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Renormalization of the singlet axial current at N^3LO in QCD

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In this talk, we discuss the calculation of the renormalization constant of the flavor-singlet axial-vector current with a non-anticommuting γ_5 in dimensional regularization determined to order α_s^3 in QCD with massless quarks. In addition we briefly comment on the subtlety in the renormalized form of an axial-current component of a given quark flavor, and the closely related renormalization formulae of the singlet contribution to an axial quark form factor, as well as the renormalization group equations.

The European Physical Society conference on high energy physics 2021
(EPS-HEP 2021, Online Conference)

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[†] L. Chen thanks the organisers of EPS-HEP 2021.

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1. Introduction

It is well known that special attention is needed in the treatment of the intrinsically 4-dimensional object γ_5 in dimensional regularization (DR) [1, 2]. At the root of the issue is that a fully anticommuting γ_5 is algebraically incompatible with the Dirac algebra in D ($\neq 4$) dimensions, which on the other hand is essential for the concept of chirality of spinors in 4 dimensions and (non-anomalous) chiral symmetries in quantum field theory. In any regularization scheme where the invariance of loop integrals under arbitrary loop-momentum shifts is ensured, an anticommuting γ_5 would lead to the absence of the axial or Adler-Bell-Jackiw (ABJ) anomaly [3, 4]. Despite these issues, various γ_5 prescriptions in DR have been developed in the literature [1, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19] over a span of nearly 50 years, albeit each with its own pros and cons. This technical point matters when computing perturbative QCD corrections to form factors (FF) of the vertex that couples an external axial current to a pair of quarks (or gluons), which are important ingredients for calculating a number of phenomenologically interesting processes. In this talk, we discuss the calculation of the renormalization constant of the singlet axial current operator with a non-anticommuting γ_5 [1, 5, 6] to the third order in QCD, in the variant as prescribed in refs. [15, 16]. Furthermore, we discuss the subtlety in the renormalized form of a manually separated axial current component of a given quark flavor, and the closely related renormalization formulae of the singlet contribution to an axial quark form factor, as well as renormalization group equations.

2. Renormalization of the singlet axial current

As summarized in refs. [15, 16], the properly renormalized singlet axial current in QCD with n_f massless quarks with a non-anticommuting γ_5 can be written as

$$\begin{aligned} \left[J_5^{\mu}\right]_R &= \sum_{\psi_B} Z_J\, \bar{\psi}^B \gamma^{\mu} \gamma_5\, \psi^B \\ &= \sum_{\psi_B} Z_5^f\, Z_5^{ms}\, \bar{\psi}^B \frac{-i}{3!} \varepsilon^{\mu\nu\rho\sigma} \gamma_\nu \gamma_\rho \gamma_\sigma\, \psi^B\,, \end{aligned} \tag{2.1}$$

where ψ^B denotes a bare quark field with mass dimension $(D-1)/2$ and the subscript R at a square bracket denotes operator renormalization. [1] The sum extends over all n_f quark fields. Here and below J_5^{μ} denotes the bare flavor-singlet axial current. It is known to renormalize multiplicatively [3, 21], as it is the only local composite current operator in the context of QCD that has the correct mass dimension and conserved quantum numbers (which are preserved under renormalization). The factor $Z_J \equiv Z_5^f\, Z_5^{ms}$ denotes the UV renormalization constant of the current, conveniently parameterized as the product of a pure $\overline{\text{MS}}$-renormalization part Z_5^{ms} and an additional finite renormalization factor Z_5^f. The latter is needed to restore the correct form of the axial Ward identity, namely, the all-order axial-anomaly equation [3, 22], which reads in terms of renormalized local

[1] In ref. [20], a factor μ^{4-D} in the mass scale μ of dimensional regularization was introduced in order for the mass dimension of the r.h.s. operator be equal to the canonical dimension of the l.h.s. in four dimensions.

