

Overview of neutrino electromagnetic properties (the theory, laboratory experiments and astrophysical probes)

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A brief overview of the electromagnetic properties of neutrinos is presented with a flash on the theory, laboratory constraints and possible consequences for astrophysics.

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1. The theory

It is usually assumed that the entirety of the electromagnetic properties of neutrinos are embodied by the structure of the amplitude corresponding to the Feynman diagramme shown in Fig. 1 that describes the interaction of a neutrino with a real photon. Two incoming and outgoing

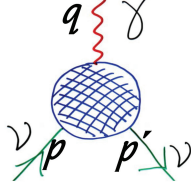


Figure 1: Neutrino effective photon coupling.

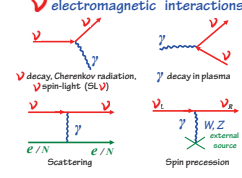


Figure 2: Basic neutrino electromagnetic processes.

lines represent initial and final neutrino states and the third line stands for the real photon. These three lines are connected by an effective vertex, which in general contains the whole set of the neutrino electromagnetic characteristics. The diagramme in Fig. 1 corresponds to the one-photon approximation for the electromagnetic interactions of a neutrino field $\nu(x)$ that can be described by the effective interaction Hamiltonian $H_{em}(x) = j_\mu(x)A^\nu(x) = \bar{\nu}(x)\Lambda_\mu\nu(x)A^\mu(x)$, where $j_\mu(x)$ is the neutrino effective electromagnetic current and Λ_μ is a matrix in the spinor space. Considering the neutrinos as free particles with the Fourier expansion of the free Dirac fields, for the amplitude corresponding to the diagramme in Fig. 1 it is possible to get (see [1] and [2] for the detailed derivations) $\langle \nu(p_f) | j_\mu(0) | \nu(p_i) \rangle = \bar{u}(p_f)\Lambda_\mu(q)u(p_i)$, where $q = p_i - p_f$.

In the most general form the neutrino electromagnetic vertex function $\Lambda_\mu^{ij}(q)$ can be expressed [1] in terms of four form factors

$$\Lambda_\mu^{ij}(q) = (\gamma_\mu - q_\mu \not{q}/q^2) \left[f_Q^{ij}(q^2) + f_A^{ij}(q^2)q^2\gamma_5 \right] - i\sigma_{\mu\nu}q^\nu \left[f_M^{ij}(q^2) + if_E^{ij}(q^2)\gamma_5 \right], \quad (1)$$

where $\Lambda_\mu(q)$ and form factors $f_{Q,A,M,E}(q^2)$ are 3×3 matrices in the space of massive neutrinos. Note that in the derivation of the decomposition (1) the demands followed from the Lorentz-invariance and electromagnetic gauge invariance are taken into account.

In the case of coupling with a real photon ($q^2 = 0$) the form factors $f(q^2)$ provide four sets of neutrino electromagnetic characteristics: 1) the electric millicharges $q_{ij} = f_Q^{ij}(0)$, 2) the dipole magnetic moments $\mu_{ij} = f_M^{ij}(0)$, 3) the dipole electric moments $\epsilon_{ij} = f_E^{ij}(0)$ and 4) the anapole moments $a_{ij} = f_A^{ij}(0)$. The expression (1) for $\Lambda_\mu^{ij}(q)$ is applicable for Dirac and Majorana neutrinos. However, a Majorana neutrino does not have diagonal electric charge and dipole magnetic and electric form factors, only a diagonal anapole form factor can be nonzero. At the same time, a Majorana neutrino can also have nonzero off-diagonal (transition) form factors.

If one considers the case of equal masses for the initial and final neutrinos, $m_i = m_f$, then the decomposition of the neutrino electromagnetic vertex function reduces to

$$\Lambda_\mu^{ii}(q) = f_Q^{ii}(q^2)\gamma_\mu - i\sigma_{\mu\nu}q^\nu \left[f_M^{ii}(q^2) + if_E^{ii}(q^2)\gamma_5 \right] + f_A^{ii}(q^2) \left(\gamma_\mu - q_\mu \not{q}/q^2 \right) \gamma_5.$$

It is also interesting to consider neutrino electromagnetic properties for the case of massless neutrinos of the Standard Model when neutrinos are described by two-component left-handed Weyl

spinors. In this case neutrinos have only one form factor which is equal to the difference of the charge $f_Q(q^2)$ and anapole $f_A(q^2)$ form factors, and the electromagnetic vertex function is given by $\Lambda_\mu(q) = (\gamma_\mu - q_\mu \not{q}/q^2)f(q^2)$, $f(q^2) = f_Q(q^2) - f_A(q^2)$. From these expressions one can expect that at least from the phenomenological point of view it is not possible to treat consequences of neutrino nonzero electric charge $f_Q(q^2)$ and anapole $f_A(q^2)$ form factors separately. This case can approximate the relation between these two form factors that arises for an ultrarelativistic massive neutrinos.

