Vector-like Quarks

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In this talk we emphasise the importance of vector-like quarks (VLQs) and their potential to solve some of the open questions of the Standard Model. These are, in some sense minimal extensions of the Standard Model, that can be probed in the next round of experiments. We also make an analogy between vector-like quarks (VLQs) and right-handed neutrinos, emphasising that in both cases some of the flavour dogmas of the SM are violated in a controlled way.

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1. Introduction

The Standard Model (SM) is about 50 years old! One may consider that the SM was finally born in 1971, with the discovery of renormalizability of spontaneously broken gauge theories by 't Hooft and Veltman [1–3]. The birth took about ten years (1961-1971). The building of the SM started in 1961 with Glashow's paper [4] suggesting the gauge group $SU(2) \times U(1)$. One may ask the question why to include 1971 as the year when the construction of the SM was completed. The best way to answer this question is by asking the following question: What were particle physicists doing in 1968? One may answer this question consulting the ICHEP(1968) Proceedings [5]. This was one of the most exceptional conferences of all times. The number of Nobel laureates (either already Nobel at the time of the conference or later) is truly impressive: 19! But Nobody among these renowned participants talked about gauge theories! The turning point was induced by 't Hooft and Veltman's revolutionary papers. This illustrates the importance of the work of 't Hooft and Veltman in completing the construction of the SM.

Strictly speaking, the SM as suggested by Glashow, Weinberg and Salam [4, 6, 7], has been ruled out by the discovery of neutrino oscillations indicating that at least two neutrinos have non-zero masses. However, most of the predictions of the SM involving the gauge sector, the quark sector and the Higgs sector have been verified experimentally to a great precision, with only some experimental hints for physics beyond the SM in these sectors, yet to be confirmed. Furthermore, there is indirect evidence for the need of new sources of CP violation in order to account for the observed baryon asymmetry of the Universe (BAU). The SM also lacks suitable dark matter candidates and leaves several questions open, in particular it does not provide an understanding of the flavour pattern of fermion masses and mixing.

At this stage, one may ask: What is the key message that "Nature" is telling us concerning the observation of neutrino oscillations? In our opinion the message is: Choose Simplicity! Indeed the SM is "almost" the simplest possibility to put together the charged and neutral gauge interactions including the electromagnetism. Why almost? The reason for this adjective has to do with the fact the new SM (NSM) is simpler. By NSM we mean the SM with the addition of right handed neutrinos. Recall that at the time one of the rules to construct gauge models was: Write down the most general Lagrangian consistent with gauge invariance and renormalizability. If one follows this rule one has to include Dirac mass terms for the neutrinos, $\overline{\nu_L^0} m \nu_R^0$ together with Majorana mass terms, $\frac{1}{2} M \nu_R^{0T} C^* \nu_R^0$. This automatically leads to the seesaw mechanism [8–12]. Peter Minkowski was the first one to write the seesaw formula [8] but had forgotten until hearing about it again in 2004. Historically there were other motivations to introduce right handed neutrinos: SO(10) grand unified theories (GUTs) and Left-Right Symmetric models, for a review see [13] and references therein. At the moment it is not known whether neutrinos are Dirac or Majorana particles. But if we follow the "Simplicity Principle" neutrinos should be Majorana particles. For a long time, there was a profound prejudice in favour of massless neutrinos based on the fact that there was no experimental evidence for neutrino masses and leptonic mixing. This prejudice was extended to GUTS. The only "reasonable" GUT was SU(5). In SU(5) right handed neutrinos are not introduced and B-L is an accidental exact symmetry. As a result, neutrinos are strictly massless in SU(5), just as in the SM.

There were other related dogmas in Flavour Physics, based on the properties of the SM and on stringent experimental limits, namely:

- Dogma 1 : There should be no Z-Mediated Flavour-Changing Neutral Currents (FCNC) at tree level either in the quark or in the leptonic sectors.

- Dogma 2: Dogma 1 was extended to the scalar sector. There should be no scalar mediated FCNC at tree level.