composite operators[2]

$$\left[\partial_\mu J_5^\mu\right]_R = a_s\, n_f\, \mathrm{T}_F \left[F\tilde{F}\right]_R\,, \tag{2.2}$$

where $\mathrm{T}_F = 1/2$, $F\tilde{F} \equiv -\varepsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma} = \varepsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma}$ denotes the contraction of the field strength tensor $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$ of the gluon field A^a_μ with its *dual* form. We use the shorthand notation $a_s \equiv \frac{\alpha_s}{4\pi} = \frac{g_s^2}{16\pi^2}$ for the QCD coupling, and f^{abc} denotes the structure constants of the non-Abelian color group of QCD. In contrast to the l.h.s. of (2.2), the renormalization of the axial-anomaly operator $F\tilde{F}$ is not strictly multiplicative (as known from ref. [3]), but involves mixing with the divergence of the axial current operator [11, 23],

$$\left[F\tilde{F}\right]_R = Z_{F\tilde{F}} \left[F\tilde{F}\right]_B + Z_{FJ} \left[\partial_\mu J_5^\mu\right]_B\,, \tag{2.3}$$

where the subscript B implies that the fields in the local composite operators are bare.

3. Deriving the result for Z_J

We choose to determine the renormalization constant $Z_J \equiv Z_5^f Z_5^{ms}$ of the singlet axial current to $\mathcal{O}(a_s^3)$ in DR by computing matrix elements of operators appearing in the axial-anomaly equation between the vacuum and a pair of off-shell gluons evaluated at a specifically chosen single-scale kinematic configuration [24, 16, 20]. Let us denote by $\langle 0|\hat{\mathrm{T}}\left[J_5^\mu(y) A_a^{\mu_1}(x_1) A_a^{\mu_2}(x_2)\right]|0\rangle|_{\mathrm{amp}}$ the amputated one-particle irreducible (1PI) vacuum expectation value of the time-ordered covariant product of the (singlet) axial current and two gluon fields in coordinate space with open Lorentz indices. Subsequently, we introduce the following rank-3 matrix element $\Gamma_{lhs}^{\mu\mu_1\mu_2}(p_1,p_2)$ in momentum space, defined by

$$\Gamma_{lhs}^{\mu\mu_1\mu_2}(p_1,p_2) \equiv \int d^4x\, d^4y\, e^{-ip_1\cdot x - iq\cdot y}\, \langle 0|\hat{\mathrm{T}}\left[J_5^\mu(y) A_a^{\mu_1}(x) A_a^{\mu_2}(0)\right]|0\rangle|_{\mathrm{amp}} \tag{3.1}$$

where $p_2 = -q - p_1$. Rather than performing the projection for the axial anomaly literally as devised in eq. (19) of ref. [16] where a derivative w.r.t the momentum q is taken before going to the limit $q_\mu \to 0$ (see also ref. [24]), we use instead the following projector

$$\mathcal{P}_{\mu\mu_1\mu_2} = -\frac{1}{6\, p_1\cdot p_1}\, \varepsilon_{\mu\mu_1\mu_2\nu}\, p_1^\nu\,, \tag{3.2}$$

and directly project $\Gamma^{\mu\mu_1\mu_2}(p_1,-p_1)$ onto this structure right at $q=0$ (i.e. $p_2 = -p_1$). This leads to the mass-dimensionless matrix element

$$\mathcal{M}_{lhs} = \mathcal{P}_{\mu\mu_1\mu_2}\, \Gamma_{lhs}^{\mu\mu_1\mu_2}(p_1,-p_1)\,. \tag{3.3}$$

Although, at first sight, the projector $\mathcal{P}_{\mu\mu_1\mu_2}$ in eq.(3.2) does not seem to have anything to do with the divergence or anomaly of the axial current, it can be shown that with the appropriate regularity condition it is indeed equivalent to the operation devised for projecting out the anomaly

[2] When inserted into a Green's function, the time component of the derivative generates *contact* terms in the respective Ward identity.

