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The muon magnetic moment: a precision test shaking the Standard Model

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The status of the measurement of the muon magnetic moment and of its prediction within the Standard Model is reviewed. Both approaches have been followed for decades with ever increasing precision. Their relative progress is a necessity in order to test the present theory and possibly find hints for new physical phenomena. Indeed, the recent measurement from Fermilab shows a tantalizing 4.2 standard-deviation excess over the up-to-date prediction. Particular emphasis is placed on the contribution from hadronic vacuum polarization which controls the theoretical precision of the prediction. It is evaluated through a dispersion relation using as input experimental results on electron-positron annihilation into hadrons which are thus playing a key role in establishing the prediction.

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1. Introduction

The release of the first result on the measurement of the muon magnetic anomaly by the Fermilab experiment in April 2021 [1] showed an excess at the level of 4.2 standard deviations when compared to the updated prediction within the Standard Model obtained by the Muon g-2 Theory Initiative [2] through a wide effort to produce the most reliable estimate [3–37]. The present excess confirms the previous deviation found by the Brookhaven experiment with an increased significance. After a general introduction and some information on the experimental method, I will review the status of the hadronic vacuum polarization contribution using a dispersion relation based on the measured cross sections for $e^+e^- \to$ hadrons. This contribution is the dominant one after the overwhelming QED contribution, while also controlling the precision of the prediction.

2. Motivation

2.1 Historical background: the electron magnetic moment

Dirac's relativistic theory of the electron in 1928 naturally accounted for quantized spin S and described elementary spin-1/2 particles, as well as predicting the existence of their anti-particles. In the classical limit the Dirac equation exhibits a Pauli magnetic term yielding a magnetic moment μ for the electron:

$$\vec{\mu} = -g_e \frac{e}{2m_e} \vec{S} \tag{1}$$

with the gyromagnetic factor $g_e = 2$. Dirac's prediction was confirmed by Kinsler and Houston in 1934 through the study of the Zeeman effect in Neon. However a deviation from $g_e = 2$ was established in 1947 by Nafe, Nels, and Rabi by comparing the hyperfine structure of hydrogen and deuterium spectra, followed by a first precision measurement by Kusch and Foley [39] using Rabi's atomic beam magnetic resonance technique, showing a distinct magnetic anomaly a_e:

$$a_e = \frac{g_e - 2}{2} = 0.00119(5) \tag{2}$$

This small deviation from relativistic quantum theory was soon understood through the development of quantum electrodynamics by Dyson, Feynman, Schwinger, and Tomonaga, a field theory describing the emission and absorption of photons by electrons, and implying the existence of quantum fluctuations induced by virtual particles (loops in the Feynman diagram language). With divergences regularized by renormalization of physical quantities, the amplitude for any QED process could be written as a perturbative expansion in the coupling constant e and visualized with Feynman diagrams at any order. While Dirac's $g_e = 2$ corresponded to the lowest-order QED graph, the first correction of order α with a virtual photon exchange was computed by Schwinger [40] to be

$$a_e^{QED}(\alpha) = \frac{\alpha}{2\pi} = 0.001161... \tag{3}$$

in agreement with the experimental value.

As precision of the measurements improved, it became necessary to include higher-order QED terms, as well as contributions from quantum fluctuations induced by other interactions, either already known (strong, weak) or possibly beyond, emphasizing the unique role of precision physics for investigating new phenomena not yet directly accessible at energy-frontier accelerators.

Progress on the direct measurement of the electron magnetic anomaly has been spectacular in the last decades using magnetic traps, first with the Seattle experiment [41], and more recently by the Harvard group [42]. Their latest value shows the fantastic level of precision currently achieved:

$$a_e^{exp} = 0.00115965218073(28) \tag{4}$$

2.2 Why the muon?

Given the situation with the accuracy obtained with the electron, one could wonder why one should bother with the muon. Its short lifetime places certainly a limit for the precision of the measurement, hence reducing the ability to search for new physics. However, this is not the case because of its heavier mass, so that the energy scale Λ that can be explored is in fact larger than for the electron, as one expects a corresponding contribution to the magnetic anomaly of a charged lepton of mass m_l

$$\Delta a_e^{new physics} \sim \left(\frac{m_l}{\Lambda}\right)^2 \tag{5}$$

providing a factor 43000 enhancement in favour of the muon. The effect would be even stronger for the τ lepton, but its very short lifetime renders the prospect of a direct measurement unlikely.