Dogma 1 is violated in the leptonic sector with Majorana neutrinos. Dogma 2 is safely violated in Branco-Grimus-Lavoura (BGL) models [14] with an extended Higgs sector and their generalisations by Botella, Branco and Rebelo [15]. Here safely refers to the natural suppression of these effects, based on a symmetry.

In the seesaw framework, with the addition of three right handed neutrinos, the leptonic mixing matrix at low energies, the Pontecorvo-Maki-Nakagawa-Sakata, U_{PMNS}, matrix, is not 3×3 unitary. By the same token Dogma 1 is also violated. These two aspects result from the mixing among the six neutrino fields appearing in the flavour basis and leading to the three light neutrinos and the three heavy neutrinos. This is a special feature of seesaw since Majorana neutrinos have fewer degrees of freedom than Dirac neutrinos, resulting in the possibility of doubling the number of physical particles with different masses. Indeed FCNC appear at tree level but are naturally suppressed by the ratio m/M where m stands for Dirac neutrino masses and M for the Majorana masses of right handed neutrinos. Since the Majorana right handed neutrino mass terms are gauge invariant they are not protected by the gauge symmetry, unlike Dirac mass terms, and thus can be much larger than v, the scale of electroweak symmetry breaking. The same suppression operates in the deviations of unitarity of the U_{PMNS} matrix.

2. Analogy between Vector-like Quarks (VLQs) and right-handed neutrinos

Consider an extension of the SM where new quark isosinglets, i.e., singlets of SU(2), of charge -1/3 and/or +2/3 are introduced, such that the right handed and the left handed components transform in the same way under the gauge group. In this case mass terms of the form $M_D \overline{D_L} D_R$ and/or $M_U \overline{U_L} U_R$ are gauge invariant and must be introduced in the Lagrangian. The simplest possibility is to assume that VLQs are isosinglets but there are other possible choices of representations that are also of interest. VLQs have in common with right handed neutrinos the fact that a new scale (let us call it V) is introduced by these mass terms. This scale can be significantly above v, the scale of electroweak symmetry breaking. Like in the leptonic sector, VLQs lead to violations of 3×3 unitarity of the Cabibbo-Kobayashi-Maskawa matrix, the V_{CKM} matrix, as well as to Z-mediated FCNC. Both effects are related. In the case of vector-like quarks these effects result from the mixing of the standard-like quarks with the new vector-like quarks and are naturally suppressed by the ratio v/V. In the quark sector all physical particles are of Dirac-type. Majorana mass terms would break any U(1) charge and therefore are forbidden for any particle with non-zero electric charge.

3. What can Vector-like Quarks (VLQs) do for you?

Next we give a partial answer to the above question, providing a list of some of the contributions that VLQs can give to an understanding of the some of the open questions in the SM.

i) VLQs provide a simple alternative solution to the Strong CP problem [16, 17], without having axions, which so far have not been seen. This solution was proposed by Barr and Nelson [18–20] and a minimal implementation in a model, was given by Bento, Branco and Parada [21].

ii) They provide the simplest extension of the SM with spontaneous CP violation, in a model consistent with experiment. At this stage it is worth recalling what are the requirements in order to have a viable model of spontaneous CP violation: The Lagrangian should be CP invariant but CP invariance should be broken by the vacuum. The minimal extension of the SM leading to spontaneous CP violation involves the extension of the scalar sector through the addition of a complex singlet. Sometimes the vacuum leads to geometrical vacuum phases. It is important to note that often these geometrical phases do not lead to spontaneous CP violation [22].