in eq.(19) of ref. [16]; see the Appendix of ref. [20]. Another useful way to appreciate the connection between this projector and the anomaly of the axial current is by examining the form factor decomposition of the matrix element $\Gamma^{\mu\mu_1\mu_2}_{lhs}(p_1,p_2)$ in eq.(3.1). It involves the following 3 tensor structures [25],

$$\begin{aligned}\Gamma^{\mu\mu_1\mu_2}_{lhs}(p_1,p_2) &= F_1\,\varepsilon^{\mu\,\mu_1\,\mu_2\,(p_2-p_1)}\\ &+ F_2\left(p_1^{\mu_1}\varepsilon^{\mu\,\mu_2\,p_1\,p_2} - p_2^{\mu_2}\varepsilon^{\mu\,\mu_1\,p_1\,p_2}\right)\\ &+ F_3\left(p_1^{\mu_2}\varepsilon^{\mu\,\mu_1\,p_1\,p_2} - p_2^{\mu_1}\varepsilon^{\mu\,\mu_2\,p_1\,p_2}\right)\end{aligned} \tag{3.4}$$

each of which is respectively parity-odd and Bose-symmetric w.r.t the two external gluons. [3] The form factors $F_{1,2,3}$ are functions of the Lorentz invariants made out of p_1, p_2. With this parameterization, the contraction of this tensor with the sum of p_1 and p_2 receives contribution from just F_1:

$$-q_\mu\Gamma^{\mu\mu_1\mu_2}_{lhs}(p_1,p_2) = 2F_1\,\varepsilon^{\mu_1\,\mu_2\,p_1\,p_2}\,,$$

which corresponds to the divergence of the axial current. With on-shell p_1 and p_2, the F_1 is known to vanish at $q^2 = 0$ in an abelian theory with all fermions massive, due to a low-energy theorem [26]. It is also clear from eq.(3.4) that there is only $\varepsilon^{\mu\,\mu_1\,\mu_2\,(p_2-p_1)}$ surviving at the chosen kinematics $q = 0, p_2 = -p_1$. Consequently, the projector $\mathscr{P}_{\mu\mu_1\mu_2}$ projects out the form factor in front of this unique structure, which is not vanishing in the limit $q = 0$ with off-shell p_1 (and p_2). Strictly speaking, the squared norm of the Lorentz structure $\varepsilon_{\mu\mu_1\mu_2\nu}p_1^\nu$ is equal to (6-11D+6D^2-D^3)$p_1\cdot p_1$ with all Lorentz indices of the space-time metric tensors resulting from contracting a pair of Levi-Civita tensors taken to be D-dimensional, but it is known [27, 28] that the parameter D here can be safely set to 4 consistently throughout the computation in DR without problem. The very same projector $\mathscr{P}_{\mu\mu_1\mu_2}$ is also used in extracting a scalar quantity from the matrix element of $\left[F\tilde{F}\right]_R$ between the vacuum and the same external (off-shell) gluon state, denoted as $\mathscr{M}_{rhs}$.

We compute the pertubative QCD corrections to $\mathscr{M}_{lhs}$ and $\mathscr{M}_{rhs}$ in terms of Feynman diagrams, which are manipulated in the usual way. We refer to ref. [20] for details on the technical aspects of the computation. Although the off-shell $\mathscr{M}_{lhs}$ and $\mathscr{M}_{rhs}$ depend on the gauge-fixing parameter ξ, the $Z_J \equiv Z_5^{ms} Z_5^f$ of the gauge-invariant axial-current operator J_5^μ is independent of ξ. The $\overline{\text{MS}}$ part Z_5^{ms} can be extracted based on the UV divergences remaining in the $\mathscr{M}_{lhs}$ after performing the renormalization of the external (off-shell) gluon fields and the QCD coupling a_s, as well as the renormalization of the covariant-gauge fixing parameter ξ. The finite renormalization constant Z_5^f is determined by demanding the equality between the fully renormalized $\mathscr{M}_{lhs}$ and $\mathscr{M}_{rhs}$, originating from eq. (2.2). It reads [20]:

$$Z_5^f = 1 + a_s\Big\{-4C_F\Big\} + a_s^2\Big\{C_A C_F\left(-\frac{107}{9}\right) + C_F^2\left(22\right) + C_F n_f\left(\frac{31}{18}\right)\Big\}$$

[3] There are 6 linearly independent rank-3 Lorentz tensor structures that one can compose in 4 dimensions out of the two momenta p_1, p_2 and one Levi-Civita tensor. They can be reduced to the three included in eq.(3.4) upon imposing Bose symmetry and parity odd conditions. We note further that the external gauge bosons are not required to be transversal in this decomposition.

