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Quantum correlations in neutrino oscillations

Massimo Blasone,[a,b,*] **Silvio De Siena**[c] **and Cristina Matrella**[a,b]

[a] *Dipartimento di Fisica, Università di Salerno, Via Giovanni Paolo II, 132 I-84084 Fisciano (SA), Italy.*

[b] *INFN, Sezione di Napoli, Gruppo collegato di Salerno, Italy.*

[c] *Università degli Studi di Salerno, Fisciano, Italy.*

E-mail: mblasone@unisa.it, silvio.desiena@gmail.com, cmatrella@unisa.it

We investigate quantum correlations in the context of neutrino oscillations, with specific reference to Daya-Bay and MINOS experiments. We compute the non-local advantage of quantum coherence – a valuable quantum resource – for the two experiments, within the wave-packet approach. We find that this kind of non-local correlation may persist at long distances, when oscillations are washed out, depending on the value of the mixing angle.

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*Speaker

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1. Introduction

The notion of coherence [1, 2] is among the basic ones in Quantum Mechanics. A state is said to be coherent provided that there are nonzero elements in the non-diagonal positions of its density matrix representation. Coherence can be considered as a resource [3] and thus, much effort has been focused on its quantitative characterization, expressing the level of quantumness of a given system. In this context, the concept of non-local advantage of quantum coherence (NAQC) has been introduced [4]: it characterizes the most valuable, non–local, portion of the quantum coherence, and is therefore object of active investigation.

Recently, these concepts, originally developed in quantum optics and in quantum information, have been applied to particle physics, with special emphasis to neutrinos and Kaons [5–15]: various kinds of quantum correlations have been explored and violation of Bell [16] and Leggett-Garg [17, 18] inequalities has been verified. In particular, neutrinos represent an ideal playground for these investigations due to their very weak interactions and small decoherence effects.

In Ref. [19] we investigated the l_1-norm based NAQC in the context of neutrino oscillations by extending the work of Ming et al. [20] in which several quantum correlations are studied in connection with Daya–Bay and MINOS neutrino experiments. In our analysis we used a wave packet description whereas in Ref. [20] a plane wave approximation has been adopted.

Here, we review the main results of the above work. In addition, we extend our study by using another measure of quantum coherence, namely the relative entropy of coherence.

In Section 2, we briefly recall the notions of quantum coherence and NAQC. In Section 3 we compute NAQC for two flavor neutrino oscillations, with specific reference to Daya-Bay and MINOS experiments. Section 4 is devoted to conclusions and perspectives of future work.

2. Quantum coherence

There are various ways to quantify the coherence of a state. However, there is a set of conditions any proper measure of coherence should satisfy. First of all, given a basis $\{|i\rangle\}$, the set of incoherent states I is the set of quantum states whose density matrices are diagonal with respect to this basis. So, a measure of quantum coherence C must satisfy the following conditions [21]:

1. $\mathrm{C}(\rho) \geq 0$ for all quantum states. $\mathrm{C}(\rho) = 0$ if and only if $\rho \in \mathrm{I}$, i.e. it belongs to the set of incoherent states.

2. Monotonicity under completely positive trace preserving incoherent maps (ICPTP) Φ, i.e. $\mathrm{C}(\rho) \geq \mathrm{C}(\Phi(\rho))$ and monotonicity under selective measurement on average: $\mathrm{C}(\rho) \geq \sum_n p_n \mathrm{C}(\rho_n)$ where $\rho_n = \hat{K}_n \rho \hat{K}_n^\dagger / p_n$ and $p_n = \mathrm{Tr}\left[\hat{K}_n \rho \hat{K}_n^\dagger\right]$ for all $\{\hat{K}_n\}$ with $\sum_n \hat{K}_n^\dagger \hat{K}_n = \mathbb{I}$ and $\hat{K}_n \mathbb{I} \hat{K}_n^\dagger \subset \mathbb{I}$.

3. Convexity: $\sum_n p_n \mathrm{C}(\rho_n) \geq \mathrm{C}(\sum_n p_n \rho_n)$ for any set of states $\{\rho_n\}$ and any $p_n \geq 0$ with $\sum_n p_n = 1$.

