PROCEEDINGS OF SCIENCE

Lattice Developments and Tools

Huey-Wen Lin [a,b,*]

[a] *Department of Physics and Astronomy, Michigan State University, East Lansing, MI, 48824, U.S.A*

[b] *Department of Computational Mathematics, Science and Engineering, Michigan State University, East Lansing, MI, 48824, U.S.A*

E-mail: hwlin@pa.msu.edu

Lattice QCD is a natural theoretical tool to study the structure of hadrons with quarks and gluons as the fundamental degrees of freedom. In this proceeding, we introduce lattice QCD and show examples of commonly used software tools from USQCD (a US lattice-QCD collaboration). We highlight a few recent lattice developments that used these shared software tools, focusing on nucleon parton distributions that are most relevant to LHC physics.
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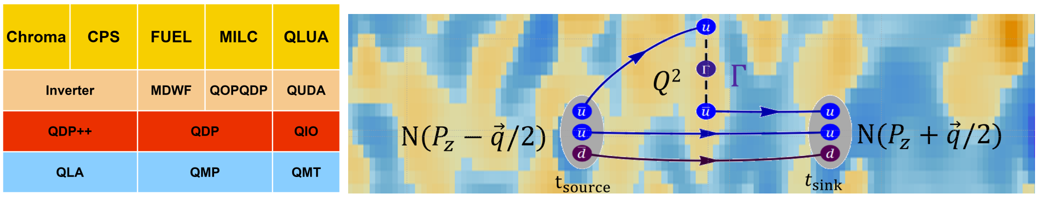

Figure 1: (left) The SciDAC Layers showing the modular software architecture; taken from USQCD software website, `https://usqcd-software.github.io` (right) Illustration of an example matrix-element calculation for a generalized parton distribution, depicted on top of the lattice QCD vacuum.

Parton distribution functions (PDFs) provide a universal description of hadronic constituents as well as critical inputs for the discovery of the Higgs boson found at the Large Hadron Collider (LHC) through proton-proton collisions. While the world waits for the next phase of LHC discovery, focused on searching for new-physics signatures, improvements in the precision with which we know Standard-Model backgrounds will be crucial to discern these signals. For example, our knowledge of many Higgs-production cross sections remains dominated by PDF uncertainties. Among the known PDFs, the strange and charm PDFs have particularly large uncertainty despite decades of experimental effort. In global PDF analyses, the assumptions $\bar{s}(x) = s(x)$ and $\bar{c}(x) = c(x)$ are often made and can agree with the data merely due to the large uncertainty. At the LHC, strangeness can be extracted through the $W + c$ associated-production channel, but their results are rather puzzling. Future high-luminosity studies may help to improve our knowledge of the strangeness, and finalize the size of the "intrinsic" charm contribution, as first raised in 1980 [1].

Lattice QCD is an ideal theoretical tool to study the parton structure of hadrons, starting from quark and gluon degrees of freedom. Lattice QCD discretizes four-dimensional continuum QCD to allow the study of the strong-coupling regime of QCD, where perturbative approaches converge poorly. As in continuum QCD, we calculate an observable of interest through a path integral:

$$\langle 0|O(\bar{\psi}, \psi, A)|0\rangle = \frac{1}{Z} \int [dA][d\bar{\psi}][d\psi] O(\bar{\psi}, \psi, A) e^{i \int dx^4 \mathcal{L}_{\text{QCD}}(\bar{\psi}, \psi, A)}, \tag{1}$$

where $\mathcal{L}_{\text{QCD}}$ is the sum of the pure-gauge and fermion Lagrangian, O is the operator that gives the correct quantum numbers for our observable, and Z is the partition function of the space-time integral of the QCD Lagrangian. It is straightforward to carry out this path integral numerically within a finite space-time volume and under an ultraviolet cutoff (the lattice spacing a). For observables that have a well-defined operator in the Euclidean path integral for numerical integration, we can find their values in continuum QCD by taking the limits lattice spacing $a \to 0$, spatial size $L \to \infty$ and quark mass $m_q \to m_q^{\text{phys}}$. These calculations require high-performance supercomputer centers with software developed by and shared among the community. The left-hand side of Fig. 1 shows an example of commonly used software from US lattice community and their dependencies.