$$+a_s^3\Big\{C_A^2 C_F\Big(56\zeta_3-\frac{2147}{27}\Big)+C_A C_F^2\Big(\frac{5834}{27}-160\zeta_3\Big)+C_A C_F n_f\Big(\frac{110}{3}\zeta_3-\frac{133}{81}\Big)$$
$$+C_F^3\Big(96\zeta_3-\frac{370}{3}\Big)+C_F^2 n_f\Big(\frac{497}{54}-\frac{104}{3}\zeta_3\Big)+C_F n_f^2\Big(\frac{316}{81}\Big)\Big\}. \tag{3.5}$$

We note that the explicit perturbative result in eq.(3.5) is given in terms of the usual $\overline{\text{MS}}$-renormalized QCD coupling a_s with n_f quark flavors. The first two orders of eq. (3.5) agree with ref. [16], while the third order terms are our new result.

4. Renormalization of $J^{\mu}_{5,q}$

We note that the difference between Z_5^f in eq. (3.5) and the additional finite renormalization constant for the non-singlet axial current computed to $\mathcal{O}(a_s^3)$ in ref. [15] starts at $\mathcal{O}(a_s^2)$, and is proportional to $n_f C_F$ just like their $\overline{\text{MS}}$ counterparts. There is actually a quite interesting point related to this, which we elaborate below, given the common practice of splitting the QCD corrections to the axial part of the quark form factors into the so-called non-singlet and singlet contribution (see, e.g., [29, 30, 31].) It starts with the following question: if there are n_f flavors of quarks active in the QCD Lagrangian, what should be the renormalized form of a manually separated axial current component with one particular quark flavor. Namely, instead of the complete singlet axial current in eq.(2.1), we consider the renormalization of an individual component $\bar{\psi}_q\,\gamma^\mu\,\gamma_5\,\psi_q$ with the subscript q denoting the quark flavor in QCD with n_f quarks active. Apparently, by definition, one should have

$$\sum_{q=1}^{n_f}\big[J^{\mu}_{5,q}\big]_R=\big[J^{\mu}_{5}\big]_R \tag{4.1}$$

with $\big[J^{\mu}_{5}\big]_R$ the renormalized complete singlet axial current in eq.(2.1) and $\big[J^{\mu}_{5,q}\big]_R$ denoting an individual component of a given quark flavor q. The point is that it would be incorrect to make the naive identification of $\big[J^{\mu}_{5,q}\big]_R$ as $Z_J\,\bar{\psi}^B_q\,\gamma^\mu\,\gamma_5\,\psi^B_q$, the later of which is actually not UV finite, although the condition (4.1) would be trivially fulfilled. Indeed, treated as an external local-composite operator, all contributing Feynman diagrams to the matrix elements of the bare $J^{\mu}_{5,q}=\bar{\psi}^B_q\,\gamma^\mu\,\gamma_5\,\psi^B_q$ must have the Z boson directly coupled to this particular current made out of field q; while on the other hand, one can have more than one quark flavors active in the QCD Lagrangian which generates all standard QCD-loop corrections to matrix elements. The correct renormalized form should read [32]

$$\begin{aligned}\big[J^{\mu}_{5,q}\big]_R &= Z_{ns}\,\bar{\psi}^B_q\,\gamma^\mu\,\gamma_5\,\psi^B_q+Z_s\sum_{i=1}^{n_f}\bar{\psi}^B_i\,\gamma^\mu\,\gamma_5\,\psi^B_i\\ &=\big(Z_{ns}+Z_s\big)\,\bar{\psi}^B_q\,\gamma^\mu\,\gamma_5\,\psi^B_q+Z_s\sum_{i=1,i\neq q}^{n_f}\bar{\psi}^B_i\,\gamma^\mu\,\gamma_5\,\psi^B_i\,,\end{aligned} \tag{4.2}$$

with $Z_s\equiv\frac{1}{n_f}\big(Z_J-Z_{ns}\big)$ and Z_{ns} denoting the full renormalization constant for the non-singlet axial current operator (i.e., including the non-$\overline{\text{MS}}$ finite piece). Owing to the aforementioned feature regarding the difference between the non-singlet and singlet axial-current renormalization constants, the so-defined Z_s starts from $\mathcal{O}(a_s^2)$ and is free of n_f in the denominator. It is interesting to observe that in the explicit renormalized form $\big[J^{\mu}_{5,q}\big]_R$ of an axial-current component with one

