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Charmonium-like resonances in coupled $D\bar{D}$ - $D_s\bar{D}_s$ scattering

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Charmonium-like resonances and bound states with isospin zero and $J^{PC} = 0^{++}, 1^{--}, 2^{++}, 3^{--}$ are extracted on the lattice. Coupled $D\bar{D}$ and $D_s\bar{D}_s$ scattering suggests three charmonium-like states with $J^{PC} = 0^{++}$ in addition to $\chi_{c0}(1P)$: a so far unobserved $D\bar{D}$ bound state just below threshold, a conventional resonance likely related to $\chi_{c0}(3860)/\chi_{c0}(2P)$ and a narrow resonance just below the $D_s\bar{D}_s$ threshold with a large coupling to $D_s\bar{D}_s$ likely related to $X(3915)/\chi_{c0}(3930)$. One-channel $D\bar{D}$ scattering renders resonances and bound states with $J^{PC} = 1^{--}, 2^{++}, 3^{--}$ related to the observed conventional charmonia. Lattice QCD ensembles from the CLS consortium with $m_\pi \simeq 280$ MeV are utilized.

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1. Introduction

We study charmonium-like resonances with isospin zero by simulating the scattering on the lattice. Studies of charmonium-like states are of particular interest as most of the exotic hadrons discovered contain a $\bar{c}c$ pair. Furthermore, all these exotic hadrons are resonances and can decay strongly. We consider the resonances and bound states with $J^{PC} = 0^{++}$, 1^{--}, 2^{++}, 3^{--} indicated in Fig. 1, where the relevant thresholds are also shown. The scalars are extracted from the coupled channel scattering $D\bar{D}$ - $D_s\bar{D}_s$ since two resonances reside near the $D_s\bar{D}_s$ threshold. The negative parity states of interest lie significantly below the $D_s\bar{D}_s$ threshold and are explored with one-channel $D\bar{D}$ scattering. The spin-two resonance is also extracted in the approximation of one-channel $D\bar{D}$ scattering. Details are provided in publications [1] and [2] which consider the negative and positive parity sectors, respectively.