Using precision moments from lattice QCD as inputs can effectively constrain the PDFs. Consider the transversely polarized PDF, the least known leading twist-2 PDF, for example. Reference [2] uses the lattice-averaged isovector g_T to constrain the global-analysis fits of SIDIS charged-pion production data from proton and deuteron targets, including their x, z and $P_\perp$ dependence, with a total of 176 data points collected from measurements at HERMES and COMPASS. This gives in principle eight linear combinations of transversity TMD PDFs and Collins TMD FFs for different quark flavors, from which we attempt to extract the u and d transversity PDFs and the unflavored Collins FFs, together with their respective transverse-momentum widths, as shown on the left-hand side of Fig. 2. Without the lattice constraints, the distribution is consistent with zero within 2 sigma; with the constraint from the lattice tensor charge, we are able to make world-best predictions for the large-x transversity for both up and down quarks.

There has been rapid progress calculating the Bjorken-x dependence of PDFs on the lattice since the proposal of Large-Momentum Effective Theory (LaMET also called "quasi-PDF") [3, 4]. LaMET relates equal-time spatial correlators, whose Fourier transforms are called "quasi-PDFs", to PDFs in the limit of infinite hadron momentum. For large but finite momenta accessible on a realistic lattice, LaMET relates quasi-PDFs to physical ones through a factorization theorem, the proof of which was developed in Refs. [5–7]. Since the first lattice x-dependent PDF calculation [8], much progress has been made and calculations done. For recent reviews, we refer readers to Refs. [9–12]. Here, we only show a few examples of the lattice progress, focusing on highlight calculations at physical pion mass. The middle of Fig. 2 shows the nucleon isovector helicity PDF results (work done by LP^3 collaboration), showing nice agreement with the global fits [13], while the right-hand side shows a summary of lattice unpolarized PDF done by multiple lattice collaborations [12]. Note that none of the current lattice calculations have taken the continuum limit ($a \to 0$) and have remaining lattice artifacts (such as finite-volume effects); disagreement in the obtained distributions is not unexpected. Further studies of the systematic uncertainties, including multiple lattice spacings and volumes, from each collaboration will be needed.

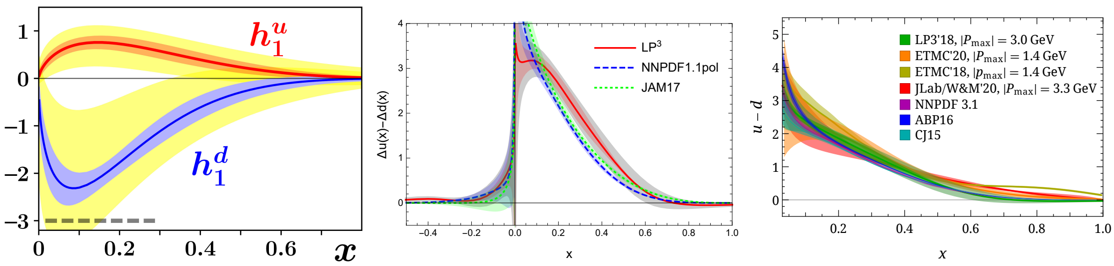

Figure 2: Example of nucleon isovector PDFs from lattice QCD in recent years. (left) Lattice-QCD g_T constraint on transversity PDFs $h_1^{u,d}$ for SIDIS with (red/blue) and without (yellow) lattice inputs at $Q^2 = 2$ GeV^2, compared with the SIDIS-only fit uncertainties (yellow bands) [2]. (middle) Nucleon boost-momentum dependence of the matched polarized helicity isovector PDFs [13]. For the quark asymmetry, the shape is consistent throughout most x regions. However, in the antiquark region, there is a significant change in the distribution as momentum increases. (Right) Summary of the lattice calculation of the isovector unpolarized quark distribution from Ref. [12]

There have also been many exploratory studies applying the quasi-PDF approach to gluon PDFs [14, 15] and first lattice-QCD calculations of the strange and charm parton distributions [16]. The left-hand side of Fig. 3 shows lattice gluon PDFs at pion masses $M_\pi = 135$ (extrapolated), 310 and 690 MeV compared with the CT18 NNLO (red band with dot-dashed line) and NNPDF3.1 NNLO (orange band with dotted line) gluon PDFs; there is a consistency between lattice and gluon fit results. Larger-momentum calculations are needed to improve the lattice results for $x < 0.3$ In Fig. 3, we found that our strange and charm renormalized real matrix elements are zero within our statistical errors for both strange and charm, supporting the strange-antistrange and