While electrons can be stored for a very long time allowing the study of their magnetic transitions, the experimental procedure for the unstable muon is quite different. It naturally follows from two striking features resulting from the maximal parity violation of the weak interaction: (1) in pion decays muons are produced fully polarized and (2), when injected into a magnetic ring as their spin precesses around the magnetic field, its orientation can be evaluated at the time of decay as it is correlated with the direction of energetic final-state electrons.

3. Six decades of muon anomaly measurements

The measurement scheme thus follows closely from the famous Garwin-Lederman-Weinrich textbook experiment which established in 1957 the maximal parity violation in pion and muon decays. Progress came with increased muon flux, higher energies, better storage and improved control of systematic effects. Most of the development was carried in successive experiments at CERN with the concept of a storage ring with electron detectors placed all around. The nice feature is that the experiment measures directly the muon magnetic anomaly a_μ proportional to the difference between the relativistic precession and cyclotron frequencies. Any deviation from $g_\mu = 2$ results into an angular mismatch over a full turn between the muon momentum vector and the spin direction, which accumulates with time. In order to keep a uniform magnetic field $\overrightarrow{B}$, strong focussing of the stored muon beam is achieved by electrostatic quadrupoles, while the unwanted motional magnetic effect $\overrightarrow{\beta} \times \overrightarrow{B}$, where β is the muon velocity, is eliminated by choosing for the muon momentum the "magic" value of 3.09 GeV/c, corresponding to $\gamma^2 = 1 + 1/a_\mu$. The anomalous precession frequency ω_a is then given by

$$\omega_a = \frac{e}{m_\mu c} a_\mu B \tag{6}$$

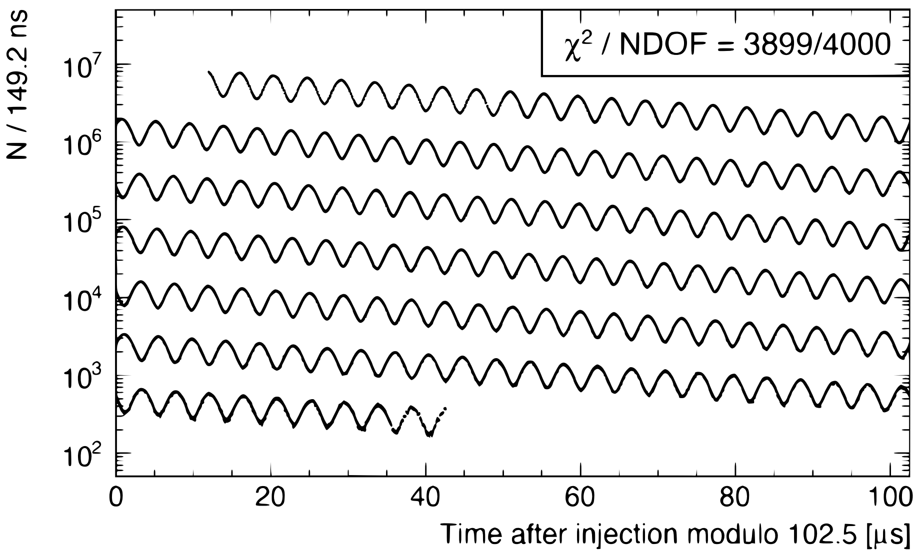

Figure 1: The decay rate of stored polarized muons in the Fermilab experiment observed in electron detectors is modulated at the anomaly frequency ω_a. Reprinted from Ref. [43].