The vacuum phase should be able to generate a complex Cabibbo-Kobayashi-Maskawa matrix, V_{CKM}, since experimentally we do know that V_{CKM}, is complex even if one allows for the presence of New Physics [23, 24]. Note that at present we do know that the phase γ where $\gamma = \arg(-V_{ud}V_{cd}V_{ub}^*V_{cd}^*)$ is not zero or pi and we also know that it cannot be generated solely through new physics beyond the SM.

iii) With the additional introduction of right handed neutrinos in the leptonic sector it is possible to build models with VLQs where there is a common origin for all CP violations [25], i.e., models providing at the same time a solution for the strong CP problem of the type described in i), spontaneous CP violation, CP violation in the quark sector in agreement with experiment of the type described in ii) and CP violation in the leptonic sector both at low energies and at hight energies leading to the possibility of leptogenesis [26]. Such models require minimal extensions of the SM with either the introduction of one down vector-like or one up vector-like quark and will be described in what follows.

iv) VLQs provide a simple framework where there are New Physics contributions to $B_d - \overline{B}_d$, $B_s - \overline{B}_s$ and/or $D^0 - \overline{D}^0$ mixing. Also in models with up-type VLQs, there are contributions to the decay $t \rightarrow cZ_\mu$ at tree level [27, 28].

v) VLQs may populate the "desert" between v and some higher mass scale like M_{GUT}, without worsening the hierarchy problem. To our knowledge, this was first emphasised in a paper by Pierre Ramond [29].

vi) VLQs may contribute to the unification of the coupling constants without requiring Super-symmetry [30, 31].

vii) VLQs may play an important rôle in providing an explanation for the V_{CKM}, unitarity problem, arising from new measurements of $|V_{us}|$ and $|V_{ud}|$ together with new theory calculations and lattice results [32–38], namely:

$$\Delta \equiv 1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2 < 1, \qquad \sqrt{\Delta} \sim 0.04 \tag{1}$$

at the level of 2 to 3 standard deviations [39–42]. One may wonder whether or not this two to three sigma deviation should be taken as a serious hint for physics beyond the SM. Several experts are taking it seriously.

4. A Common Origin for all CP Violations

Let us illustrate how VLQs can address the first three open questions listed above [25]. The model is based on the Bento, Branco and Parada framework [21] together with the leptonic sector of the SM extended with the inclusion of three right handed neutrinos. CP invariance is imposed on the Lagrangian so that all coefficients are taken to be real. A $\mathbb{Z}_2$ symmetry is imposed to the quark and to the Higgs sectors enlarged with the addition of one down-type isosinglet vector-like quark (D_L^0, D_R^0) and a complex scalar singlet S.

$$\begin{pmatrix} u^0 \\ d^0 \end{pmatrix}_{iL}, u_{iR}^0, \ d_{\alpha R}^0, \ D_L^0, \ i = 1, 2, 3, \ \alpha = 1, ..., 4, \ \phi, \ S \tag{2}$$

All standard like particles in these two sectors are invariant under this $\mathbb{Z}_2$ symmetry while D_L^0, D_R^0 and S are odd under $\mathbb{Z}_2$. The $SU(2) \times U(1) \times \mathbb{Z}_2$ invariant potential is of the form:

$$V = V_0 \ (\phi, S) + (\mu^2 + \lambda_1 \ S^* S + \lambda_2 \ \phi^\dagger \phi)(S^2 + S^{*2}) + \lambda_3 \ (S^4 + S^{*4}) \tag{3}$$

where V_0 contains all terms that are phase independent and includes the SM Higgs potential. The vevs of the neutral scalar fields will be of the form:

$$\langle \phi^0 \rangle = \frac{v}{\sqrt{2}}, \quad \langle S \rangle = \frac{V \exp(i\beta)}{\sqrt{2}} \tag{4}$$

and will in general violate CP. There are two mass scales v, the electroweak breaking scale and V which can be much larger than v. The Yukawa interactions of the quarks are given by:

$$\mathcal{L}_Y = -\sqrt{2}(\overline{u^0} \ \overline{d^0})_L^i (g_{ij}\phi \ d_{jR}^0 + h_{ij}\tilde{\phi} \ u_{jR}^0) - M_d \overline{D_L^0} \ D_R^0 - \sqrt{2}(f_i S + f_i' \ S^*) \ \overline{D_L^0} d_{iR}^0 + \text{h.c.} \tag{5}$$

with (i,j = 1, 2, 3) and the down quark mass matrix is now of the form:

$$\mathcal{M}_d = \begin{pmatrix} m_d & 0 \\ \overline{M_d} & M_d \end{pmatrix} \tag{6}$$

the zero block of $\mathcal{M}_d$ results from the $\mathbb{Z}_2$ symmetry. Only the block $\overline{M_d}$ can be complex. Therefore arg det $\mathcal{M}_d$ is zero. The up quark mass matrix is real. As a result arg(det $\mathcal{M}_d \times \det m_u$) is zero and the strong CP problem is solved à la Barr and Nelson [21]. Mixing of the four down type quarks can lead to a complex V_{CKM} with deviations from unitarity suppressed by the ratio v/M_D,

where M_D is the mass of the heavy down quark, as was shown in [21]. ZFCNC also feel the same type of suppression [21]. However the strength of CP violation does not suffer the same type of suppression provided that the new mass scale of M_d is not higher than that of $\overline{M_d}$ [21], even in the limit when both go to infinity. This is an example of non-decoupling of very massive particles. In this framework the introduction of the complex scalar singlet S together with the Higgs doublet leads to spontaneous CP violation. The CP violating phase β generated at high energies leads to a complex V_{CKM} matrix due to the presence of the isosinglet vector-like quark which has couplings to the scalar S together with SM-like right handed quarks. This rôle can be played by either a down or an up-type vector-like quark.

In the extension of this model to the leptonic sector the $\mathbb{Z}_2$ is promoted to a $\mathbb{Z}_4$ symmetry [25] under which the fields that are not invariant transform as:

$$
\begin{aligned}
D^0 &\to -D^0, \quad S \to -S, \\
\psi_L^0 &\to i\psi_L^0, \quad e_R^0 \to ie_R^0, \quad \nu_R^0 \to i\nu_R^0,
\end{aligned}
\tag{7}
$$

where ψ_L^0 are the left handed lepton doublets, e_R^0 are the right handed charged leptons and ν_R^0 stand for the three right handed neutrinos. The Yukawa terms for the leptonic sector are:

$$
\mathcal{L}_l = \overline{\psi_L^0} G_l \phi\, e_R^0 + \overline{\psi_L^0} G_\nu \tilde{\phi}\, \nu_R^0 + \frac{1}{2} \nu_R^{0T} C(f_\nu S + + f_\nu' S^*) \nu_R^0 + h.c.
\tag{8}
$$

again all new coefficients are taken to be real. After spontaneous symmetry breakdown one obtains the following mass terms for the charged leptons and for the neutrinos:

$$
\begin{aligned}
\mathcal{L}_m &= -\left[\overline{\nu_L^0} m \nu_R^0 + \frac{1}{2} \nu_R^{0T} CM \nu_R^0 + \overline{l_L^0} m_l l_R^0 \right] + h.c. \\
&= -\left[\frac{1}{2} n_L^T C\mathcal{M}^* n_L + \overline{l_L^0} m_l l_R^0 \right] + h.c. .
\end{aligned}
\tag{9}
$$

The leptonic mass matrices are now given by:

$$
\begin{aligned}
\mathcal{M} &= \begin{pmatrix} 0 & m \\ m^T & M \end{pmatrix}, \quad m_l = \frac{v}{\sqrt{2}} G_l, \quad m = \frac{v}{\sqrt{2}} G_\nu \\
M &= \frac{V}{\sqrt{2}} (f_\nu^+ \cos(\alpha) + i f_\nu^- \sin(\alpha))
\end{aligned}
\tag{10}
$$

with $f_\pm^\nu \equiv f_\nu \pm f_\nu'$. In Ref. [25] it is shown how this framework can lead to a complex U_{PMNS} matrix. It is also shown that this framework contains the necessary ingredients allowing for the possibility of leptogenesis [26]. Although CP violation in the leptonic sector has not yet been observed it is natural to expect that it should also be present in this sector since it is broken in the quark sector and therefore it is not a symmetry of nature.