If ρ is a state in the reference basis $\{|i\rangle\}$, a typical measure of coherence takes the form:

$$C_D(\rho) = \min_{\delta \in \mathrm{I}} D(\rho, \delta), \tag{1}$$

i.e. the minimum distance between ρ and the set of incoherent states. $D(\rho,\delta)$ represents any distance measure between two quantum states. One can consider $D(\rho,\delta) = ||\rho - \delta||$, with $||.||$ any matrix norm – in particular the l_1-norm – or $D(\rho,\delta) = S(\rho||\delta)$, that is the quantum relative entropy. The first one can be written as $||\rho - \delta||_{l_1} = \sum_{i,j} |\rho_{i,j} - \delta_{i,j}|$. By minimizing over the set of incoherent states, one can obtain a measure of coherence that satisfies the previous conditions, called l_1*-norm of coherence*:

$$C_{l_1}(\rho) = \sum_{i \neq j} |\langle i|\rho|j\rangle| \tag{2}$$

where the sum is extended to the absolute values of the off diagonal elements.

By considering the second distance measure, we can write $S(\rho||\delta) = S(\rho_{diag}) - S(\rho) + S(\rho_{diag}||\delta)$, where ρ_{diag} is the matrix of the diagonal elements of ρ. Remembering that $S(\rho_{diag}||\delta) = \mathrm{Tr}\big(\rho_{diag} \ln \rho_{diag} - \rho_{diag} \ln \delta\big)$, the minimization procedure leads to $\delta = \rho_{diag}$, so we can define the *relative entropy of coherence* as:

$$C_{re}(\rho) = S(\rho_{diag}) - S(\rho), \tag{3}$$

where $S(\rho)$ is the von Neumann entropy of ρ.

A further important distinction can be done between local and non-local coherence. We can introduce the total coherence $\mathrm{C}(\rho_{AB})$ of a bipartite state with respect to local reference bases $\{|i\rangle_A\}$, $\{|j\rangle_B\}$, and we can interpret $\mathrm{C}(\rho_A)$ and $\mathrm{C}(\rho_B)$ as the local coherences of the subsystem A and B, respectively, where ρ_A and ρ_B are the reduced density matrices.

The interesting thing is that the sum of the local coherences is not necessarily equal to the total coherence [22]. Hence, it is possible that there is a portion of quantum coherence related to the correlations between subsystems, which is called *correlated* or *non-local coherence*:

$$\mathrm{C}_{nl}(\rho_{AB}) \equiv \mathrm{C}(\rho_{AB}) - \mathrm{C}(\rho_A) - \mathrm{C}(\rho_B). \tag{4}$$

Non-local coherence arises if in a system made by two subsystems A and B, A(B) cannot be considered to have a own individuality that is separated by B(A). It has a crucial rôle in the definition of a resource suitable for quantum protocols.

2.1 Non-local Advantage of Quantum Coherence

Mondal et al. [4] refers to Non-Local Advantage of Quantum Coherence (NAQC) as an interesting effect: it occurs in a bipartite system when the average coherence of the conditional state of a subsystem B – after a local measurements on subsystem A – exceeds the coherence limit of the single subsystem measured on mutually unbiased bases. In Refs. [20, 23] it is shown as NAQC captures a kind of quantum correlation which is stronger than Bell non-locality. Therefore, in the already known hierarchy manifested by the inclusion relations Discord ⊃ Entanglement ⊃ Steering ⊃ Bell non-locality [24], a further step towards understanding the quantum world has been taken, by including the NAQC as a quantum correlation stronger than the others – see Fig. 1.

Mondal et al. formulated more than one definition of NAQC. In Ref. [19] we considered in particular the one based on the l_1-norm of coherence. If $C_i^{l_1}$ $(i = x, y, z)$ is the coherence in i-basis, it is proven that for a general state of a single system the upper bound of $C^{l_1} = C_x^{l_1} + C_y^{l_1} + C_z^{l_1}$ is