The determination of a_μ is given *in fine* by the ratio of two frequencies: ω_a as obtained from the modulation of the electron counting as function of time and ω_p, the magnetic resonance frequency of protons in probes surveying the value of the magnetic field on the muon orbits by carefully averaging over the beam transverse distribution. The counting rate in electron detectors, modulated at the frequency ω_a, is shown in Fig. 1 for the Fermilab experiment.

The best determination of a_μ before Fermilab was obtained at Brookhaven in 2006

$$a_\mu^{BNL} = 0.00116592080(54_{stat})(33_{syst}) \tag{7}$$

The Fermilab set-up is a copy of the previous CERN and BNL experiments, reconstructed anew except for the ring superconducting magnet moved over from the Brookhaven site. It benefits from a cleaner and $\times 20$ more intense beam allowing for more muons to be stored, and an improved electron detection incorporating a tracking system. The latter feature also permits a close monitoring of the position and shape of the stored bunched beam throughout the fills. A strong asset of the experiment is the double blinding of the two frequencies entering the final ratio, each one obtained and studied with unknown clock offsets, the final values being only disclosed after the completion of all consistency tests and correction of systematic effects.

While the goal of the experiment is to improve the precision of a_μ by a factor of 4 over a few years, thus a final relative uncertainty of 250 ppb, the first results presented in 2021 are based on only 6% of the foreseen data set. An important achievement is that, already at this level, systematic uncertainties (150 ppb) are consistent with their final expectation. The obtained result [1]

$$a_\mu^{FNAL} = 0.00116592040(43_{stat})(16_{syst}) \tag{8}$$

is in agreement with the Brookhaven value with comparable precision, both dominated by statistical uncertainties. Therefore, they can be combined, providing the best experimental determination to-date [1]

$$a_\mu^{exp} = 0.00116592061(41) \tag{9}$$

4. The theoretical prediction

4.1 Introduction

The longstanding discrepancy observed between the measured and the predicted values of the anomalous magnetic moment of the muon represents a potential hint of new physics in particle physics and called for an improvement of the theoretical prediction, in view of the significant gain in precision foreseen for the Fermilab measurement. A series of workshops organised by the Muon g-2 Theory Initiative has allowed for detailed discussions on the various approaches that are being considered for computing the different contributions to a_μ. A consensus has been reached for a realistic, yet conservative, evaluation of the theoretical prediction and its uncertainty. The results presented here are based mainly on the White Paper of the Muon g-2 Theory Initiative [2] and on the inputs used therein from Refs. [3–37].

4.2 Status of the theoretical prediction

4.2.1 Different components

The evaluation involves QED contributions, currently known up to O(α^5) [34, 35], which corresponds to a precision of 0.001 ppm. Then there are electroweak contributions, currently known with a precision of 0.01 ppm. The dominant uncertainties originate from non-perturbative contributions, namely the hadronic vacuum polarisation (HVP) and the hadronic light-by-light (HLbL), with a precision of 0.34 ppm and 0.15 ppm respectively. All these contributions have to be well under control before one can make any claim about the observation of potential contributions from new physics, beyond the Standard Model.

The dominant uncertainty for the theoretical prediction originates from lowest-order HVP contribution ($a_\mu^{\text{HVP, LO}}$), which cannot be calculated based on perturbative QCD because of the low mass scale. Instead, one can use experimental data on hadronic production cross section ($\sigma\,(\mathrm{e^+e^-} \to \text{hadrons})$) under first-principles assumptions. Indeed, the optical theorem relates by unitarity the imaginary part of the two-point correlator to the hadronic cross section and by analyticity a dispersion integral allows to compute $a_\mu^{\text{HVP, LO}}$. The kernel of this integral strongly enhances the low-energy region and therefore precise measurements of the $\sigma\,(\mathrm{e^+e^-} \to \text{hadrons})$ in this region are very important. We do not use anymore data from hadronic τ decays, because they are less precise and are also impacted by theory uncertainties on isospin-breaking corrections [7].