5. Conclusions

We have emphasised that vector-like quarks (VLQs) are one of the simplest extensions of the SM and yet they provide, on the one hand a possible solution to some of the open questions of

the SM and on the other hand, they predict various new phenomena so that their predictions may be probed in the next round of experiments. Interesting aspects of VLQs include the possibility of having a realistic model with spontaneous CP violation and one of the simplest solutions to the strong CP problem without the introduction of axions. VLQS also provide a framework where one can have a common origin for all manifestations of CP violation, including CP breaking in the quark and lepton sectors and the generation of the Baryon Asymmetry of the Universe through leptogenesis.

In section 3 we have summarised several aspects of physics beyond the Standard Model that can be addressed via the introduction of VLQs. Many of these are related to recent experimental results from collider experiments. This list is far from complete.

Recent measurements of $|V_{us}|$ and $|V_{ud}|$, together with recent theoretical improvements in calculations, indicate that unitarity of the first row of the V_{CKM} may be violated at the level of two or three standard deviations. It is possible to account for these deviations with simple extensions of the Standard Model with vector-like quarks. This possibility was analysed in detail in [39–42].

We have addressed in different works of ours, involving VLQs, other recent experimental hints for new physics. However, we have opted to illustrate in section 4 of this paper some of the mechanisms at work in models with VLQs rather than to deal with very specific phenomena.

Models with VLQs have always attracted attention in the literature. At present many more authors are turning their attention to the study of this class of models. VLQs naturally arise in the context of some Grand Unified models like E_6.

Acknowledgments

References

[1] G. 't Hooft, "Renormalization of Massless Yang-Mills Fields," Nucl. Phys. B **33** (1971), 173-199 doi:10.1016/0550-3213(71)90395-6

[2] G. 't Hooft, "Renormalizable Lagrangians for Massive Yang-Mills Fields," Nucl. Phys. B **35** (1971), 167-188 doi:10.1016/0550-3213(71)90139-8

[3] G. 't Hooft and M. J. G. Veltman, "Regularization and Renormalization of Gauge Fields," Nucl. Phys. B **44** (1972), 189-213 doi:10.1016/0550-3213(72)90279-9

[4] S. L. Glashow, "Partial Symmetries of Weak Interactions," Nucl. Phys. **22** (1961), 579-588 doi:10.1016/0029-5582(61)90469-2

[5] J. Prentki and J. Steinberger, "Proceedings, 14th International Conference on High-Energy Physics (ICHEP 68): Vienna, Austria, 28 Aug-5 Sep, 1968"

[6] S. Weinberg, "A Model of Leptons," Phys. Rev. Lett. **19** (1967), 1264-1266 doi:10.1103/PhysRevLett.19.1264

[7] A. Salam, "Weak and Electromagnetic Interactions," Conf. Proc. C **680519** (1968), 367-377 doi:10.1142/9789812795915_0034

[8] P. Minkowski, "$\mu \to e\gamma$ at a Rate of One Out of 10^9 Muon Decays?," Phys. Lett. B **67** (1977), 421-428 doi:10.1016/0370-2693(77)90435-X

[9] T. Yanagida, "Horizontal gauge symmetry and masses of neutrinos", Conf. Proc. C **7902131** (1979) 95.

[10] S. L. Glashow, "The Future of Elementary Particle Physics", in Quarks And Leptons. Proceedings, Summer Institute, Cargese, France, 9 - 29 July 1979, M. Levy, J.L. Basdevant, D. Speiser, J. Weyers, R. Gastmans and M. Jacob eds., pp. 687- 713 [NATO Sci. Ser. **B 61** (1980) 687] doi:10.1007/978-1-4684-7197-7_15

[11] M. Gell-Mann, P. Ramond and R. Slansky, "Complex Spinors and Unified Theories", Conf. Proc. C **790927** (1979) 315 [arXiv:1306.4669 [hep-th]].

[12] R. N. Mohapatra and G. Senjanovic, "Neutrino Mass and Spontaneous Parity Nonconservation", Phys. Rev. Lett. **44** (1980) 912. doi:10.1103/PhysRevLett.44.912

[13] R. N. Mohapatra and A. Y. Smirnov, "Neutrino Mass and New Physics," Ann. Rev. Nucl. Part. Sci. **56** (2006), 569-628 doi:10.1146/annurev.nucl.56.080805.140534 [arXiv:hep-ph/0603118 [hep-ph]].