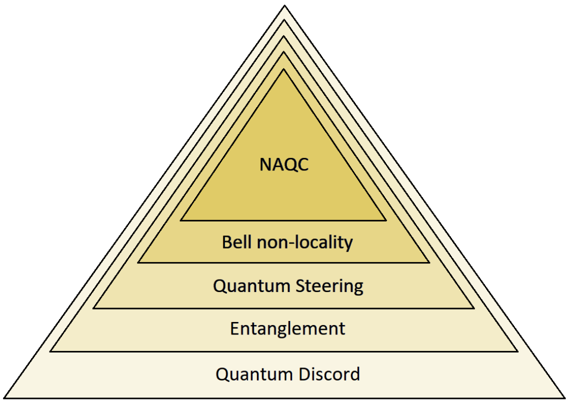

Figure 1: Hierarchy of quantum correlations (Figure adapted from Ref.[24]).

state independent and is given by:

$$\sum_{i=x,y,z} C_i^{l_1}(\rho) \leq \sqrt{6} \tag{5}$$

where the equality sign holds for a pure state. Therefore, Eq. (5) can be considered as a coherence complementarity relation implying that C^{l_1} has to be lower than $\sqrt{6}$ for a single system description. A state acquires a NAQC if this inequality is violated.

Let us consider [4] a bipartite system shared by Alice and Bob, described by the following decomposition:

$$\rho_{AB} = \frac{1}{4}(\mathrm{I}_4 + \vec{r}\cdot\vec{\sigma}\otimes\mathrm{I}_2 + \mathrm{I}_2\otimes\vec{s}\cdot\vec{\sigma} + \sum_{i,j} t_{ij}\sigma_i\otimes\sigma_j), \tag{6}$$

where $\vec{r} \equiv (r_x, r_y, r_z)$, $\vec{s} \equiv (s_x, s_y, s_z)$ and t_{ij} are the correlation matrix elements. The decomposition coefficients can be found as: $r_i = \mathrm{Tr}[\rho_{AB}(\sigma_i\otimes\mathrm{I}_2)]$, $s_i = \mathrm{Tr}[\rho_{AB}(\mathrm{I}_2\otimes\sigma_i)]$ and $t_{ij} = \mathrm{Tr}\big[\rho_{AB}(\sigma_i\otimes\sigma_j)\big]$, $(i, j = x, y, z)$, where σ_i are the Pauli matrices.

Suppose that Alice performs a local measurement on subsystem A in the eigenbasis of Pauli matrix σ_k with outcome $b = 0, 1$. The conditional state of Bob is $\rho_{B|\Pi_k^b} = [(\Pi_k^b\otimes\mathrm{I}_2)\rho_{AB}(\Pi_k^b\otimes\mathrm{I}_2)]/p(\rho_{B|\Pi_k^b})$ with probability $p(\rho_{B|\Pi_k^b}) = \mathrm{Tr}\big[(\Pi_k^b\otimes\mathrm{I}_2)\rho_{AB}\big]$ due to the Alice measurement.

Then, Alice tells Bob her measurement choice and the corresponding result: then he measures the coherence of his side (B) randomly in the eigenbasis of either σ_i or σ_j, $(i, j \neq k)$.

The coherence of the conditional state of Bob, $\rho_{B|\Pi_k^b}$ in the basis of σ_i is given by:

$$\mathrm{C}_i^{l_1}(\rho_{B|\Pi_k^b}) = \sqrt{\frac{\sum_{j\neq i}\alpha_{jk_b}^2}{\gamma_{k_b}^2}}, \tag{7}$$

where $\alpha_{ij_b} = s_i + (-1)^b t_{ji}$, $\gamma_{k_b} = 1 + (-1)^b r_k$, $i, j, k \in \{x, y, z\}$.

Since there are six local measurement settings Alice can choose from, the criterion for achieving NAQC can be obtained via all possible probabilistic averagings:

$$N_{l_1}(\rho_{AB}) = \frac{1}{2}\sum_{i,j,b} p(\rho_{B|\Pi_{j\neq i}^b})C_i^{l_1}(\rho_{B|\Pi_{j\neq i}^b}) > \sqrt{6}, \tag{8}$$

where $p(\rho_{B|\Pi_j^b}) = \frac{\gamma_{j_b}}{2}$.

3. NAQC in neutrino oscillations

Recently, several authors have dealt with the study of the NAQC in various physical contexts [20, 25–27]. In Ref. [19] we have explored the NAQC associated to neutrino oscillations in the wave packet approach, thus extending the work of Ming et al. [20] based on plane waves.