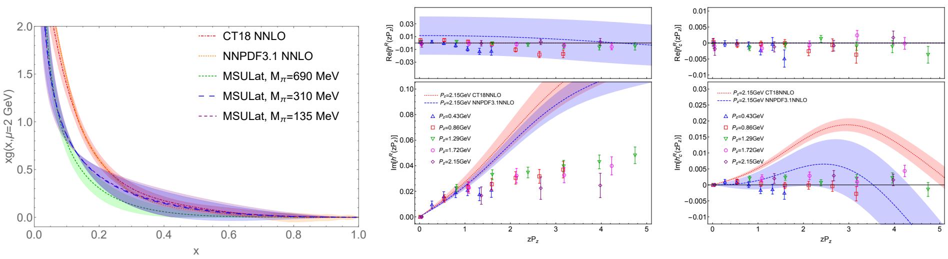

Figure 3: (left) The first x-dependent unpolarized gluon PDF, $xg(x, \mu)$, from lattice and compared with global fits. (middle/right) The real (top) and imaginary (bottom) parts of the strange (left) and charm (right) quasi-PDF matrix elements in coordinate space from our calculations at physical pion mass with $P_z \in [0.43, 2.15]$ GeV [16], along with those from CT18 and NNPDF NNLO.

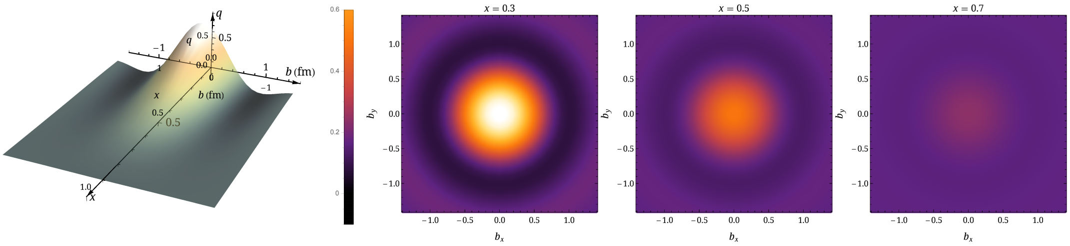

Figure 4: (left) Nucleon tomography: three-dimensional impact parameter–dependent parton distribution as a function of x and b using lattice H at physical pion mass. (right) Two-dimensional impact-parameter–dependent isovector nucleon GPDs for $x = 0.3$, 0.5 and 0.7 from the lattice at physical pion mass [17].

charm-anticharm symmetry assumptions commonly adopted by most global PDF analyses. The CT18 analysis assumes $s(x) = \bar{s}(x)$, so their results are exactly zero after matching and Fourier transformation. Our real matrix elements at $P_z > 1$ GeV are consistent with zero, supporting strange-antistrange symmetry, while our imaginary ones are smaller than global-fit results. Our imaginary matrix elements are proportional to the sum of the quark and antiquark distribution, and we clearly see that the strange contribution is about a factor of 5 larger than the charm ones. They are consistently smaller than those from CT18 and NNPDF3.1, possibly due to the missing contributions from the mixing with gluon matrix elements in the renormalization. Higher statistics will be needed to better constrain the quark-antiquark asymmetry. Work on revealing the three-dimensional nucleon structure via the generalized parton distribution (GPD) at physical pion mass was attempted for the first time [17] (see Fig. 4).

This is an exciting era for using lattice QCD to study PDFs. The well-studied systematics of the traditional moment method allow us to provide precision structure quantities directly at the physical pion masses with multiple lattice spacings. We are now also able to provide information on the Bjorken-x dependence of parton distributions, and they are widely studied with more than just the "quasi-PDF" method mentioned in this proceeding. Some lattice studies have begun to control systematics by using multiple lattice spacings and volumes; many such calculations are planned for near-future updates to improve the current calculations. We are also starting to address neglected disconnected contributions and taking a look into flavor-dependent quantities, but much work remains ahead. Stay tuned for more updates from lattice QCD in the near future.

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