,

$$g^{\mu\nu}_J = g^{\mu\nu}_E(1+\xi\phi^2) , \tag{17}$$

which results in a non-canonical kinetic term of the scalar ϕ and the rescaled scalar potential.

A canonical scalar kinetic term is obtained via a field redefinition $\varphi = \varphi(\phi)$ according to

$$\frac{d\varphi}{d\phi} = \frac{\sqrt{1+\xi(1+6\xi)\phi^2}}{1+\xi\phi^2} \tag{18}$$

that gives rise to the standard Lagrangian

$$\mathcal{L}_E = \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right] \tag{19}$$

with the scalar potential

$$V(\varphi) = \frac{V_H(\phi(\varphi))}{\left[1+\xi\phi^2(\varphi)\right]^2} . \tag{20}$$

In the *large* field approximation, $\phi^2 \gg v^2$, a solution to Eq. (18) is given by

$$\varphi \approx \sqrt{\frac{3}{2}}\ln\left(1+\xi\phi^2\right) \tag{21}$$

which yields the scalar potential

$$V(\varphi) = \frac{\lambda}{4\xi^2}\left(1 - e^{-\sqrt{2/3}\varphi}\right)^2 \tag{22}$$

that *coincides* with the Starobinsky potential (9).

The CMB observations require $\xi/\sqrt{\lambda} \approx 5\cdot 10^4$ with the inflaton mass $m = \sqrt{\frac{\lambda}{3}}\xi^{-1} \approx 10^{-5}$. Assuming the Higgs coupling constant λ to be of the order one implies that ξ is of the order 10^5.

Unlike the renormalizable scalar potential (16) in the Jordan frame, an expansion of the scalar potential (20) in powers of φ in the Einstein frame leads to the non-renormalizable (marginal) terms multiplied by powers of ξ. Therefore, after restoring the Planck scale $M_{\rm Pl}$ by dimensional considerations, the *UV-cutoff* of the Higgs inflation model is given by $\Lambda_{\rm Higgs.UV} = M_{\rm Pl}/\xi$, in agreement with [17]. The UV-cutoff $\Lambda_{\rm Higgs.UV}$ is much closer to the Hubble value of inflation and is much lower than the UV-cutoff of the Starobinsky model $\Lambda_{\rm Star.UV} = M_{\rm Pl}$ by the large factor ξ, which implies that the Higgs inflation is considerably more sensitive to quantum corrections than the Starobinsky inflation.

4. Large-field equivalence between the Starobinsky and Higgs inflation models

The same inflaton potential in the Starobinsky and Higgs models implies the same physical predictions for large-field inflation. Given the obvious differences between the models, it is surprising. A simple explanation is possible after taking into account the slow roll condition [16, 18].

The Higgs field H of the Standard Model is a charged doublet. One can choose the unitary gauge $H = \phi/\sqrt{2}$ in the Higgs Lagrangian ($M_{\rm Pl} = 1$)

$$\mathcal{L}_H = \sqrt{-g}\left[\frac{1}{2}R + \xi H^\dagger H R - g^{\mu\nu}\partial_\mu H^\dagger \partial_\nu H - \lambda\left(H^\dagger H - \frac{1}{2}v^2\right)^2\right] . \tag{23}$$

In the *large* field approximation one can ignore v in the Higgs scalar potential, whereas during *slow roll* inflation one can also ignore the scalar kinetic term against the scalar potential. Taken together, these assumptions greatly simplify the above Lagrangian to

$$\mathcal{L}_H \approx \sqrt{-g}\left[\frac{1}{2}(1+\xi\phi^2)R - \frac{\lambda}{4}\phi^4\right] , \tag{24}$$

where the field ϕ becomes auxiliary. Varying the action with respect to ϕ yields $\xi\phi R = \lambda\phi^3$ or just

$$\phi^2 = \frac{\xi}{\lambda}R . \tag{25}$$

Substituting this result back into the Lagrangian (24) gives the Starobinsky model with

$$\mathcal{L}_H \approx \sqrt{-g}\left(\frac{1}{2}R + \frac{\xi^2}{4\lambda}R^2\right) . \tag{26}$$

There is *no* equivalence in the small field approximation. Reheating is also different. For instance, the reheating temperature $T_{\rm Higgs} \approx 10^{13}$ GeV, whereas $T_{\rm Star.} \approx 10^9$ GeV [6].