Neutrino oscillations (NOs) is a quantum mechanical phenomenon, suggested for the first time by Bruno Pontecorvo [28], which has found numerous experimental confirmations. For simplicity, we limit ourselves to the case of two-flavor oscillations and to the relativistic approximation of the full field theoretical description [29, 30]. We suppose to have a neutrino of flavor α at the initial time $t = 0$. The time evolution of the state gives us:

$$|\nu_\alpha(t)\rangle = a_{\alpha\alpha}(t)\,|\nu_\alpha\rangle + a_{\alpha\beta}(t)\,|\nu_\beta\rangle\,, \qquad \alpha,\beta = e,\mu. \tag{9}$$

It's convenient to introduce the occupation number states of neutrinos by establishing the following correspondence[1] [5, 6]:

$$\begin{aligned} |\nu_\alpha\rangle &\equiv |1\rangle_\alpha \otimes |0\rangle_\beta \equiv |10\rangle \\ |\nu_\beta\rangle &\equiv |0\rangle_\alpha \otimes |1\rangle_\beta \equiv |01\rangle \end{aligned} \tag{10}$$

Hence, Eq.(9) can be written as:

$$|\nu_\alpha(t)\rangle = a_{\alpha\alpha}(t)\,|10\rangle + a_{\alpha\beta}(t)\,|01\rangle\,. \tag{11}$$

The survival probability to find a neutrino of flavor α after a time t is given by $P_{\alpha\alpha}(t) = |a_{\alpha\alpha}(t)|^2$, and the transition probability is $P_{\alpha\beta}(t) = |a_{\alpha\beta}(t)|^2$, with $P_{\alpha\alpha}(t) + P_{\alpha\beta}(t) = 1$. The survival probability for relativistic neutrinos, in the plane-wave approximation, is given by:

$$P_{\alpha\alpha}(L) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4c\hbar}\frac{L}{E}\right) \tag{12}$$

where θ is the mixing angle, Δm^2 is the mass-squared difference, $L = ct$ is the distance between the production and the detection points after a time t and E is the neutrino energy.

Obviously, the plane wave approach is only an approximation. Since the production and detection processes in neutrino oscillation experiments are localized in space-time, a more realistic description requires a wave packet approach [32, 33]. In Ref. [34] the conditions under which a plane wave approximation is sufficient to adequately describe the phenomenon of neutrino oscillations are studied.

By using a wave packet approach, Eq. (9) becomes:

$$|\nu_\alpha(x,t)\rangle = \sum_j U^*_{\alpha j}\psi_j(x,t)\,|\nu_j\rangle\,, \tag{13}$$

where $U_{\alpha j}$ are the PMNS mixing matrix elements and $\psi_j(x,t)$ is the wave function of the mass eigenstate $|\nu_j\rangle$ of mass m_j. By assuming a Gaussian distribution for the momentum of the massive

[1] The proper mathematical structure for the representation space of mixed neutrinos is that of tensor product of Hilbert spaces. Indeed, in Quantum Field Theory, the Hilbert spaces associated to fields with different masses are unitarily inequivalent to each other [31].

PoS(DISCRETE2020-2021)003

neutrino ν_j, following Refs. [32, 33], the transition probability, in the wave packet approach, is given by:

$$P_{\nu_\alpha\to\nu_\beta}(L) = \sum_{j,k} U^*_{\alpha j}U_{\alpha k}U^*_{\beta j}U_{\beta k}\exp\left[-2\pi i\frac{L}{L^{osc}_{jk}} - \left(\frac{L}{L^{coh}_{jk}}\right)^2 - 2\pi^2(1-\xi)^2\left(\frac{\sigma_x}{L^{osc}_{jk}}\right)^2\right], \qquad (14)$$

where L^{osc}_{jk} is the oscillation length and L^{coh}_{jk} the coherence length, defined by:

$$L^{osc}_{jk} = \frac{4\pi E}{\Delta m^2_{jk}}, \qquad L^{coh}_{jk} = \frac{4\sqrt{2}E^2}{|\Delta m^2_{jk}|}\sigma_x, \qquad (15)$$

with $\sigma_x^2 = \sigma_x^{P\,2} + \sigma_x^{D\,2}$ and $\xi^2\sigma_x^2 = \xi_P^2\sigma_x^{P\,2} + \xi_D^2\sigma_x^{D\,2}$.