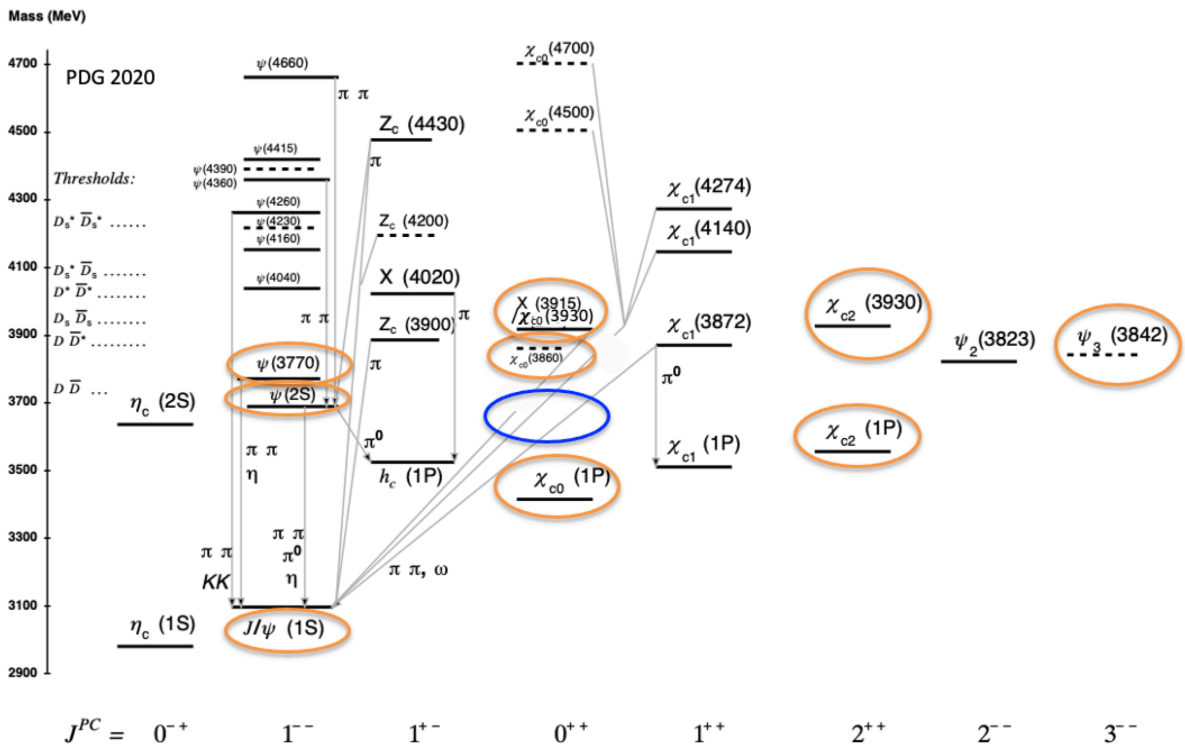


Figure 1: Charmonium-like states from PDG [3] are shown in black. The ellipses indicate states considered in the present lattice study: $J^{PC} = 0^{++}$, 2^{++} and $J^{PC} = 1^{--}$, 3^{--} were extracted in [2] and [1] respectively. The state indicated in blue is the lattice prediction of yet-unobserved unconventional state that couples strongly to $D\bar{D}$ and lies near this threshold. Other states have been experimentally discovered: all of them seem to be conventional charmonia, except for $X(3915)/\chi_{c0}(3930)$ which is unconventional state that couples strongly to $D_s\bar{D}_s$ according to this lattice study.

m_π [MeV]	m_K [MeV]	m_D [MeV]	m_{D^*} [MeV]	m_{D_s} [MeV]	M_{av} [MeV]
280(3)	467(2)	1927(2)	2050(2)	1981(1)	3103(3)

Table 1: Hadron masses for the gauge configurations used in this study, where $M_{av} = (m_{\eta_c} + 3m_{J/\psi})/4$.

2. Extracting scattering amplitudes from lattice

The infinite-volume scattering matrices $t_{ij}(E_{cm})$ are determined from the finite-volume eigen-energies via Lüscher's formalism. The eigen-energies are obtained from the correlation matrices based on a number of $\bar{c}c$, $D\bar{D}$, $D_s\bar{D}_s$ and $J/\psi\omega$ interpolating fields with appropriate quantum numbers. The channel $J/\psi\omega$ is treated as decoupled, while the channel $\eta_c\eta$ is omitted from the simulation. We consider the systems with total momenta $|\vec{P}| = 0,\ 2\pi/L,\ \sqrt{2}(2\pi/L)$ on lattices with $L = 24a,\ 32a$ in order to constrain the scattering matrix at more values of the center-of-momentum energy E_{cm}. The $c\bar{c}$ annihilation is omitted like in most of the charmonium studies on the lattice, while other Wick contractions as evaluated using the distillation method. The simulation is performed on the $N_f = 2 + 1$ ensembles U101 and H105 generated by the CLS consortium [4] with $a = 0.08636(98)(40)$ fm and unphysical $m_{u/d} > m_{u/d}^{exp}$ and $m_s < m_s^{exp}$, such that $2m_{u/d} + m_s$ is close to the physical value. The value of the charm quark mass (related to $\kappa_c = 0.12315$) is slightly higher than physical and the resulting masses of the relevant stable hadrons are collected in Table 1. In particular, the $D\bar{D}$ and $D_s\bar{D}_s$ thresholds are closer to each other than in experiment as shown in Fig. 3.

The energy dependence of scattering matrices $t_{ij}(E_{cm})$ is parametrized and the parameters are fitted as to best satisfy the Lüscher quantization condition for all lattice eigen-energies simultaneously. The *TwoHadronsInBox* package [5] is utilised for this. The typical parametrizations used for the real part of the inverse scattering amplitude are of Breit-Wigner type $\tilde{K}^{-1}/E_{cm} = p^{2l+1}\cot\delta/E_{cm} = a + bE_{cm}^2$ and $\tilde{K}_{ij}^{-1}/E_{cm} = a_{ij} + b_{ij}E_{cm}^2$ for one-channel and two-channel scattering, respectively, with more details given in [1, 2]. The study makes several simplifying assumptions (detailed in Section 5 of [2]) necessary for a first investigation of this coupled-channel system.