[14] G. C. Branco, W. Grimus and L. Lavoura, "Relating the scalar flavor changing neutral couplings to the CKM matrix," Phys. Lett. B **380** (1996), 119-126 doi:10.1016/0370-2693(96)00494-7 [arXiv:hep-ph/9601383 [hep-ph]].

[15] F. J. Botella, G. C. Branco and M. N. Rebelo, "Minimal Flavour Violation and Multi-Higgs Models," Phys. Lett. B **687** (2010), 194-200 doi:10.1016/j.physletb.2010.03.014 [arXiv:0911.1753 [hep-ph]].

[16] G. 't Hooft, "Symmetry Breaking Through Bell-Jackiw Anomalies," Phys. Rev. Lett. **37** (1976), 8-11 doi:10.1103/PhysRevLett.37.8

[17] G. 't Hooft, "Computation of the Quantum Effects Due to a Four-Dimensional Pseudoparticle," Phys. Rev. D **14** (1976), 3432-3450 [erratum: Phys. Rev. D **18** (1978), 2199] doi:10.1103/PhysRevD.14.3432

[18] A. E. Nelson, "Naturally Weak CP Violation," Phys. Lett. B **136** (1984), 387-391 doi:10.1016/0370-2693(84)92025-2

[19] A. E. Nelson, "Calculation of θ Barr," Phys. Lett. B **143** (1984), 165-170 doi:10.1016/0370-2693(84)90827-X

[20] S. M. Barr, "Solving the Strong CP Problem Without the Peccei-Quinn Symmetry," Phys. Rev. Lett. **53** (1984), 329 doi:10.1103/PhysRevLett.53.329

[21] L. Bento, G. C. Branco and P. A. Parada, "A Minimal model with natural suppression of strong CP violation," Phys. Lett. B **267** (1991), 95-99 doi:10.1016/0370-2693(91)90530-4

[22] G. C. Branco, J. M. Gerard and W. Grimus, "Geometrical T Violation," Phys. Lett. B **136** (1984), 383-386 doi:10.1016/0370-2693(84)92024-0

[23] F. J. Botella, G. C. Branco, M. Nebot and M. N. Rebelo, "New physics and evidence for a complex CKM," Nucl. Phys. B **725** (2005), 155-172 doi:10.1016/j.nuclphysb.2005.07.006 [arXiv:hep-ph/0502133 [hep-ph]].

[24] M. Bona *et al.* [UTfit], "The UTfit collaboration report on the status of the unitarity triangle beyond the standard model. I. Model-independent analysis and minimal flavor violation," JHEP **03** (2006), 080 doi:10.1088/1126-6708/2006/03/080 [arXiv:hep-ph/0509219 [hep-ph]].

[25] G. C. Branco, P. A. Parada and M. N. Rebelo, "A Common origin for all CP violations," [arXiv:hep-ph/0307119 [hep-ph]].

[26] M. Fukugita and T. Yanagida, "Baryogenesis Without Grand Unification," Phys. Lett. B **174** (1986), 45-47 doi:10.1016/0370-2693(86)91126-3

[27] F. J. Botella, G. C. Branco and M. Nebot, "The Hunt for New Physics in the Flavour Sector with up vector-like quarks," JHEP **12** (2012), 040 doi:10.1007/JHEP12(2012)040 [arXiv:1207.4440 [hep-ph]].

[28] A. Crivellin, M. Kirk, T. Kitahara and F. Mescia, "Large t→cZ as a sign of vectorlike quarks in light of the W mass," Phys. Rev. D **106** (2022) no.3, L031704 doi:10.1103/PhysRevD.106.L031704 [arXiv:2204.05962 [hep-ph]].