5. Starobinsky and Higgs inflation in the minimal supergravity

A local supersymmetrisation of Eq. (1) is possible in curved superspace of supergravity, see e.g., Refs. [19, 20], see also Ref. [21] for a connection to no-scale supergravity. However, it leads to multi-field inflation. Instead, one can assign inflaton to a *massive* $N = 1$ vector multiplet V that has only one physical scalar, and supersymmetrise the potential (9). A generic scalar potential of the vector multiplet is given by the squared derivative of arbitrary real potential $J(V)$ [22, 23]. The manifestly supersymmetric Lagrangian in curved superspace is governed by a potential $J(V)$ (see Ref. [24] for the notation) as follows:

$$\mathcal{L} = \int d^2\theta 2\mathcal{E}\left\{\frac{3}{8}(\overline{\mathcal{D}}\overline{\mathcal{D}} - 8\mathcal{R})e^{-\frac{2}{3}J} + \frac{1}{4}W^\alpha W_\alpha\right\} + \text{h.c.} , \tag{27}$$

while its bosonic part in the Einstein frame (after Weyl rescalings) reads [22, 23]

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{4}F_{mn}F^{mn} - \frac{1}{2}J''\partial_m C\partial^m C - \frac{1}{2}J'' B_m B^m - \frac{g^2}{2}J'^2 , \tag{28}$$

where $C = V|$ is the real scalar inflaton field, $J = J(C)$, and $M_{\rm Pl} = 1$.

The D-type scalar potential of the Starobinsky model is obtained after choosing the potential J as

$$J(C) = \frac{3}{2}(C - \ln C) \quad \text{with} \quad C = \exp\left(\sqrt{2/3}\phi\right) . \tag{29}$$

6. Large-field equivalence of the Starobinsky and Higgs inflation in supergravity

Let us replace the master function (potential) $J(V)$ by a function $\tilde{J}(He^{2V}\overline{H})$, where we have introduced the chiral Higgs superfield H and have chosen $g = 1$ for simplicity. The argument of the function $\tilde{J}$ and the function itself are invariant under the supergauge transformations

$$H \to e^{-iZ}H\,, \quad \overline{H} \to e^{i\overline{Z}}\overline{H}\,, \quad V \to V + \frac{i}{2}(Z - \bar{Z})\,, \tag{30}$$

whose gauge parameter Z itself is a chiral superfield. The original theory of the massive vector multiplet governed by the master function J is recovered in the supersymmetric gauge $H = 1$.

We can choose another (Wess-Zumino) supersymmetric gauge in which $V = V_1$, where V_1 describes the irreducible *massless* vector multiplet minimally coupled to the *dynamical* Higgs chiral multiplet H. The standard Higgs mechanism appears when choosing the canonical function $J = \frac{1}{2}He^{2V}\bar{H}$ that corresponds to a linear function $\tilde{J}$ [25, 26]. This phenomenon is known in the literature as the super-Higgs effect [24]. The supergravity theory in terms of the superfields H and V_1 defines the Higgs inflation in supergravity, which is equivalent to the Starobinsky inflation by construction, because both appear in the two different gauges of the *same* supergravity model.

The difference against the standard approach, where inflaton belongs to a chiral (charged) superfield H, is due to the supergauge invariance. In particular, the scalar superpartner of inflaton is a gauge (unphysical) degree of freedom in the construction above.