3. Summary of the resulting hadrons and their relation to experiment

Let us summarize the properties of the charmonium-like hadrons found in this simulation. All results were obtained at unphysical quark masses and are not extrapolated to the continuum limit. The resonances and bound states are related to the poles of the scattering matrix. The locations of the poles in the complex energy plane are given in Fig. 2. The masses of the charmonium-like states correspond to Re(E_{cm}^{pole}) and are compared to the experimental spectrum in Fig. 3. The resonance decay widths Γ are given by 2Im(E_{cm}^{pole}). We don't compare them directly to experiment since they depend on the phase space and therefore on the position of the threshold, which is different in the simulation. Figure 4 shows the coupling g that parametrizes the full width of a resonance and compares it to experiment

$$\Gamma \equiv g^2 p_D^{2l+1}/m^2 \quad \text{with} \quad l = 0,1,2,3 \quad \text{for } J^{PC} = 0^{++}, 1^{--}, 2^{++}, 3^{--}\ . \tag{1}$$

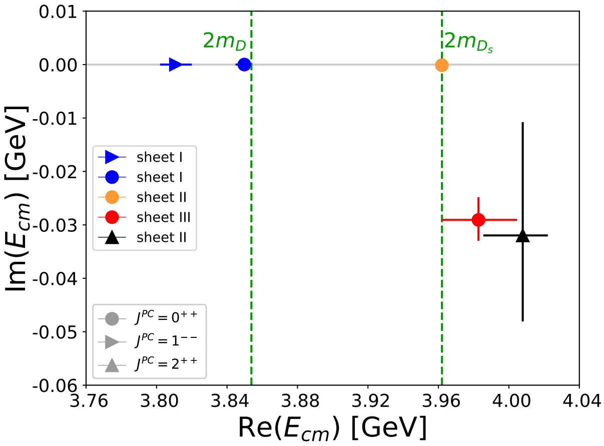


Figure 2: Pole singularities of the scattering amplitude/matrix in the complex energy plane that are extracted from this lattice study. These singularities are associated with the charmonium-like states and various symbol shapes denote the corresponding J^{PC}. The 3^{--} state is not presented as we did not extract its width.

Due to the unphysical quark masses in the simulation, the difference $m - E^{ref}$ is compared to experiment below and $m - E^{ref} + E^{ref}_{exp}$ is shown in Fig. 3b. The reference energy E^{ref} is the spin-averaged charmonium mass $M_{av} = \frac{1}{4}(3m_{J/\psi} + m_{\eta_c})$ for conventional candidates and a nearby threshold for states dominated by $D_{(s)}\bar{D}_{(s)}$.

Below we review the spectroscopic properties of the charmonium-like states, which are summarized also in Figs. 2, 3 and 4:

- **Ground states J/ψ, $\psi(2S)$, $\chi_{c0}(1P)$, $\chi_{c2}(1P)$**

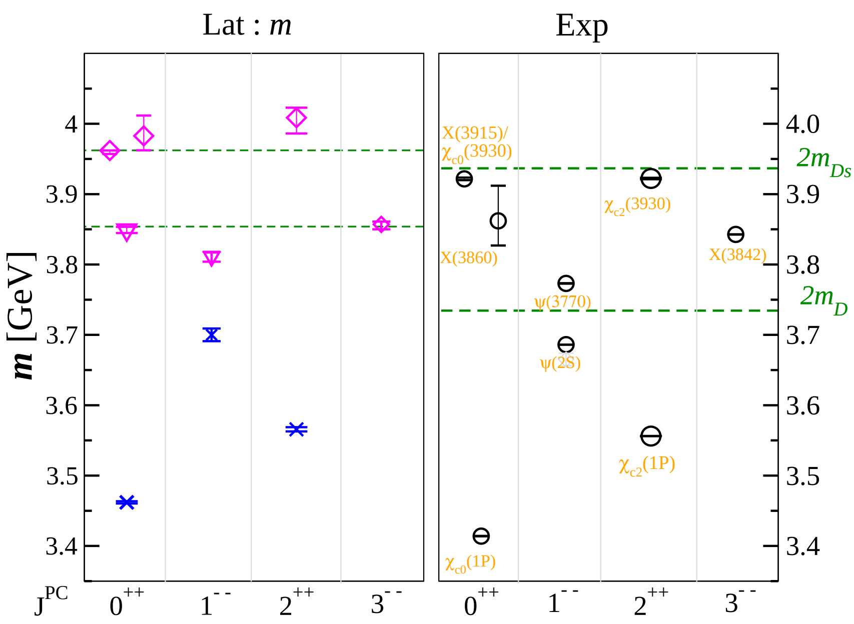
  These states lie significantly below the $D\bar{D}$ threshold and their masses are extracted from the energies $m = E(P = 0)$, resulting in a reasonable agreement with experimental masses.