4.2.2 The data-based dispersive approach for the HVP contribution

The reliabilility and precision of the HVP contribution computed with the dispersion integral is completely controlled by the quality of the input e^+e^- data from different experiments and their proper combination which has to take into account a detailed analysis of their systematic uncertainties and correlations.

Figure 2 shows examples of experimental spectra for the dominant $\pi^+\pi^-$ channel. For CMD-2 [44, 45] and SND [46] the measurements are performed through a scan of the energy in the centre-of-mass of the collider. KLOE used the ISR technique, selecting events with a hard photon emitted from the initial state in different topologies [47–49]. The measurement performed at BABAR also uses the ISR approach, but selecting events with the hard photon reconstructed in the

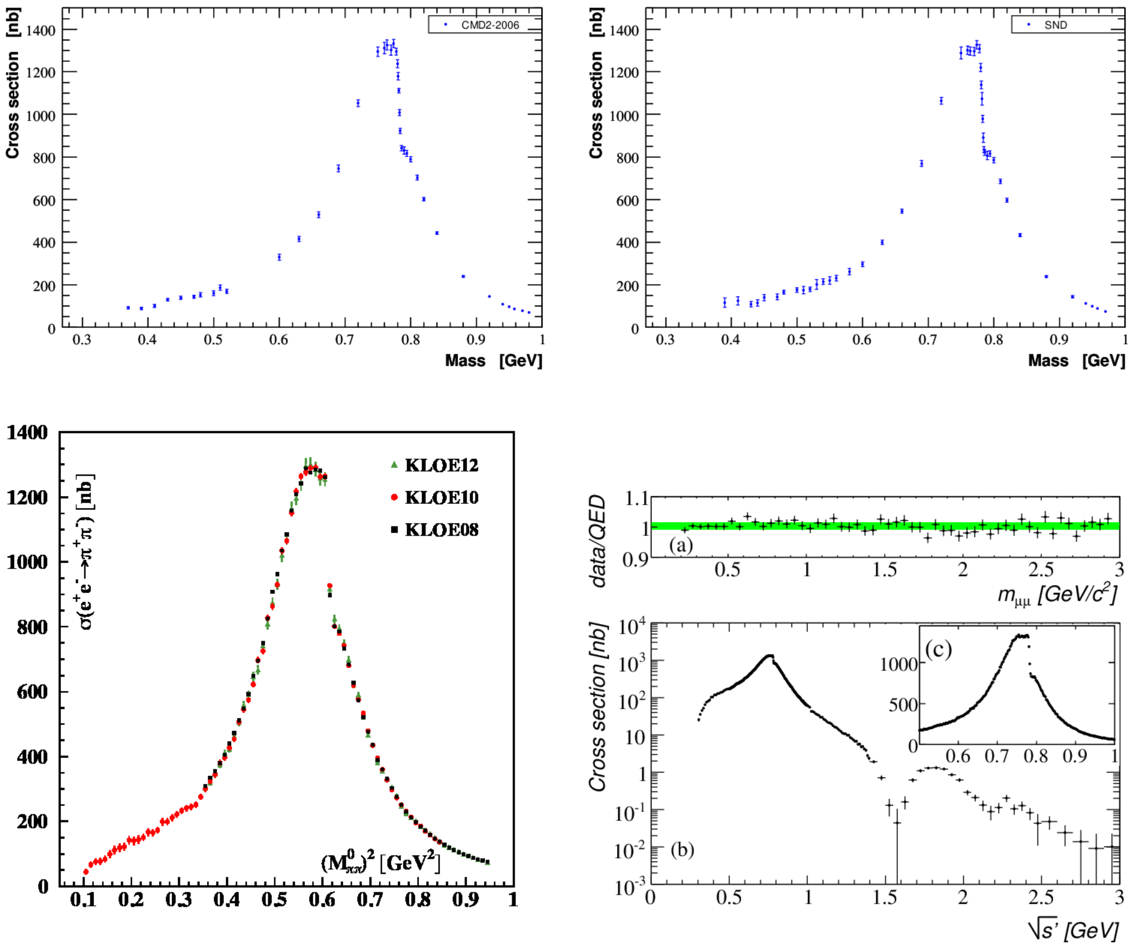

Figure 2: The top plots show data from CMD-2 [44, 45] (left) and SND [46] (right) on $e^+e^- \to \pi^+\pi^-$ in the ρ region. Bottom-left: the plot, adapted from Ref. [49], shows the KLOE data sets on $e^+e^- \to \pi^+\pi^-$ in the ρ region obtained in the three experimental configurations described in the text [47–49]. Bottom-right: the plot, reprinted from Ref. [50], displays the results from BABAR [50, 51] using the large-angle ISR method: $e^+e^- \to \mu^+\mu^-$ compared to NLO QED (top frame) and $e^+e^- \to \pi^+\pi^-$ from threshold to 3GeV using the $\pi\pi/\mu\mu$ ratio (bottom frame). The insert shows the ρ region.