Here, σ_x^P is the spatial width of the wave packet, i.e. the position uncertainty determined by the production process. σ_x^D is the uncertainty of the detection process. E is the neutrino energy and $\Delta m^2_{jk} = m_j^2 - m_k^2$ is the mass-squared difference. ξ_P and ξ_D are dimensionless quantities that depend on the characteristics of the production and detection process, respectively. For simplicity, in our ananlysis, we neglect these corrections, setting $\xi = 0$.

We note that the wave packet description confirms the standard value of the oscillation length. The coherence length is the distance beyond which the interference of the massive neutrinos ν_j and ν_k is suppressed. The last term in the exponential of Eq. (14) implies that the interference of the neutrinos is observable only if the localization of the production and detection processes is smaller than the oscillation length.

From Eq. (11), the neutrino density matrix in the orthonormal basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, is given by:

$$\rho^\alpha_{AB}(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |a_{\alpha\beta}(t)|^2 & a_{\alpha\beta}(t)a^*_{\alpha\alpha}(t) & 0 \\ 0 & a_{\alpha\alpha}(t)a^*_{\alpha\beta}(t) & |a_{\alpha\alpha}(t)|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad (16)$$

3.1 l_1–norm NAQC

From Eq. (16), we evaluate NAQC in NOs, using the prescription showed in Subsection 2.1:

$$N_{l_1}(\rho^\alpha_{AB}(t)) = 2 + 2a_{\alpha\alpha}(t)a_{\alpha\beta}(t). \qquad (17)$$

From this, remembering that $|a_{\alpha\alpha}(t)|^2$ and $|a_{\alpha\beta}(t)|^2$ are nothing more than the survival and transition probabilities, NAQC is expressed in terms of oscillation probabilities as:

$$N_{l_1}(\rho^\alpha_{AB}(t)) = 2 + 2\sqrt{P_{\alpha\alpha}(t)P_{\alpha\beta}(t)}. \qquad (18)$$

In Ref. [19] we studied the different behavior of the NAQC obtained in the plane–wave and wave–packet approaches, referring to two different experiments: the Daya Bay reactor neutrino experiment [35–37] and the Main Injector Neutrino Oscillation Search (MINOS) experiment [38, 39]. The neutrino parameters are $\sin^2 2\theta_{13} = 0.084 \pm 0.005$, $\Delta m^2_{ee} = 2.42^{+0.10}_{-0.11} \times 10^{-3} eV^2$ for Daya-Bay and $\sin^2 2\theta_{23} = 0.95^{+0.035}_{-0.036}$, $\Delta m^2_{32} = 2.32^{+0.12}_{-0.08} \times 10^{-3} eV^2$ for MINOS.