In order to illustrate our conclusions by a simple argument, let us consider only large scalar fields and ignore their kinetic terms. Then the relevant part of the supergravity action (27) *before* the transformation to the Einstein frame by Weyl rescaling with $M_{\rm Pl} = 1$ reads [25]

$$e^{-1}\mathcal{L} = \exp\left(-\frac{2}{3}J\right)\left(\frac{1}{2}R\right) - \frac{1}{2}g^2\exp\left(-\frac{4}{3}J\right)(J')^2\ , \tag{31}$$

where by using Eq. (29) we have

$$e^{-\frac{2}{3}J} = Ce^{-C} \equiv \Omega > 0\ . \tag{32}$$

Therefore, Eq. (31) can be rewritten to the form

$$e^{-1}\mathcal{L} = \Omega\left(\frac{1}{2}R\right) - \frac{1}{2}\left(\frac{3}{2}g\right)^2\Omega^2\left(1 - C^{-1}\right)^2\ , \tag{33}$$

where the field Ω is auxiliary, and the field $C = C(\Omega)$ is the special (Lambert) function of Ω.

Varying the Lagrangian (33) with respect to Ω yields

$$\frac{1}{2}R = \left(\frac{3}{2}g\right)^2\Omega\left(1 - \frac{2}{C(\Omega)}\right) \approx \left(\frac{3}{2}g\right)^2\Omega\left(1 + \frac{2}{\ln\Omega}\right)\ , \tag{34}$$

where in the large field approximation we have $C^{-1} \ll 1$ and $|1/\ln\Omega| \ll 1$. Hence, in the leading order we find

$$\frac{1}{2}R \approx \left(\frac{3}{2}g\right)^2\Omega\,. \tag{35}$$

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Substituting it back into the Lagrangian (35) yields the leading R^2-term,

$$e^{-1}\mathcal{L} \approx \frac{1}{8}\left(\frac{3}{2}g\right)^{-2} R^2 \ . \tag{36}$$

Having included the *next-to-leading term* in (34), we find

$$e^{-1}\mathcal{L} \approx \frac{1}{8}\left(\frac{3}{2}g\right)^{-2} R^2 \left[1 + \frac{2}{\ln\left(\frac{2}{9}R/g^2\right)}\right] \ . \tag{37}$$

This gives rise to the modified R^2 inflation in the gauge $H = 1$. Similarly, when using the Wess-Zumino gauge $V = V_1$ with the charged Higgs (Stueckelberg) superfield H and the function $\tilde{J}(\bar{H}e^{2gV_1}H)$, the R^2 inflation is reproduced after introducing another function $\exp[-\frac{2}{3}\tilde{J}(\bar{H}H)] = \tilde{\Omega}$ as the auxiliary field and ignoring both the H-kinetic term and the gauge field dependence in V_1.

7. Conclusion

The Starobinsky and Higgs models of inflation are distinguished by their simplicity: both have only one relevant parameter, namely, the inflaton mass whose value is fixed by observations. Therefore, both models do not have free parameters for describing inflation and thus have the maximal predictive power, being in the same universality class. Nevertheless, they remain viable by providing the best fit to all observational data available at present. Their sharp predictions for the value of the CMB tensor-to-scalar ratio offer an exciting opportunity to verify or falsify them in a near future when this crucial ratio is expected to be measured by several Collaborations (BICEP/Keck Array, Simons Observatory, LiteBIRD).

Both models are extendable to supergravity in the minimal supergravity framework, while keeping their equivalence.

Though an apparently low nongaussianity of the CMB radiation favors single-field inflationary models, it does not exclude possible mixing of inflaton with other scalars. However, this mixing should be properly suppressed, which is non-trivial in supergravity. Small deviations from the basic R^2-inflation are expected, while mixing the Starobinsky inflaton (called scalaron) with the Higgs field of the Standard Model is also possible [27–29].

Mixing of scalaron with other scalars during inflation can be exploited in the scenarios of double inflation leading to the formation of primordial black holes due to tachyonic instabilities of scalars and isocurvature pumping of scalar perturbations [30]. This mechanism is particularly natural in supergravity [31, 32].

Acknowledgements

The author was supported in part by Tokyo Metropolitan University, the Development Program of Tomsk Polytechnic University within the assignment of the Ministry of Science and Higher Education of the Russian Federation, the World Premier International Research Center Initiative (WPI Initiative), MEXT, and Yamada Foundation in Japan.

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