detector, allowing to cover the full energy range of interest. Furthermore, in the case of BABAR, the ISR luminosity is evaluated in-situ based on $e^+e^- \to \mu^+\mu^-$ events, which allows to reduce several systematic uncertainties, and in both the $\pi\pi$ and $\mu\mu$ channels events with extra photons are considered, thus achieving an "NLO measurement". In the $\mu\mu$ channel, the comparison of the measured spectrum with the NLO QED-based prediction provides an excellent test of the procedure [50, 51]. While the KLOE '08 [47] and KLOE '12 [49] measurements consider events with the hard ISR photon along the beam, the KLOE '10 measurement uses large-angle ISR photons [48]. The KLOE '12 measurement also uses the muon spectrum for the normalisation [49]. Still, for the time being, only the BABAR study includes reconstructed extra photons, a necessary feature given the sub-percent level precision aimed for the analysis.

Thanks to an extensive program with the ISR method BABAR has provided a complete set

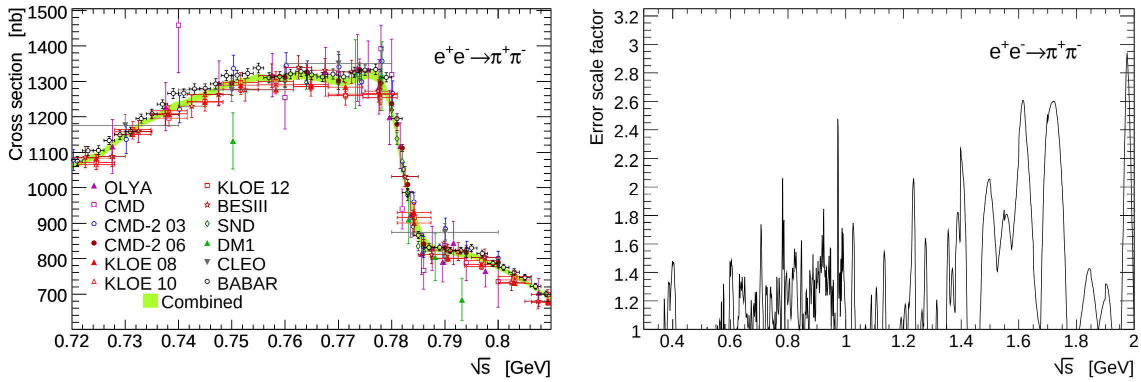

Figure 3: Left: Cross section for $e^+e^- \to \pi^+\pi^-$ annihilation measured by the different experiments for the entire energy range. The error bars contain both statistical and systematic uncertainties, added in quadrature. The shaded (green) band represents the average of all the measurements obtained by HVPTools, which is used for the numerical integration following the procedure discussed in Ref. [7]. Right: Rescaling factor accounting for inconsistencies among experiments versus $\sqrt{s}$. Reprinted from Ref. [7].

