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The Flavour Puzzle as a Vacuum Problem

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Symmetry principles have long been applied to the flavour puzzle. In a bottom-up approach, the variety of possible symmetry groups and symmetry breaking sectors is huge, the predictability is reduced and the number of allowed models diverges. A relatively well-motivated and more constrained framework is provided by supersymmetric theories where a discrete subgroup Γ of a non-compact Lie group G plays the role of flavour symmetry and the symmetry breaking sector spans a coset space G/K, K being a compact subgroup of G. For a general choice of G, K, Γ and a generic matter content, we show how to construct a minimal Kähler potential and a general superpotential, for both rigid and local $N = 1$ supersymmetric theories.

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PoS(DISCRETE2020-2021)007

A fresh look into an old matter

Traditional (linearly realized) flavour symmetries act in generation space. In their simplest implementation the flavour group G_f commutes with both the Poincaré and the gauge groups, but it can be also combined with CP resulting in an additional non-trivial action in flavour space. The most important fact about this type of symmetries is that in any realistic construction they need to be broken [1]. Broken symmetries are well understood and ubiquitous in particle physics and, at first sight, do not represent a problem in their implementation. Why should we be worried about them? A first unpleasant aspect is that the freedom is huge: the flavour group can be abelian or not, continuous or discrete, global or local. There is no accepted baseline model in a bottom-up approach and in most of the existing constructions the predictability is very limited. Usually the symmetry breaking sector consists of a set $\{\tau_\alpha\}$ of dimensionless, gauge-invariant fields charged under G_f. When space-time coordinates are varied, these fields span a moduli space $\mathscr{M}$ describing the possible vacua of the system. We can expand a fermion mass matrix $m_{ij}(\tau)$ in powers of τ_α [1]:

$$m_{ij}(\tau) = m_{ij}^{(0)} + m_{ij}^{(1)\ \alpha}\tau_\alpha + m_{ij}^{(1)\ \bar{\alpha}}\bar{\tau}_{\bar{\alpha}} + m_{ij}^{(2)\ \alpha\beta}\tau_\alpha\tau_\beta + ... \tag{1}$$

A realistic model requires at least few terms in the series (1). Additional parameters are brought in by the renormalization group evolution needed to translate the high-energy predictions into low-energy physical parameters. If the theory is supersymmetric, extra parameters associated to supersymmetry breaking are needed. Most of realistic models depend on a large number of free parameters to the detriment of predictability.

On top of that, a very unattractive feature is the need of a mechanism delivering τ_α with appropriate size and orientation in flavour space. This alignment problem is typically solved at the expenses of enlarging both the symmetry group, including additional "shaping" factors in G_f, and the symmetry breaking sector, including a pletora of driving fields, not directly entering the expression (1). In model building the usual path proceeds from the choice of G_f and its representations $\rho^{(f)}(g)$ in field space, to an ad hoc and often baroque construction of the symmetry breaking sector $\{\tau_\alpha\}$. In this way the central ingredient of the whole construction is relegated to the very last step.

Can we reverse the logic? If the symmetry breaking sector is so crucial, why not look for physically and/or mathematically motivated symmetry breaking sectors and inspect their symmetry properties? Consider the following simple example. Imagine that the moduli space $\mathscr{M}$ describes the (non-oriented) lines of the plane passing through the origin. To parametrize this set we can choose points lying on the unit circle centered at the origin of the complex plane: $\mathscr{M} = \{\tau \in C, |\tau| = 1\}$, with the agreement that τ and $\gamma\tau = -\tau$ should be identified, since they describe the same line. The $\Gamma \equiv Z_2$ parity symmetry $\tau \to \gamma\tau$ is a gauge symmetry, since it reflects the redundancy of the adopted parametrization. The moduli space $\mathscr{M}$ is "too large" and a one-to-one correspondence with the lines of the plane is obtained by considering the quotient $\mathscr{M}/\Gamma$ [2]. In a putative field theory where τ is a scalar field, we should also assign matter fields $\Psi(x)$ to (possibly non-linear and projective) Γ representations. By consistency, the low-energy EFT should satisfy the gauge symmetry under Γ. Following this procedure, the flavour group Γ and its representations are derived

[1] Here we make no distinction between a field τ_α and its VEV.

[2] In the string terminology, the moduli space is $\mathscr{M}/\Gamma$. Here we remain closer to the QFT dictionary: we distinguish $\mathscr{M}$ and Γ, call moduli space the whole $\mathscr{M}$ and interprete Γ as a gauge symmetry.