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particular quark flavor, there appears some mixing terms made out of the fields of the remaining quark flavors. It is straightforward to see that the condition (4.1) is satisfied by the formula (4.2). Consequently, one can also derive the RG equation for $\left[J^{\mu}_{5,q}\right]_R$ based on eq.(4.2), which reads

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \left[J^{\mu}_{5,q}\right]_R = \gamma_s \left[J^{\mu}_{5}\right]_R , \tag{4.3}$$

where γ_s is defined by $\mu^2 \frac{\mathrm{d}Z_s}{\mathrm{d}\mu^2} = \gamma_s \big(Z_{ns} + n_f\, Z_s\big)$. We note that it is the renormalized complete axial current $\left[J^{\mu}_{5}\right]_R$ that appears in the r.h.s. of eq.(4.3).

5. Renormalization of the singlet contribution to quark FFs

The 3-loop singlet contribution to the axial part of the quark FF in purely massless QCD was determined recently in ref. [30] where the result (3.5) entered. However, for physical application of the result for the axial quark FF, such as for theoretical predictions of the Z-mediated Drell-Yan processes to the third order in a_s (similar as for those mediated by a virtual photon [33, 34] or a W boson [35]), one should incorporate the singlet QCD contribution with top quark loops [36, 37, 38, 39, 40] due to the presence of the axial-anomaly type diagrams [3, 4]. Here we would like to briefly comment on why this is necessary from the point of view of renormalization scale dependence [37, 38]. We refer to ref. [31] for the detail of the definition of the so-called singlet-type QCD contribution to the (massless) quark FFs, in particular the axial part, considered in the remainder of this proceeding.

As far as the following discussion is concerned, it is sufficient to know that the renormalized axial quark FF can be formally viewed as the projection of the matrix element of the current operator $\left[J^{\mu}_{5,q}\right]_R$ between an external pair of on-shell (massless) quarks $|q\bar{q}\rangle$ and the vacuum onto a specifically constructed form-factor projector which we denote as $\hat{\mathscr{P}}$. Based on eq.(4.2), one can directly read off the renormalized total axial quark FF:

$$\begin{aligned}
\hat{\mathscr{P}}\, \langle 0| \left[J^{\mu}_{5,q}\right]_R |q\bar{q}\rangle \,&=\, Z_{ns}\, Z_2 \left(F^A_{ns}(\hat{a}_s, \hat{m}_t) + F^A_{s,q}(\hat{a}_s, \hat{m}_t)\right) \,+\, Z_s\, Z_2 \left(F^A_{ns}(\hat{a}_s, \hat{m}_t) + \sum_{i=1}^{n_f} F^A_{s,i}(\hat{a}_s, \hat{m}_t)\right), \\
&=\, \big(Z_{ns} + Z_s\big) Z_2 \left(F^A_{ns}(\hat{a}_s, \hat{m}_t) + F^A_{s,q}(\hat{a}_s, \hat{m}_t)\right) \,+\, Z_s\, Z_2 \Big(\sum_{i=1, i\neq q}^{n_f} F^A_{s,i}(\hat{a}_s, \hat{m}_t)\Big) \\
&=\, Z_{ns}\, Z_2\, F^A_{ns}(\hat{a}_s, \hat{m}_t) \,+\, \mathbf{F}^A_{s,q}(a_s, m_t, \mu)\,,
\end{aligned} \tag{5.1}$$

where we have adopted the notations used in ref. [31]. In short, $F^A_{ns}(\hat{a}_s, \hat{m}_t)$ denotes the bare non-singlet QCD corrections where the Z boson coupled directly to the open fermion line of the external quark q. While the bare singlet contribution $F^A_{s,i}(\hat{a}_s, \hat{m}_t)$ features a closed fermion loop which contains the quark i coupling to the Z boson. The bare QCD coupling $\hat{a}_s$ is to be renormalized conventionally in the $\overline{\mathrm{MS}}$ scheme and the bare mass $\hat{m}_t$ is to be renormalized on-shell by $\hat{m}_t = Z_m\, m_t$. The Z_2 is the on-shell wavefunction renormalization constant of the external light quark, which differs from one due to the presence of massive top loops starting from 2-loop order. In the last line of eq.(5.1), we have introduced $\mathbf{F}^A_{s,q}(a_s, m_t, \mu)$ denoting the renormalized a_q-tagged singlet contribution after subtracting the renormalized purely non-singlet part $Z_{ns}\, Z_2\, F^A_{ns}(\hat{a}_s, \hat{m}_t)$ from the total. Specifically, one arrives at the following renormalization formulae for this individual a_q-tagged singlet contribution [31]:

$$\mathbf{F}^A_{s,q}(a_s,m_t,\mu) = Z_{ns} Z_2 F^A_{s,q}(\hat{a}_s,\hat{m}_t) + Z_s Z_2 \Big(F^A_{ns}(\hat{a}_s,\hat{m}_t) + \sum_{i=1}^{n_f} F^A_{s,i}(\hat{a}_s,\hat{m}_t) \Big) . \tag{5.2}$$

Once expanded up to $\mathcal{O}(a_s^3)$ in question, only

$$Z_{ns} Z_2 F^A_{s,q}(\hat{a}_s,\hat{m}_t) + Z_s Z_2 F^A_{ns}(\hat{a}_s,\hat{m}_t)$$

contribute, because the singlet quantities, Z_s and $F^A_{s,i}$, all start from $\mathcal{O}(a_s^2)$. The remaining terms in eq.(5.2), which are quite non-trivial due to mixing with singlet diagrams featuring quarks of other flavors (with potentially different masses), should get involved but only starting from 4-loop order. Furthermore, the difference between the two subsets of singlet contributions in eq.(5.2), say the a_b-tagged and a_t-tagged ones from the third quark generation, requires only the non-singlet axial current renormalization:

$$\mathbf{F}^A_{s,b}(a_s,m_t) - \mathbf{F}^A_{s,t}(a_s,m_t) = Z_{ns} Z_2 \left(F^A_{s,b}(\hat{a}_s,\hat{m}_t) - F^A_{s,t}(\hat{a}_s,\hat{m}_t) \right) , \tag{5.3}$$

as expected. Starting with eq.(5.2) and noting the non-zero anomalous dimension of the singlet axial-current operator in eq.(4.3), one can then derive the RG equation for the renormalized a_q-dependent singlet contribution, which reads

$$\begin{aligned} \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \mathbf{F}^A_{s,q}(a_s,m_t,\mu) &= \mu^2 \frac{\partial}{\partial \mu^2} \mathbf{F}^A_{s,q}(a_s,m_t,\mu) + (\beta - \varepsilon) a_s \frac{\partial}{\partial a_s} \mathbf{F}^A_{s,q}(a_s,m_t,\mu) \\ &= \gamma_s \Big(\mathbf{F}^A_{ns}(a_s,m_t,\mu) + \sum_{i=1}^{n_f} \mathbf{F}^A_{s,i}(a_s,m_t,\mu) \Big) , \end{aligned} \tag{5.4}$$

where all FFs on both sides are the UV renormalized ones. Just like the pure non-singlet contribution $\mathbf{F}^A_{ns}(a_s,m_t,\mu)$, one sees that the "physical" combination $\mathbf{F}^A_{s,b}(a_s,m_t,\mu) - \mathbf{F}^A_{s,t}(a_s,m_t,\mu)$ has a zero anomalous dimension as a direct consequence of eq.(5.3), which is also clear from eq.(5.4). Therefore, the net μ dependence in the a_t-dependent singlet contribution $\mathbf{F}^A_{s,t}(a_s,m_t,\mu)$ is necessary to cancel that of $\mathbf{F}^A_{s,b}(a_s,m_t,\mu)$, such that the remaining explicit μ dependence is related to the $\overline{\text{MS}}$ renormalization of a_s in the usual way. Based on this, one anticipates already that the top-quark contribution $\mathbf{F}^A_{s,t}(a_s,m_t,\mu)$ can not completely decouple in the naive sense in the large top mass or low energy limit, because the "massless" contribution $\mathbf{F}^A_{s,b}(a_s,m_t,\mu)$ still has a non-zero anomalous dimension to be compensated [36, 37, 38, 39, 40].

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