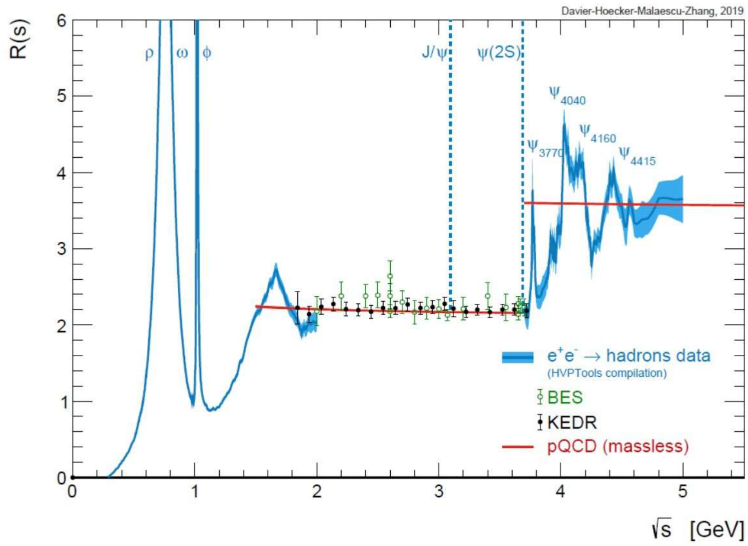

Figure 4: The R ratio from the sum of exclusive hadronic cross sections up to 2 GeV, perturbative QCD between 2 and 3.7 GeV, cross section data from 3.7 to 5 GeV, and perturbative QCD above. From the DHMZ analysis [7]

of measurements of some 40 hadronic cross sections up to multiplicities of 6-7 of pions, kaons and η mesons. Some channels have also been measured with the scan method at Novosibirsk, in good agreement with BABAR. This procedure allows one to reconstruct R, the ratio of the inclusive hadron cross section to the point-like cross section, up to 1.8 GeV with a negligible loss of high-multiplicity channels. Figure 4 displays the R reconstruction from this sum of exclusive cross sections, complemented with the perturbative QCD prediction in the 2-3.7 GeV range and data in the charm region up to 5 GeV. Above this value perturbative QCD is used.