[29] P. Ramond, "Fermions in the Desert," Contribution to: Europhysics Study Conference: Unification of Fundamental Interactions II, 6-14 October 1981 Erice, Italy UFTP-81-27

[30] D. Choudhury, T. M. P. Tait and C. E. M. Wagner, "Beautiful mirrors and precision electroweak data," Phys. Rev. D **65** (2002), 053002 doi:10.1103/PhysRevD.65.053002 [arXiv:hep-ph/0109097 [hep-ph]].

[31] R. Dermisek, "Unification of gauge couplings in the standard model with extra vector-like families," Phys. Rev. D **87** (2013) no.5, 055008 doi:10.1103/PhysRevD.87.055008 [arXiv:1212.3035 [hep-ph]].

[32] C. Y. Seng, M. Gorchtein, H. H. Patel and M. J. Ramsey-Musolf, "Reduced Hadronic Uncertainty in the Determination of V_{ud}," Phys. Rev. Lett. **121** (2018) no.24, 241804 doi:10.1103/PhysRevLett.121.241804 [arXiv:1807.10197 [hep-ph]].

[33] C. Y. Seng, M. Gorchtein and M. J. Ramsey-Musolf, "Dispersive evaluation of the inner radiative correction in neutron and nuclear β decay," Phys. Rev. D **100** (2019) no.1, 013001 doi:10.1103/PhysRevD.100.013001 [arXiv:1812.03352 [nucl-th]].

[34] A. Czarnecki, W. J. Marciano and A. Sirlin, "Radiative Corrections to Neutron and Nuclear Beta Decays Revisited," Phys. Rev. D **100** (2019) no.7, 073008 doi:10.1103/PhysRevD.100.073008 [arXiv:1907.06737 [hep-ph]].

[35] C. Y. Seng, X. Feng, M. Gorchtein and L. C. Jin, "Joint lattice QCD–dispersion theory analysis confirms the quark-mixing top-row unitarity deficit," Phys. Rev. D **101** (2020) no.11, 111301 doi:10.1103/PhysRevD.101.111301 [arXiv:2003.11264 [hep-ph]].

[36] L. Hayen, "Standard model $O(\alpha)$ renormalization of g_A and its impact on new physics searches," Phys. Rev. D **103** (2021) no.11, 113001 doi:10.1103/PhysRevD.103.113001 [arXiv:2010.07262 [hep-ph]].

[37] K. Shiells, P. G. Blunden and W. Melnitchouk, "Electroweak axial structure functions and improved extraction of the Vud CKM matrix element," Phys. Rev. D **104** (2021) no.3, 033003 doi:10.1103/PhysRevD.104.033003 [arXiv:2012.01580 [hep-ph]].

[38] Y. Aoki, T. Blum, G. Colangelo, S. Collins, M. Della Morte, P. Dimopoulos, S. Dürr, X. Feng, H. Fukaya and M. Golterman, *et al.* "FLAG Review 2021," [arXiv:2111.09849 [hep-lat]].

[39] B. Belfatto, R. Beradze and Z. Berezhiani, Eur. Phys. J. C **80** (2020) no.2, 149 doi:10.1140/epjc/s10052-020-7691-6 [arXiv:1906.02714 [hep-ph]].

[40] B. Belfatto and Z. Berezhiani, "Are the CKM anomalies induced by vector-like quarks? Limits from flavor changing and Standard Model precision tests," JHEP **10** (2021), 079 doi:10.1007/JHEP10(2021)079 [arXiv:2103.05549 [hep-ph]].

[41] G. C. Branco, J. T. Penedo, P. M. F. Pereira, M. N. Rebelo and J. I. Silva-Marcos, "Addressing the CKM unitarity problem with a vector-like up quark," JHEP **07** (2021), 099 doi:10.1007/JHEP07(2021)099 [arXiv:2103.13409 [hep-ph]].

[42] F. J. Botella, G. C. Branco, M. N. Rebelo, J. I. Silva-Marcos and J. F. Bastos, Eur. Phys. J. C **82** (2022) no.4, 360 doi:10.1140/epjc/s10052-022-10299-9 [arXiv:2111.15401 [hep-ph]].