- **0^{++} state dominated by D$\bar{\text{D}}$ slightly below threshold**

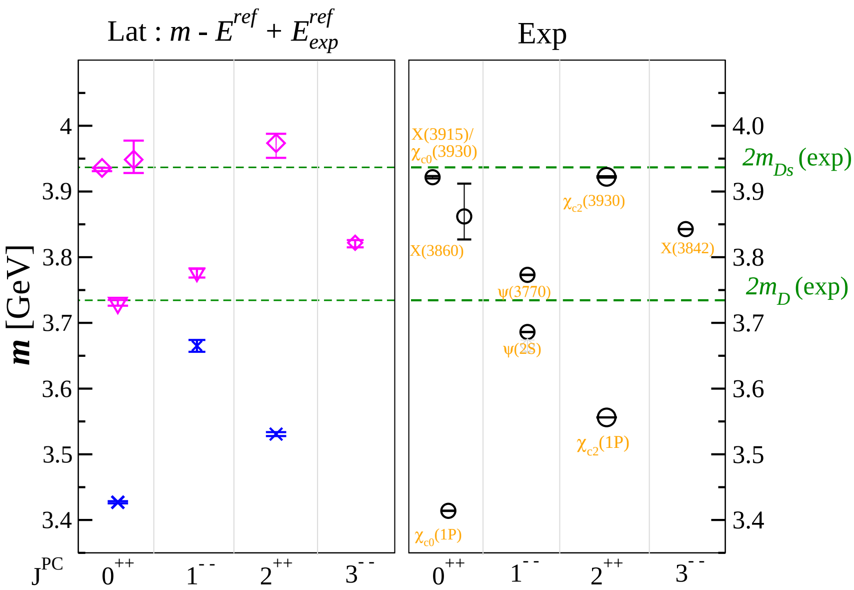
  The $D\bar{D}$ scattering near threshold in Fig. 5 indicates the presence of a shallow $D\bar{D}$ bound state with the binding energy $m - 2\,m_D = -4.0^{+3.7}_{-5.0}$ MeV. Our results suggest this state is likely not a conventional charmonium $\bar{c}c$, but owes its existence to a large interaction in the $D\bar{D}$ channel near threshold. Such a state has not been claimed by experiments (yet). It would feature as a sharp increase of the $D\bar{D}$ rate just above the threshold if it has a binding energy of a few MeV. Various strategies for its experimental search were proposed by E. Oset et al. and are listed in Section 8.1 of [2].

  The existence of a shallow $D\bar{D}$ bound state dubbed $X(3720)$ was already suggested by an effective phenomenological model based on the exchanges of light pseudoscalar and vector mesons [7][1]. In a molecular picture, a 0^{++} state is expected as a partner of $X(3872)$ via

[1]This state with $m \simeq 3.718$ GeV is listed in Table 4 of Ref. [7].



(a) Left pane: The masses m obtained from our lattice study with unphysical quark masses $m_{u/d} > m_{u/d}^{exp}$, $m_s < m_s^{exp}$ and $m_c \gtrsim m_c^{exp}$. The green lines denote positions of the $D\bar{D}$ and $D_s\bar{D}_s$ thresholds on our lattice with $m_D \simeq 1927$ MeV and $m_{Ds} \simeq 1981$ MeV.



