PROCEEDINGS OF SCIENCE

Impact of soft photons on $B \to K\ell^+\ell^-$

Dayanand Mishra[a,b,*] **and Namit Mahajan**[a]

[a] *Physical Research Laboratory, 380009, Ahmedabad, India*
[b] *Indian Institute of Technology, Gandhinagar, India*

E-mail: dayanand@prl.res.in, nmahajan@prl.res.in

In this era of precision, it has become necessary to have precise expressions and construct theoretically clean observables to match with the experiments. The $O(\alpha)$ QED corrections to $B \to K\ell^+\ell^-$ modes are discussed here. The structure of the contact term is fixed by demanding gauge invariance of the real emission amplitude. A fictitious photon mass (λ) acts as IR regulator, and the results are shown to be independent of it. The QED effects in the individual channel are found to be negative. The electron channel is shown to receive a large correction of $O(20\%)$. The impact of soft photons on the lepton flavour universality (LFU) ratio ($R_K^{\mu e}$) is also discussed.

The Ninth Annual Conference on Large Hadron Collider Physics - LHCP2021
7-12 June 2021
Online

*Speaker

https://pos.sissa.it/

PoS(LHCP2021)205

1. Introduction

Quark transitions due to Flavour Changing Neutral Currents (FCNCs) are both loop and CKM suppressed, making them important candidates to test the Standard Model (SM) [1]. The decay modes $B \to K\ell^+\ell^-$ allow to test the lepton flavour universality (LFU) [2]:

$$R_K^{\mu e} \equiv \frac{\int_{1GeV^2}^{6GeV^2} dq^2 \frac{d\Gamma(B^0\to K^0\mu^+\mu^-)}{dq^2}}{\int_{1GeV^2}^{6GeV^2} dq^2 \frac{d\Gamma(B^0\to K^0e^+e^-)}{dq^2}}$$

The SM and experimental values [3] are $R_K^{\mu e}|_{SM}$ $(R_K^{\mu e}|_{exp}) = 1.00 \pm 0.01$ $\left(0.846^{+0.060}_{-0.054}{}^{+0.016}_{-0.014}\right)$. Within the SM, if the kinematical range is chosen such that the dilepton invariant mass is way larger than the mass of the considered leptons, then it is expected with high accuracy that the ratio of the two branching fractions is unity. The strong interaction effects in the ratio are included via RGEs and form factors. In this kinematical range, $R_K^{\mu e}$ is almost insensitive to form factors. In the present study, the focus is on the $O(\alpha)$ soft photon QED corrections to the decay width of $B \to K\ell^+\ell^-$ and $R_K^{\mu e}$. The cancellation of collinear divergences is also shown.

2. QED corrections

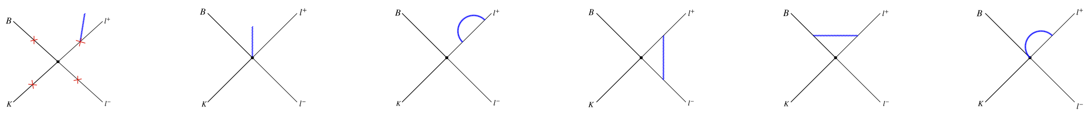
The effective Hamiltonian for $b \to s\ell^+\ell^-$ transition [4, 5] is $H_{\text{eff}} \propto \sum_i C_i(\mu)O_i(\mu)$, where $C_i(\mu)$ contains the information of short-distance physics and can be determined perturbatively, while $O_i(\mu)$ contains the information of long-distance physics. The matrix elements of these operators can be parameterized in terms of form factors using Dirac and Lorentz structure. The non-radiative differential decay width is given by:

$$\frac{d^2\Gamma_0(B \to Kl^+l^-)}{dq^2ds} = \frac{1}{256\pi^3 m_B^3}\frac{G_F^2\alpha^2}{8\pi^2}|V_{ts}|^2|V_{tb}|^2|M_0(B \to Kl^+l^-)|^2 \tag{1}$$

where $q^2 = (p_2 + p_3)^2$, $s = (p' + p_2)^2$ and the explicit form of the matrix element is:

$$M_0 = \left[\left(\left\{C_9^{\text{eff}} f_+ + C_7^{\text{eff}}\frac{2f_T m_b}{m_B + m_k}\right\} p^\mu + \left\{C_9^{\text{eff}} f_- - C_7^{\text{eff}}\frac{2f_T m_b}{q^2}(m_B - m_k)\right\} q^\mu\right)(\bar{l}\gamma_\mu l) - C_{10}\left(f_+p^\mu + f_-q^\mu\right)(\bar{l}\gamma_\mu\gamma_5 l)\right]$$

The QED corrections are now considered. Fig. (1) shows the photon emission and virtual



(a) **(b)** **(c)** **(d)** **(e)** **(f)**

Figure 1: Representative diagram for real photon emission (X :photon emission) and virtual corrections

corrections diagrams. Fig 1(b) is the so called *contact term* (CT) and arises due to the gauge invariance of QED. The diagrams involving the contact term ensure cancellation of the infrared divergences and having a gauge invariant result. The photon is labeled by the polarization vector $\epsilon_\alpha(k)$. The charges of the B and K meson are denoted by Q_B and Q_K.