Taking into account the above-mentioned feature of the anapole form factor $f_A(q^2)$ (the ambiguity of its allocation against the charge form factor $f_Q(q^2)$ which manifests itself even in the Standard Model), an alternative decomposition of the electromagnetic vertex function $\Lambda_\mu(q)$ has been proposed in [3]. In [3] the toroidal dipole form factor $f_T(q^2)$ is introduced as a characteristic of the neutrino instead of the anapole form factor $f_A(q^2)$. In this case, the neutrino electromagnetic vertex can be written in the so-called toroidal parametrization

$$\Lambda_\mu^{ij}(q) = f_Q^{ij}(q^2)\gamma_\mu - i\sigma_{\mu\nu}q^\nu \left[f_M^{ij}(q^2) + i f_E^{ij}(q^2)\gamma_5 \right] + i f_T^{ij}(q^2)\epsilon_{\mu\nu\lambda\rho}P^\nu q^\lambda \gamma^\rho,$$

where $P = p_i + p_f$ and $\epsilon_{\mu\nu\lambda\rho}$ is the Levi-Civita unit antisymmetric tensor. The toroidal parametrization of the neutrino vertex has a more clear physical interpretation than the anapole one, because it provides a one-to-one correspondence between the form factors and the multipole moments in expansion of electromagnetic fields. The corresponding toroidal dipole moment $f_T(q^2 = 0)$, for the first time introduced in [4], is more convenient than the anapole moment $f_A(q^2 = 0)$ for the description of T-invariant interactions with nonconservation of the P and C symmetries.

From the identity $\bar{u}(p_f) \left[(m_i - m_f)\sigma_{\mu\nu}g^\mu + (g^2\gamma_\mu - \not{q}q_\mu) - \epsilon_{\mu\nu\lambda\rho}P^\nu q^\lambda \gamma^\rho \gamma_5 \right] \gamma_5 u_i(p_i) = 0$ it follows [3] that in the static limit the toroidal $f_T(q^2 = 0)$ and anapole $f_A(q^2 = 0)$ dipole moments coincide when the masses of the initial and final neutrino states are equal to each other, $m_i = m_f$.

2. Laboratory experiments

The possible electromagnetic characteristics of neutrinos can manifest themselves in astrophysical environments, where neutrinos propagate in strong magnetic fields and dense matter, and also in ground-based laboratory measurements of neutrino fluxes from various sources. The available constraints are discussed in the review paper [1] (see also [5], [6], [7], [8], [9] and [10] for the latest developments and progress in this field).

The most consistent approach to the theoretical description of the electromagnetic properties of neutrinos involves the initial introduction of nonzero electromagnetic characteristics for the mass states of neutrinos. At the same time, since in neutrino scattering experiments flavour neutrinos are registered in detectors, it is necessary to translate electromagnetic properties from the mass into the flavour neutrino basis. Therefore, due to the neutrino mixing and oscillations along the neutrino path from the source to the detector the observed (constrained) neutrino electromagnetic characteristics depends on the neutrino flavour composition in the detector. The recent and most comprehensive study of neutrino electromagnetic properties in the neutrino electron scattering with account for neutrino mixing and oscillations can be found in [11].

Consider the most stringent constraints on the effective neutrino magnetic moments that are obtained with the reactor antineutrinos: $\mu_\nu < 2.9 \times 10^{-11} \mu_B$ (GEMMA Collaboration [12]), and

solar neutrinos: $\mu_\nu \leq 2.8 \times 10^{-11} \mu_B$ (Borexino Collaboration [13]). Here μ_B is the Bohr magneton. Both these constraints are obtained with investigations of the elastic scattering of a flavour neutrino ν_l (or an antineutrino $\bar{\nu}_l$) on an electron at rest: $\nu_l + e^- \rightarrow \nu_l + e^-$, $l = e, \mu, \tau$. There are two contributions, one from the Standard Model weak interaction and another one from the neutrino magnetic moment interaction, to the electron neutrino cross section [14],

$$\frac{d\sigma_{\nu_l e^-}}{dT_e} = \left(\frac{d\sigma_{\nu_l e^-}}{dT_e} \right)_{SM} + \left(\frac{d\sigma_{\nu_l e^-}}{dT_e} \right)_\mu. \quad (2)$$