PoS(DISCRETE2020-2021)007

from the moduli space $\mathscr{M}$, that in turns describes the allowed vacua. We also notice that the gauge symmetry Γ is always realized in the broken phase, since there is no point on the unit circle that is left invariant by Γ.

A less trivial example is that of a theory where the physically inequivalent vacua are in a one-to-one correspondence with classes of conformally equivalent metrics on the torus [2]. The moduli space is the upper half-plane $\mathscr{M} = SL(2,\mathbb{R})/SO(2) = \{\tau | \, \Im(\tau) > 0\}$. Since tori related by a transformation γ of $\Gamma = SL(2,\mathbb{Z})$ are conformally equivalent, we can adopt as candidate flavour symmetry $SL(2,\mathbb{Z})$. Indeed the most general transformation of matter fields under this group is:

$$\Psi(x) \xrightarrow{\gamma} (c\tau + d)^{k_\Psi} \rho_\Psi(\gamma) \Psi(x) \quad , \tag{2}$$

where $\rho_\Psi(\gamma)$ is a unitary representation of a finite modular group $SL(2,\mathbb{Z}_N)$ [3], k_Ψ is the weight and N is the level of the representation. When non-vanishing weights are present, Yukawa couplings should be functions of the modulus τ with the appropriate transformation property to enforce invariance under Γ. In a supersymmetric construction Yukawa couplings $Y(\tau)$ are modular forms of given weight k_Y and level N_Y. Since such forms span a finite-dimensional linear space, we have a limited number of allowed couplings and mass/mixing parameters are sharply constrained.

In this approach we can consistently incorporate CP as a nontrivial automorphism of the group $SL(2,\mathbb{Z})$ [3, 4, 5]. In CP and modular invariant models, CP violation is spontaneous, arising from values of the modulus away from the boundary of the fundamental domain and from the imaginary τ axis. Also in this case the flavour symmetry is always realized in the broken phase, since there is no point of the upper half-plane that is left invariant by the action of the group $SL(2,\mathbb{Z})$. There are however points enjoying residual invariance under finite subgroups of $SL(2,\mathbb{Z})$, which can be responsible for degeneracies or hierarchies in the fermion mass spectrum. A small departure of τ from one of these points can explain the observed fermion mass hierarchies [6, 7, 8]. Finally, modular invariance can also be interpreted as the outer automorphism of an ordinary flavour group, leaving the modulus invariant, thus enhancing the whole set of transformations acting nontrivillay in flavour space to an *eclectic group* [9].

Symplectic modular invariance

To generalize the above framework we can adopt an Hermitian Symmetric Space (HSS) as moduli space $\mathscr{M}$ [10]. HSS have several attractive features. They have been completely classified [11, 12]. They are Kähler and therefore support supersymmetric realizations. Non-compact HSS naturally arise as moduli space in supergravity and string compactifications. They are nicely related to the theory of automorphic forms, a generalization of modular forms, that are the building blocks of Yukawa couplings. Every HSS is a coset space of the type $\mathscr{M} = G/K$ for some connected Lie group G and a compact subgroup K of G [4]. The generic element τ of $\mathscr{M}$ can be obtained by

[3] The group $SL(2,\mathbb{Z}_N)$, N being an integer, can be view as an unfaithful finite copy of $SL(2,\mathbb{Z})$. While the latter is infinite and does not possess finite unitary representations, the former is finite and its representations are unitary and finite dimensional.

[4] The Lie algebra $\mathscr{G}$ of G decomposes as $\mathscr{G} = \mathscr{V} \oplus \mathscr{A}$, $\mathscr{V}$ being the Lie algebra of K. The algebra $\mathscr{G}$ is invariant under $V + A \to V - A$ and satisfies $[\mathscr{V},\mathscr{V}] \subset \mathscr{V}$, $[\mathscr{V},\mathscr{A}] \subset \mathscr{A}$ and $[\mathscr{A},\mathscr{A}] \subset \mathscr{V}$. An hermitian symmetric space M can

performing a generic G transformation on an element τ_0 left invariant by K:

$$\tau = g\ \tau_0 \qquad g \in G \quad \& \quad h\ \tau_0 = \tau_0 \quad \text{ for any } h \in K. \tag{3}$$