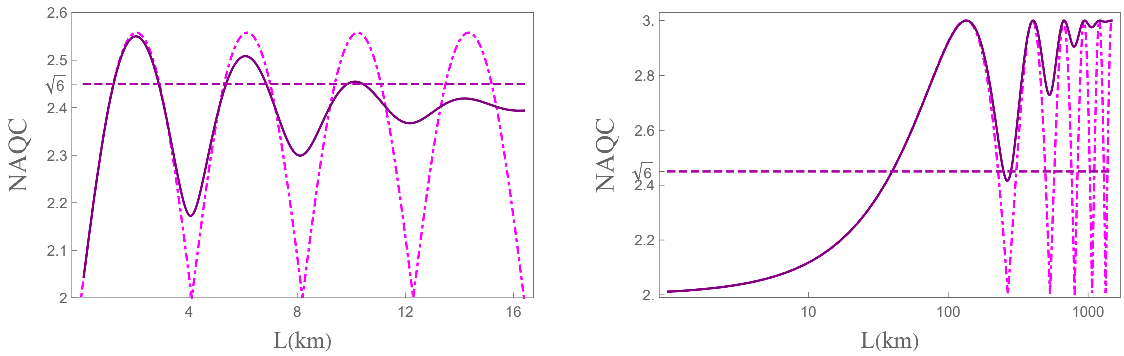

Figure 2: NAQC inequality as a function of the distance using the l_1-norm as a measure of coherence. On the left side the plot is made using the data from Daya Bay experiment with $\sigma_x = 1.25 \times 10^{-6} m$ and $E = 4MeV$. On the right side the plot is made using the data from MINOS experiment with $E = 0.5GeV$ and $\sigma_x = 7 \times 10^{-9} m$. The darker magenta horizontal line is the bound of the NAQC inequality. The solid and dot-dashed lines stand for wave packet and plane wave approach, respectively.

Referring to the Daya Bay experiment, where an electron neutrino is given at the initial time, on the left side of the Fig. 2 it is shown a comparison between the plot of the NAQC inequality obtained with the plane-wave approximation and the one obtained with the wave-packet approach. It is evident the difference at long distances between the two approaches. We note that, in the wave packet case, a non-local advantage of quantum coherence is reached only up to a certain distance, beyond which the plot remains below the threshold value $\sqrt{6}$.

In the MINOS case, we start to consider a muon neutrino at time $t = 0$. We note that there is a large difference between the two approaches: on the right side of Fig. 2 we see that for the wave-packet treatment NAQC is always reached beyond some distance. This doesn't occur in the plane wave treatment.

We also observe that the different long-distance behaviour of NAQC in the two experiments is due to the different values of the mixing angles. Indeed, NAQC equals $\sqrt{6}$ if $\sin^2(2\theta) = 0.106$ corresponding to an angle of about $10°$. Thus, for Daya Bay the value of the mixing angle is below this one, while for MINOS is well above.

3.2 Entropy based NAQC

Let us analyze the NAQC in neutrino oscillations using the relative entropy of coherence as a measure of quantum coherence. In this case, following Ref. [4], it is possible to show that the sum of coherences of a single qubit system is bounded by:

$$\sum_{i=x,y,z} \mathrm{C}_i^E(\rho) \leq 2.23. \tag{19}$$

To derive the criterion to reach a NAQC, we will follow a procedure similar to that described above for the l_1-norm of coherence. The relative entropy of coherence of the conditional state of Bob, after an Alice measurement in Π_k^b is given by:

$$\mathrm{C}_i^E(\rho_{B|\Pi_k^b}) = \sum_{p=+,-} \lambda_{k_b}^p \log_2 \lambda_{k_b}^p - \beta_{ik_a}^p \log_2 \beta_{ik_a}^p, \tag{20}$$

PoS(DISCRETE2020-2021)003

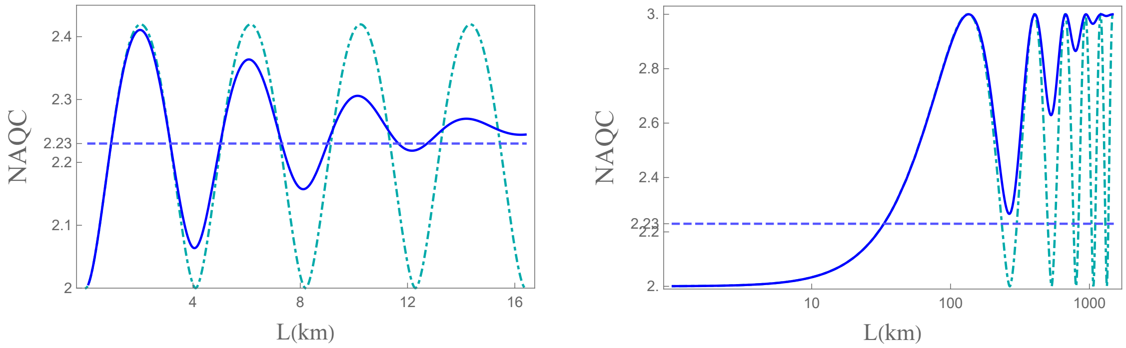

Figure 3: NAQC inequality as a function of the distance using the relative entropy as a measure of coherence. On the left side the plot is made using the data from Daya Bay experiment with $\sigma_x = 1.25 \times 10^{-6} m$ and $E = 4MeV$. On the right side the plot is made using the data from MINOS experiment with $E = 0.5GeV$ and $\sigma_x = 7 \times 10^{-9} m$. The lighter blue horizontal line is the bound of the NAQC inequality. The solid and dot-dashed lines stand for wave packet and plane wave approach, respectively.