The weak-interaction cross section is

$$\left(\frac{d\sigma_{\nu_l e^-}}{dT_e} \right)_{SM} = \frac{G_F^2 m_e}{2\pi} \left\{ (g_V^{\nu_l} + g_A^{\nu_l})^2 + (g_V^{\nu_l} - g_A^{\nu_l})^2 \left(1 - \frac{T_e}{E_\nu} \right)^2 + [(g_A^{\nu_l})^2 - (g_V^{\nu_l})^2] \frac{m_e T_e}{E_\nu^2} \right\}, \quad (3)$$

with the standard coupling constants g_V and g_A given by: $g_V^{\nu_e} = 2 \sin^2 \theta_W + 1/2$, $g_A^{\nu_e} = 1/2$, $g_V^{\nu_{\mu,\tau}} = 2 \sin^2 \theta_W - 1/2$, $g_A^{\nu_{\mu,\tau}} = -1/2$. The cross section depends on the initial neutrino energy E_ν and also contains the electron recoil energy T_e . For antineutrinos one must substitute $g_A \rightarrow -g_A$. A neutrino born in the source as the flavour state $|\nu_l\rangle$ arrives to the detector in the flavour state given by

$$|\nu_l(L)\rangle = \sum_{k=1}^3 U_{lk}^* e^{-\frac{m_k^2}{2E_\nu} L} |\nu_k\rangle, \quad (4)$$

then the neutrino magnetic-moment contribution to the cross section is given by

$$\left(\frac{d\sigma_{\nu_l e^-}}{dT_e} \right)_\mu = \frac{\pi \alpha^2}{m_e^2} \left(\frac{1}{T_e} - \frac{1}{E_\nu} \right) \left(\frac{\mu_{\nu_l}}{\mu_B} \right)^2, \quad (5)$$

where the cross section contains the effective magnetic moment μ_{ν_l} [1, 11]

$$\mu_{\nu_l}^2(L, E_\nu) = \sum_j \left| \sum_k U_{lk}^* e^{-i\Delta m_{kj}^2 L/2E_\nu} (\mu_{jk} - i\epsilon_{jk}) \right|^2 \quad (6)$$

that indeed receives equal contributions from the neutrino magnetic and electric dipole moments, both the diagonal and transition, which are given by the static values of the corresponding form factors, $\mu_{jk} = f_M^{jk}(q^2 = 0)$, $\epsilon_{jk} = f_E^{jk}(q^2 = 0)$.

It is just straightforward that scattering experiments based on detection of neutrinos arriving from different distances probe (or constraint) different combinations of the fundamental magnetic moments. Now for simplicity we omit possible contributions from the dipole electric moments. In the reactor short-baseline experiments (for instance, in the GEMMA experiment) one studies (and constrains) the effective magnetic moments in the flavor basis. Whereas in the case of long-baseline experiments (such as the Borexino experiment with the solar neutrinos) a more convenient interpreting the results is based on effective magnetic moments in the fundamental mass basis.

Note that with the inclusion [6] of the effect from possible nonzero neutrino millicharge to the analysis of the GEMMA collaboration data on the antineutrino-electron scattering provides the most severe reactor upper bound on the neutrino millicharge: $q_\nu < 1.5 \times 10^{-12} e_0$. For a more detailed and recent discussion on the main processes of the electromagnetic interaction of neutrinos in astrophysics and the corresponding limitations on millicharges and effective magnetic moments of neutrinos see [10].

3. Astrophysical probes

Neutrinos with nonzero electromagnetic characteristics generate processes that can occur in various astrophysical conditions and be the cause of important phenomena that are fundamentally observable. The most important neutrino electromagnetic processes shown in Fig. 2 are the following: 1) a heavier neutrino decay to a lighter mass state in vacuum, 2) the Cherenkov radiation by a neutrino in matter or an external magnetic field, 3) the spin-light of neutrino in matter, 4) the plasmon decay to a neutrino-antineutrino pair in matter, 5) the neutrino scattering on an electron or a nuclei, and 6) the neutrino spin precession in an external magnetic field or the transversally moving (or the transversally polarized) matter. These processes can be of great interest in astrophysics and astrophysics can really be considered as a laboratory for studying the electromagnetic properties of neutrinos (see [1, 5] and [9, 10]).

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