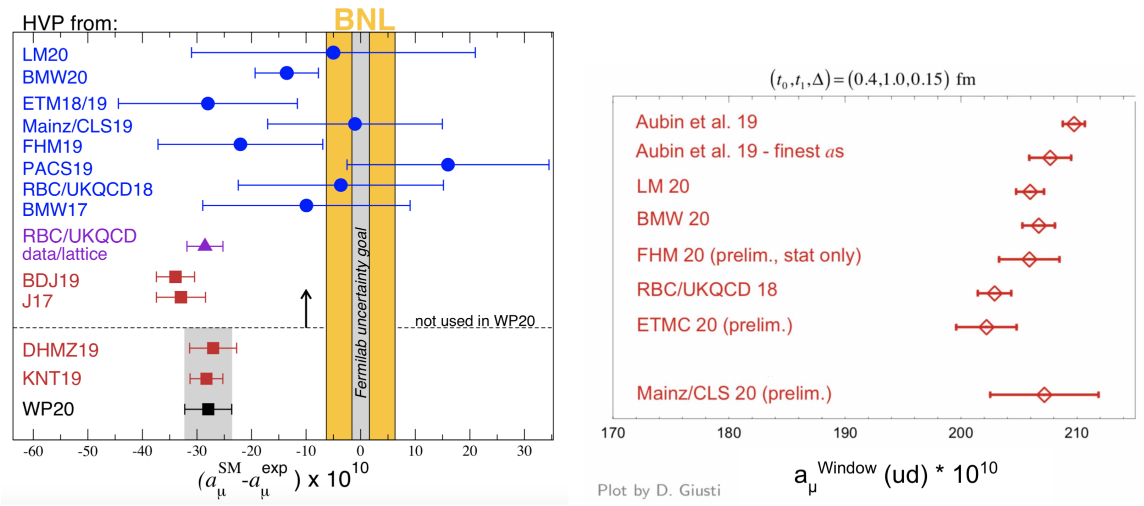

Figure 5: Left: The theoretical predictions based on various determinations of $a_\mu^{\text{HVP, LO}}$ are compared with the BNL experimental result [55], indicated by the yellow vertical band. The result of the Theory Initiative White Paper [2] (WP20), as well as the DHMZ19 [7] and KNT19 [8] inputs, are indicated below the dashed horizontal line. The uncertainty goal for the Fermilab measurement is also displayed. Right: Comparison of the values for a "Window" moment defined in Ref. [12], for various lattice QCD studies.

Computing the $a_\mu^{\text{HVP, LO}}$ contribution requires combining the experimental spectra measured by various experiments, as performed e.g. by the DHMZ [3, 7] (see Fig. 3 left) and KNT [4, 8] teams respectively, and summing the contributions of the various exclusive channels. While the channel-by-channel combinations are performed taking into account the information on the uncertainties and their correlations between bins/points and between experiments, the DHMZ approach (implemented in the HVPTools software) also accounts for correlations between different channels. This induces a necessary enhancement of the total uncertainty.

Furthermore, a method for taking into account the local tensions between the measurements (based on the computation of a χ^2/ndof in fine energy ranges, used for the rescaling of the uncertainties) has been implemented by DHMZ in Refs. [52, 53] and is still being used [3, 7] (see Fig. 3 right). Such an approach has also been used in the more recent KNT studies [4, 8]. Still, this local rescaling of the uncertainties is not sufficient in presence of systematic tensions, as the ones between the BABAR and the KLOE measurements in the $\pi\pi$ channel. These tensions are taken into account in the most recent DHMZ study [7] by treating half of the difference between integrals without either BABAR or KLOE as an extra uncertainty. This yields the dominant uncertainty in the study, amounting to $2.8 \cdot 10^{-10}$, degrading the final accuracy of the HVP contribution by 30%. In order to reach a level of precision commensurate to the final goal of the Fermilab experiment, it will be necessary to remove this tension, either by new precise analyses which are underway with BABAR, Belle II, BES, CMD-3 and SND, and/or, preferably, by identifying some systematic bias not accounted for in either BABAR and/or KLOE analyses.

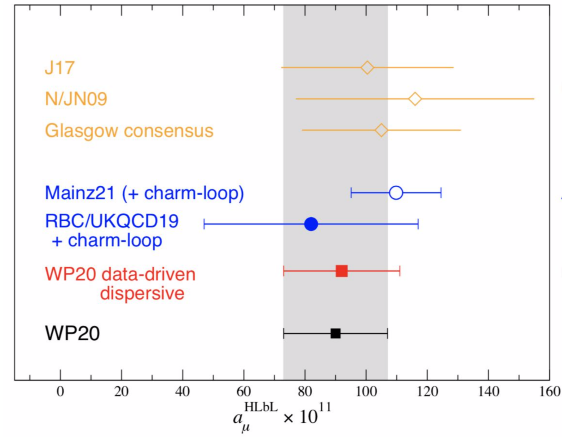

Figure 6: Summary of the $a_\mu^{\rm HLbL}$ values obtained through various approaches: hadronic model + perturbative QCD (yellow diamond), lattice QCD+QED (blue circles) and data-driven (red square). The result of the Theory Initiative White Paper [2] (WP20) indicated by the vertical grey band uses only the inputs marked with filled symbols.

5. Results for the prediction and comparison to the Fermilab result

Figure 5 left shows the comparison between the theoretical predictions based on various determinations of $a_\mu^{\rm HVP,\,LO}$ and the BNL experimental result [55]. The result of the Theory Initiative White Paper [2] is obtained through the merging of model independent results, as obtained by DHMZ [7] and KNT [8] (and CHHKS for the $\pi^+\pi^-$ [5] and $\pi^+\pi^-\pi^0$ [6] channels). The corresponding central value is obtained from the simple average of the input integrals. The largest of the DHMZ and KNT uncertainties is used in each channel. The treatment of the BABAR-KLOE tension in the $\pi\pi$ channel and of the correlations between the various channels is based on the DHMZ result. The importance of properly accounting for these two aspects, as done in the DHMZ study, is indeed visible when comparing the uncertainties of the DHMZ and KNT results (see Fig. 5 left).