We choose as flavour symmetry group a discrete subgroup Γ of G, whose action on τ is given by

$$\tau \xrightarrow{\gamma} \gamma\tau \equiv (\gamma g)\tau_0 \qquad \gamma \in \Gamma \quad . \tag{4}$$

To build a (supersymmetric) model incorporating a local symmetry under Γ and possessing physically inequivalent vacua described by $\mathscr{M}/\Gamma$, we need the transformation laws of the matter fields $\Psi(x)$ under Γ. To this purpose we introduce an automorphic factor $j(g,\tau)$ ($g \in G$) with the property:

$$j(g_1 g_2, \tau) = j(g_1, g_2\tau) j(g_2, \tau) \quad . \tag{5}$$

In general Γ is an infinite group and does not admit finite unitary representations. These can be recovered by building an unfaithful finite copy of Γ. Given a normal subgroup G_n of Γ with finite index, we define the finite group $\Gamma_n = \Gamma/G_n$. A general transformation law for matter fields under Γ reads:

$$\begin{cases} \tau \xrightarrow{\gamma} \gamma\tau \\ \Psi^{(I)}(x) \xrightarrow{\gamma} j(\gamma,\tau)^{k_I} \rho^{(I)}(\gamma) \Psi^{(I)}(x) \end{cases} , \tag{6}$$

where we have separated the matter fields $\{\Psi^{(I)}(x)\}$ in subsets with a common weight k_I and $\rho^{(I)}(\gamma)$ is a unitary representation of Γ_n. The property (5) guarantees that the transformation is a (non-linear) realization of Γ.

We consider the case of rigid $\mathscr{N} = 1$ supersymmetry and collect all chiral superfields in a multiplet $\Phi = (\tau, \Psi^{(I)})$. The action $\mathscr{S}$ describing the Yukawa interactions is defined in terms of a Kähler potential $K(\Phi, \bar{\Phi})$ and a superpotential $w(\Phi)$:

$$\mathscr{S} = \int d^4x d^2\theta d^2\bar{\theta}\ K(\Phi, \bar{\Phi}) + \left[\int d^4x d^2\theta\ w(\Phi) + \text{h.c.} \right], \tag{7}$$

where the Kähler potential $K(\Phi, \bar{\Phi})$, is a real gauge-invariant function of the chiral superfields Φ and their conjugates and the superpotential $w(\Phi)$ is a holomorphic gauge-invariant function of the chiral superfields Φ. The invariance of the action $\mathscr{S}$ under eq. (6) requires the invariance of the superpotential $w(\Phi)$ and the invariance of the Kahler potential up to a Kahler transformation [5]

$$\begin{cases} w(\Phi) \to w(\Phi) \\ K(\Phi, \bar{\Phi}) \to K(\Phi, \bar{\Phi}) + f(\Phi) + f(\bar{\Phi}) \end{cases} . \tag{8}$$

be of compact type, of noncompact type or of euclidean type. In general none of these cases applies and M decomposes as a product $M = M_c \times M_{nc} \times M_e$, where the three factors are hermitian symmetric spaces of compact, noncompact and euclidean type, respectively. A hermitian symmetric space is irreducible if it is not the product of two hermitian symmetric spaces of lower dimension. Irreducible hermitian symmetric spaces of compact type can be obtained from the noncompact ones, by means of a transformation on the generators of the Lie algebra $\mathscr{G}$: $(V, A) \to (V, iA)$.

[5]In $\mathscr{N} = 1$ local supersymmetry these requirements are relaxed and replaced by the invariance of the real gauge-invariant function $\mathscr{G} = K + \log|w|^2$. The superpotential is not necessarily invariant and its variation under Γ can be compensated by the transformation of K [10].

A candidate minimal Kähler potential is given by:

$$K_{\tt min}(\Phi,\bar\Phi)=-c\log Z(\tau,\bar\tau)+\sum_I Z(\tau,\bar\tau)^{k_I}|\Psi^{(I)}|^2 \quad . \tag{9}$$

Here

$$Z(\tau,\bar\tau)\equiv[j^\dagger(g,\tau_0)j(g,\tau_0)]^{-1} \quad , \tag{10}$$

where the dependence on τ is through the element g via the correspondence in eq. (3) and c is a real constant whose sign is chosen to guarantee local positivity of the metric for the moduli τ. By construction, the above potential is invariant under Γ up to a Kähler transformation for a general choice of G, K, Γ, G_n and $j(g,\tau)$. This is not the most general Kähler potential invariant under Γ. Invariance under Γ allows to add to $K_{\tt min}(\Phi,\bar\Phi)$ other terms, that cannot be excluded or constrained in a pure bottom-up approach. In general these terms can modify the flavour properties of the theory such as physical fermion masses and mixing angles. Additional assumptions or inputs from a top-down approach are needed in order to reduce the arbitrariness of the predictions [13].