2.1 Contact Term

Employing scalar QED for B mesons, the amplitude for $B(p_0) \to K(p_1)\ell^+(p_2)\ell^-(p_3)\gamma(k)$ reads

$$\begin{aligned} \tilde{M} &= -e\epsilon_\alpha(k)\left[\bar{u}(p_2)\Gamma^\mu_A\frac{(\not{p}_3+\not{k})-m_l}{2p_3.k}\gamma^\alpha v(p_3) - \bar{u}(p_2)\gamma^\alpha\frac{(\not{p}_2+\not{k})+m_l}{2p_2.k}\Gamma^\mu_A v(p_3)\right]\otimes H_{A\mu}(p_0,p_1) \\ &+ e\epsilon_\alpha(k)\bar{u}(p_2)\Gamma^\mu_A v(p_3)\otimes\left[Q_B\frac{2p_0^\alpha}{2p_0.k}H_{A\mu}(p_0-k,p_1) - eQ_K\frac{2p_1^\alpha}{2p_1.k}H_{A\mu}(p_0,p_1+k)\right] \end{aligned} \tag{2}$$

Emission of a photon from the lepton legs does not modify the hadronic part, $H_{A\mu}$, as in non-radiative decay, while it is appropriately modified in the case of emission from the meson legs. Gauge invariance of the amplitude necessitates an additional term in the effective Hamiltonian at the hadronic level given by

$$\mathcal{H}^{CT}_{eff} = ie\xi_A(Q_B+Q_K)\left[\bar{u}(p_2)\Gamma^\alpha_A v(p_3)\right]A_\alpha\phi^+_K\phi^-_B \tag{3}$$

The way the contact terms are determined [6] is very different from that adopted in [7].

2.2 Total $O(\alpha)$ QED corrections to Γ_0 and observable $R_{k^{\mu e}}$

The total decay rate is

$$d\Gamma_{real} = d\Gamma_0\left(1+2\alpha\mathcal{H}_{ij}+\frac{\alpha}{\pi}\right)\Omega_c + d\Gamma' \tag{4}$$

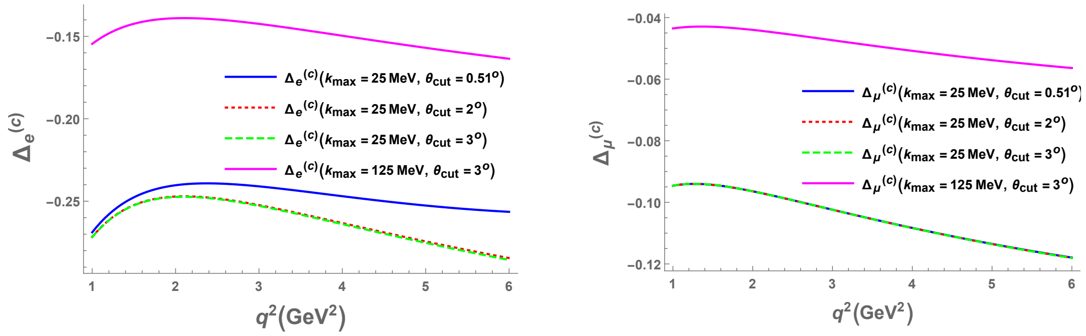
where the explicit form of the Coulomb factor (Sommerfeld enhancement factor) Ω_c and the function $\mathcal{H}_{ij}$, can be found in [6]. The corrected second order differential decay rate with Δ^i denoting the correction factor, is

$$\frac{d^2\Gamma^i}{dsdq^2} = \frac{d^2\Gamma_0}{dsdq^2}\left(1+\Delta^i\right). \tag{5}$$

The other relevant quantity is the shift in $R_K^{\mu e}$: $\Delta^i_{R_K^{\mu e}} = R_K^{0\mu e}\left(\frac{\Delta\Gamma^i_\mu}{\Gamma^i_\mu} - \frac{\Delta\Gamma^i_e}{\Gamma^i_e}\right)$, with $i = 0(c)$ for neutral (charged) B decay.