During the last few years there has been excellent progress on the lattice QCD (+QED) calculations [10–18, 54]. In particular, while the BMW20 [54] result is still to be cross-checked by other lattice groups, its precision approaches that of the dispersive method. There are indeed ongoing cross-checks using Euclidean time windows (related to the HVP with suppression of very low and high energies [12]) for which various groups achieved similar precision (see Fig. 5 right). If the BMW20 result is confirmed, the difference with respect to the dispersive results will have to be understood.

A summary of the $a_\mu^{\rm HLbL}$ values obtained through various approaches, as well as the result of the Theory Initiative White Paper [2], are indicated in Fig. 6. One can notice that there is indeed good progress on systematically improvable approaches. Furthermore, the g-2 Theory Initiative provides an adequate environment for cross-checks among these results. The uncertainty of $a_\mu^{\rm HLbL}$ is currently controlled with a precision of 0.15 ppm. The result obtained by the Mainz group [56], with a somewhat better precision than the result of the Theory Initiative White Paper [2], became publicly available after the latter and is to be considered in future iterations.

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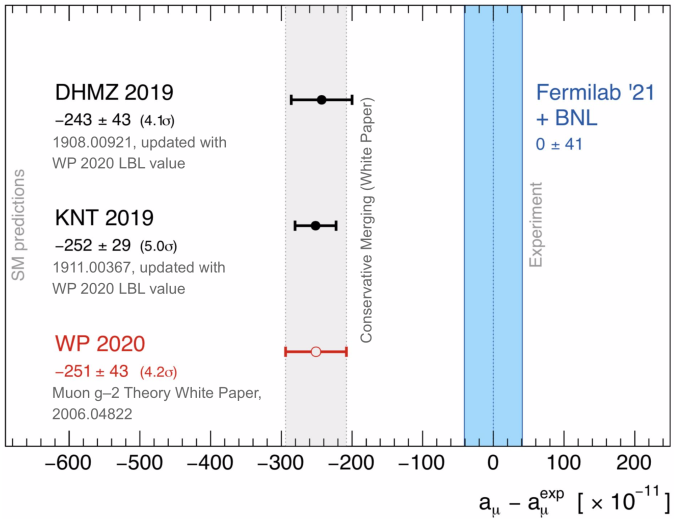

Figure 7: The theoretical predictions based on various data-driven determinations of $a_\mu^{\text{HVP, LO}}$ are compared with the Fermilab [1] and BNL [38] combined experimental result, indicated by the blue vertical band. The result of the Theory Initiative White Paper [2] is indicated by the red empty circle and the gray vertical band. The DHMZ19 [7] and KNT19 [8] results, updated to use the a_μ^{HLbL} from the White Paper, are indicated by the filed black circles.

Putting together all the contributions to the theoretical prediction, the dominant uncertainty originates from $a_\mu^{\text{HVP, LO}}$, based on the merging of model-independent methods, while the a_μ^{HLbL} is impacted by an important uncertainty too [2]. At the same time, lattice QCD results become more and more interesting. In Fig. 7 the theoretical predictions based on various data-driven determinations of $a_\mu^{\text{HVP, LO}}$ are compared with the Fermilab [1] and BNL [38] combined experimental result. A tension between the BNL measurement and the reference SM prediction, at the level of 3.7 σ is observed. The tension increases to 4.2 σ when including the FNAL result. The tension is significantly smaller when using BMW20 for the $a_\mu^{\text{HVP, LO}}$, a result which is still to be confirmed by other lattice groups.

The next couple of years promises to be an exciting time for the continuation of this investigation.

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