The conditions for the invariance of the superpotential under Γ can be deduced by expanding $w(\Phi)$ in powers of the supermultiplets $\Psi^{(I)}$:

$$w(\Phi)=\sum_p Y_{I_1...I_p}(\tau)\;\Psi^{(I_1)}\dots\Psi^{(I_p)} \quad . \tag{11}$$

The p-th order term is invariant provided the functions $Y_{I_1...I_p}(\tau)$ obey:

$$Y_{I_1...I_p}(\gamma\tau)=j(\gamma,\tau)^{k_Y(p)}\rho^{(Y)}(\gamma)\;Y_{I_1...I_p}(\tau) \quad , \tag{12}$$

with $k_Y(p)$ and $\rho^{(Y)}$ such that:

i) The weight $k_Y(p)$ compensates the total weight of the product $\Psi^{(I_1)}\dots\Psi^{(I_p)}$:

$$k_Y(p)+k_{I_1}+....+k_{I_p}=0 \quad . \tag{13}$$

ii) The product $\rho^{(Y)}\times\rho^{(I_1)}\times...\times\rho^{(I_p)}$ contains an invariant singlet.

The field-dependent Yukawa couplings $Y_{I_1...I_n}(\tau)$ are closely related to automorphic forms. Indeed when we restrict to transformations γ of the group G_n in eq. (12), we obtain:

$$Y_{I_1...I_n}(\gamma\tau)=j(\gamma,\tau)^{k_Y(n)}\;Y_{I_1...I_n}(\tau)\,, \qquad \gamma\in G_n \quad , \tag{14}$$

Thus the function

$$\mathscr{A}(g)\equiv j(g,\tau_0)^{-k_Y(n)}Y_{I_1...I_n}(g\tau_0) \tag{15}$$

is an automorphic form for G, K and G_n: a smooth complex function $\mathscr{A}(g)$ that is invariant under the action of the discrete group G_n:

$$\mathscr{A}(\gamma g)=\mathscr{A}(g), \qquad \gamma\in G_n \quad , \tag{16}$$

and that under K transforms as

$$\mathscr{A}(gh) = j(h,\tau_0)^{-1}\,\mathscr{A}(g)\,, \qquad h \in K \quad . \tag{17}$$

Moreover $\mathscr{A}(g)$ is required to be an eigenfunction of the algebra $\mathscr{D}$ of invariant differential operators on G, that is an eigenfunction of all the Casimir operators of G. The definition is completed by suitable growth conditions [14].

As an example of the general framework outlined above, we analyze the case $G = Sp(2m,\mathbb{R})$, $K = U(m)$ and $\Gamma = Sp(2m,\mathbb{Z})$. The related automorphic forms are provided by Siegel modular forms. The elements of the symplectic group $Sp(2m,\mathbb{R})$ are $2m \times 2m$ real matrices of the type:

$$g = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \qquad\qquad g^T\, J\, g = J \qquad J \equiv \begin{pmatrix} 0 & \mathbb{1}_m \\ -\mathbb{1}_m & 0 \end{pmatrix} \quad . \tag{18}$$

The symplectic group $Sp(2m,\mathbb{R})$ has a maximal compact subgroup, $K = U(m)$. An element g of $Sp(2m,\mathbb{R})$ can be uniquely decomposed as:

$$g = \begin{pmatrix} \sqrt{Y} & X\sqrt{Y^{-1}} \\ 0 & \sqrt{Y^{-1}} \end{pmatrix} h \quad , \tag{19}$$

where X and Y are real symmetric $m \times m$ matrices, Y is positive definite $(Y > 0)$ and h is an element of K. We see that the moduli space $\mathscr{M} = G/K$, of complex dimension $m(m+1)/2$, can be parametrized by a symmetric complex $m \times m$ matrix τ with positive definite imaginary part, $\tau = X + iY$. This space is called Siegel upper half-plane, $\mathscr{H}_m$, a natural generalization of the complex upper half-plane. The integer m is the genus. The action of $Sp(2m,\mathbb{R})$ on τ is given by:

$$\tau \to g\tau = (A\tau + B)(C\tau + D)^{-1} \quad . \tag{20}$$

As automorphy factor, satysfying the cocycle condition of eq. (5), we can choose:

$$j(g,\tau) = [\mathtt{det}(C\tau + D)] \quad . \tag{21}$$

A natural candidate for the discrete gauge group is the Siegel modular group $\Gamma_m = Sp(2m,\mathbb{Z})$. Other discrete subgroups of $G = Sp(2m,\mathbb{R})$ relevant to our purposes are the principal congruence subgroups $\Gamma_m(n)$ of level n, defined as:

$$\Gamma_m(n) = \left\{ \gamma \in \Gamma_m \,\middle|\, \gamma \equiv \mathbb{1}_{2m} \,\mathtt{mod}\, n \right\}, \tag{22}$$

where n is a generic positive integer, and $\Gamma_m(1) = \Gamma_m$. The group $\Gamma_m(n)$ is a normal subgroup of Γ_m, and the quotient group $\Gamma_{m,n} = \Gamma_m/\Gamma_m(n)$, which is known as finite Siegel modular group, has finite order [15]. By keeping both the genus m and the level n fixed throughout our construction, the supermultiplets $\Psi^{(I)}$ of each sector I are assumed to transform in a representation $\rho^{(I)}(\gamma)$ of the finite Siegel modular group $\Gamma_{m,n}$, with a weight k_I. Under a discrete gauge transformation $\gamma \in \Gamma_m$ we have:

$$\begin{cases} \tau \xrightarrow{\gamma} \gamma\tau = (A\tau + B)(C\tau + D)^{-1}\,, \\ \Psi^{(I)} \xrightarrow{\gamma} [\det(C\tau + D)]^{k_I} \rho^{(I)}(\gamma)\, \Psi^{(I)} \;, \end{cases} \qquad \gamma = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in \Gamma_m \quad . \tag{23}$$

PoS(DISCRETE2020-2021)007

Due to the cocycle condition in eq. (5) and the properties of $\rho^{(I)}(\gamma)$, the above definition satisfies the group law. A minimal Kähler potential is given by:

$$K = -c\,\Lambda^2 \log\det(-i\tau + i\tau^\dagger) + \sum_I [\det(-i\tau + i\tau^\dagger)]^{k_I} |\Psi^{(I)}|^2 \quad c > 0. \tag{24}$$

For the p-th order term of the expansion (11) to be modular invariant, the functions $Y_{I_1...I_p}(\tau)$ should transform as Siegel modular forms with weight $k_Y(p)$ in the representation $\rho^{(Y)}(\gamma)$ of $\Gamma_{m,n}$:

$$Y_{I_1...I_p}(\gamma\tau) = [\det(C\tau + D)]^{k_Y(p)} \rho^{(Y)}(\gamma)\, Y_{I_1...I_p}(\tau) \quad , \tag{25}$$

with $k_Y(p)$ and $\rho^{(Y)}(\gamma)$ satisfying the conditions i) and ii) of the previous section.

This general construction encompasses the special case where the moduli space is the direct product of upper half-planes, $\prod_{k=1}^N SL(2,\mathbb{R})/SO(2)$, and the flavour group is the direct product of $SL(2,\mathbb{Z})$ factors [16]. Symplectic modular invariance and/or modular invariance arise also in orbifold compactification of heterotic string theory [17, 18, 19], where strong restrictions on bottom-up flavor model building can guide the search of realistic models [20].

In a generic point τ of the moduli space $\mathscr{H}_m$ the discrete symmetry Γ_m is completely broken (i.e. $\gamma\tau = \gamma$ has no solution for $\gamma \in \Gamma_m$), but there can be regions where a part of Γ_m is preserved. The invariant locus Ω_H is a region of $\mathscr{H}_m$ whose points τ are individually left invariant by some subgroup H of Γ_m. The group that, as a whole, leaves the region Ω_H invariant is the normalizer $N(H)$ of H, whose elements γ_N satisfy $\gamma_N^{-1} H \gamma_N = H$. As a consequence, in our supersymmetric action we can restrict the moduli τ to the region Ω_H, which supersedes the full moduli space $\mathscr{H}_m$, and replace the group Γ_m with $N(H)$. Consistent CP transformations can be defined on moduli, matter multiplets and modular forms [21]. These tools allow the construction of viable and predictive models of lepton masses and mixing angles.

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