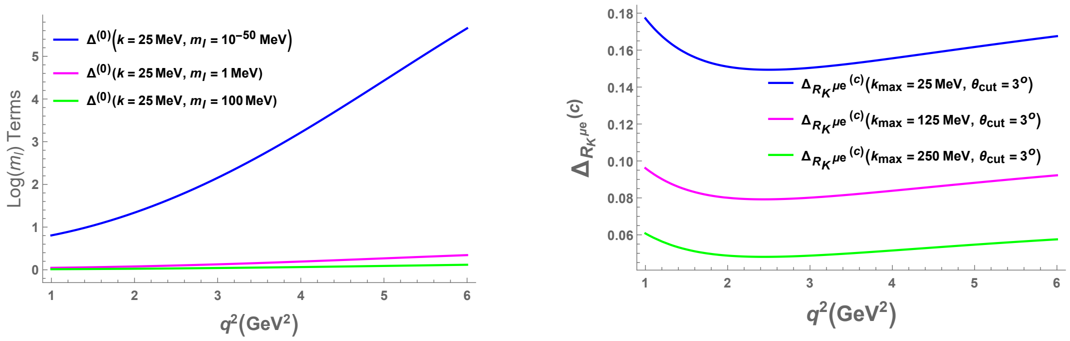
3. Result

Fig (2) shows the impact of QED corrections captured by Δ^i. The correction factor for the electrons is three times larger than that for the muons due to the mass difference between the two. There is also a dependence on the photon energy cut k_{max} chosen. Another important feature is the sensitivity to θ_{cut}(the angle between lepton and photon), particularly for the case of electrons. By choosing $\theta_{cut} \sim$ few degrees, this sensitivity essentially disappears. Fig (3a) shows the sensitivity on m_l. We can see that the lower two curves are what we would have expected for the case of muon and electron mass. These $\ln(m_l)$ terms correspond to hard collinear logs. We can see the explicit cancellation of the collinear logs by choosing a different set of kinematical variables: $t = (p_B - p_K)^2$, $s = (p_K + p_2)^2$, $x = (p_K + k)^2$, $q^2 = (p_2 + p_3)^2$ and E_k in the rest frame of $(q+k)^2$. Fig.(3b) shows the impact of QED effects on $\Delta^i_{R_K^{\mu e}}$ for $\theta_{cut} = 3°$ as a function of q^2. As the QED effects are negative for both electrons and muons, $R_K^{\mu e}$ therefore increases. However, as mentioned



(a) $\Delta_e^{(c)}$ vs. q^2 **(b)** $\Delta_\mu^{(c)}$ vs. q^2

Figure 2: $O(\alpha)$ corrections to charged $B \to K\ell\ell$ modes. Left: electrons, Right: muons



(a) Behaviour of $\ln(m_l)$ terms **(b)** $\Delta_{R_K^{\mu e}}^{(c)}$ vs. q^2

before, all the quantities are sensitive to k_{max} and θ_{cut}. The shift in $R_K^{\mu e}$ over the q^2 range decreases with an increase in k_{max}. This is expected since with an increase in k_{max}, the muons also start to effectively behave as electrons i.e. when $m_e,\ m_\mu << k_{max}$, both are affected in a similar manner. We have checked that for such a case, $\Delta^i_{R_K^{\mu e}}$ is very close to zero. The electron modes receive large QED corrections, $O(20\%)$, whereas the muon modes receive smaller corrections. We have also checked that choosing different k_{max} values for the muons and electrons changes the shift in $R_K^{\mu e}$ such that the final value of $R_K^{\mu e}$, including the QED effects, deviates from unity by a few percent.

4. Conclusion

In conclusion, we have shown that the QED effects to $B \to K\ell^+\ell^-$ are an important source of correction and should be systematically included. The individual rates for electron and muon receive significant correction with appropriate cut. The LFU ratio $R_K^{\mu e}$ receives nominal shifts and depends upon the cuts imposed. The present study [6], and also [7], leave open the question of leftover UV divergences. This is important to have an unambiguous comparison with the experiments, particularly given that observables like $R_K^{\mu e}$ are heralded as very clean probes of the SM, and therefore of new physics beyond it.

References

[1] A. Ali, G. Kramer, and Guo-huai Zhu, $B \to K\ell\ell$ decay in soft-collinear effective theory.Eur. Phys. J. C, 47:625–641, (2006).

[2] G. Hiller and F. Kruger. More model-independent analysis of $b \to s$ processes. Phys Rev D 69, 074020 (2004).

[3] Roel Aaij et al. Search for lepton-universality violation in $B^+ \to K^+\ell^+\ell^-$ decays. Phys. Rev. Lett. 122(19), 191801 (2019).

[4] Mikolaj Misiak. Radiative and rare semileptonic B decays. PoS, FPCP2010, 025 (2010).

[5] Andrzej J. Buras. Weak Hamiltonian, CP violation and rare decays. InLes HouchesSummer School in Theoretical Physics, Session 68: Probing the Standard Model ofParticle Interactions, pages 281–539, 6 1998

[6] Dayanand Mishra and Namit Mahajan. Impact of soft photons on $B \to K\ell^+\ell^-$. Phys. Rev. D 103(5), 056022 (2021).

[7] G. Isidori, S. Nabeebaccus and R. Zwicky. QED corrections in $B \to K\ell^+\ell^-$ at the double-differential level, JHEP, 12, 104 (2020).