where, in terms of the coefficients of Eq.(6), $\lambda^{\pm}_{i_b} = \frac{1}{2} \pm \frac{\sqrt{\sum_j \alpha^2_{ji_b}}}{2\gamma_{i_b}}$ are the eigenvalues of the conditional state of B and $\beta^{\pm}_{ij_b} = \frac{1}{2} \pm \frac{\alpha_{ij_b}}{2\gamma_{j_b}}$ are the diagonal elements of the conditional state $\rho_{B|\Pi^b_j}$ in the σ_i basis.

Hence, the criterion to reach the NAQC is:

$$\frac{1}{2} \sum_{i,j,b} p(\rho_{B|\Pi^b_{j\neq i}}) C^E_i (\rho_{B|\Pi^b_{j\neq i}}) \geq 2.23. \tag{21}$$

Let us see what happens in this case when we consider neutrino oscillations. By starting from the density matrix in Eq. (16), we find that is possible to express the NAQC in terms of oscillation probability as:

$$N_{re}(\rho^{\alpha}_{AB}(t)) = 2 - P_{\alpha\beta}(t) \log_2 P_{\alpha\beta}(t) - P_{\alpha\alpha}(t) \log_2 P_{\alpha\alpha}(t). \tag{22}$$

Again, we do a comparison between plane-wave and wave-packet approaches, using the parameters offered by DAYA-Bay and MINOS experiments.

From Fig. 3, apart from the analogies with l_1-norm treatment, we find that at long distances NAQC is reached also for the DAYA-Bay parameters. Indeed, in both experiments the values of the mixing angles exceed the threshold for which the NAQC saturates at bound 2.23. This corresponds to $\sin^2(2\theta) = 0.075$, i.e. to an angle of 7.93°.

This fact signals that NAQCs based on different coherence quantifiers can display an internal hierarchy. Our results show that in the case of neutrino oscillations the l_1-norm based NAQC is a stronger quantifier than the one based on the relative entropy. It is an interesting question if such a hierarchy has a more general character.

4. Conclusions

The study of quantum coherence as a physical resource is a very active area of research. In this context, the non-local advantage of quantum coherence has emerged as an important concept on which quantum information protocols can be based. We have studied NAQC associated to neutrino oscillations, in the framework of a wave-packet description, for the case of two experiments: Daya Bay reactor neutrino experiment and MINOS. We found that NAQC is achieved for both cases, although it is more persistent (even at long distances) for the case of MINOS experiment. We have considered two definitions of NAQC, one based on l_1-norm of coherence and the other based on the relative entropy of coherence. When adopting the first definition (as already done in Ref. [19]), we found that for long distances NAQC is present only in the MINOS case. On the other hand, using the second definition, the NAQC is non-zero for both experiments, due to the different value of the bound. This indicates the presence of an internal hierarchy between the difference coherence measures based NAQC. In particular the relative entropy based NAQC is a weaker quantifier with respect the l_1-norm based NAQC. On the other hand, these results accurately establish different levels of quantumness in neutrino systems.

The fact that quantum coherence persists beyond the oscillation regime, may appear surprising at a first sight. However, this is only a manifestation of the strong non-local nature of flavor neutrino states. In this respect, in Ref.[6] the static (time-independent) entanglement associated to neutrino mixing was studied, with distinct features with respect to the dynamical (time-dependent) entanglement associated to neutrino oscillations [5].

Directions for future work include both the extension to the case of three flavor oscillations and also the study of matter effects (the celebrated MSW effect [40, 41]), as well as the conceptualization of possible implementations of (long-range) quantum information protocols based on the quantum correlations embodied in neutrino oscillations.

Finally, we plan to extend the study of quantum correlations in particle systems in the framework of quantum field theory. In the case of neutrinos, this will be done by means of the QFT approach developed in Refs. [29, 30] and in line with the results of Refs. [42, 43].

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PoS(DISCRETE2020-2021)003

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