(b) Left pane: The same masses m as above, but shifted to $m - E^{ref} + E^{ref}_{exp}$ in order to account for the dominant effect of unphysical quark masses in the simulation. The reference energy is $E^{ref} = 2m_D$ ($2m_{D_s}$) for the state closest to the $D\bar{D}$ ($D_s\bar{D}_s$) threshold, while $E^{ref} = M_{av} = \frac{1}{4}(3m_{J/\psi} + m_{\eta_c})$ for the remaining four states. The green lines denote experimental thresholds.

Figure 3: Masses of charmonium-like states with isospin zero from a lattice simulation [1, 2] (left) compared to experiment (right). Left: The magenta symbols correspond to hadrons extracted via the scattering analysis on the lattice: diamonds represent resonances and triangles represent bound states. The blue crosses are extracted directly from the lattice energies. Right: experimental spectrum from the PDG [3], where $\chi_{c0}(3930)$ [6] and $X(3915)$ are now identified as the same state.

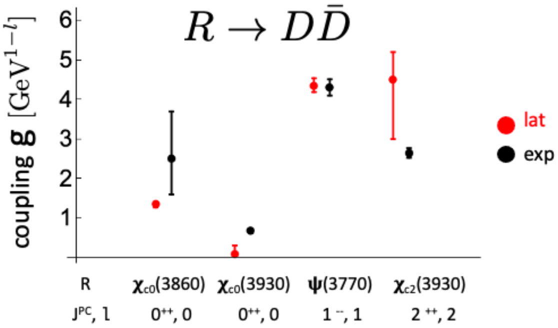


Figure 4: The coupling g that parametrizes the decay width for various resonances $R \to D\bar{D}$ via $\Gamma \equiv g^2 p_D^{2l+1}/m^2$ from this lattice simulation and from experiment. The quantum numbers of R and partial waves l in the decay $R \to D\bar{D}$ are provided at the bottom.

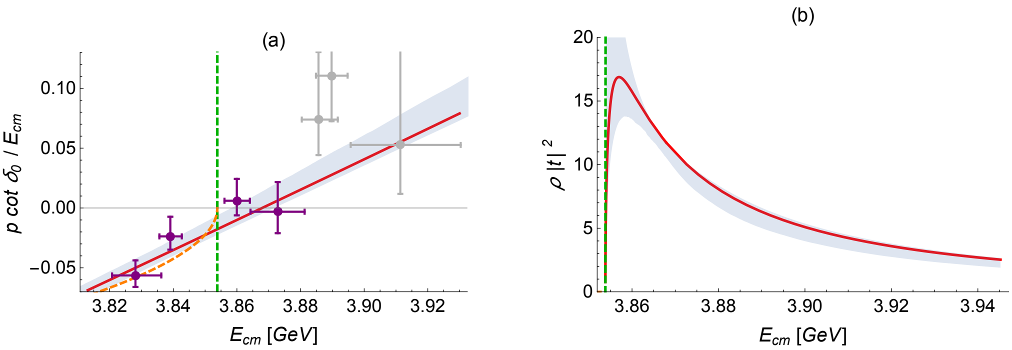


Figure 5: $D\bar{D}$ scattering in partial wave $l = 0$, where the threshold position is shown in green. Left: The bound state occurs at the energy where the red and orange curves intersect. Right: The shallow bound state is responsible for a sharp rise of the $D\bar{D}$ rate $N_{D\bar{D}} \propto p\sigma \propto \rho|t|^2$ ($\rho = 2p/E_{cm}$ and t is the scattering amplitude).

heavy-quark symmetry [8, 9, 17]. A similar virtual bound state with a binding energy of 20 MeV follows from the data of the only previous lattice simulation of $D\bar{D}$ scattering [10][2].

- **2^{++} resonance and relation to χ_{c2}(3930)**

  We find a resonance with $J^{PC} = 2^{++}$ $D\bar{D}$ scattering with $l = 2$ that is most likely related to the conventional $\chi_{c2}(3930)$ aka $\chi_{c2}(2P)$ discovered by Belle [11]

$$\begin{aligned} \text{lat}: \quad m - M_{av} &= 904\,^{+14}_{-22}\ \text{MeV}, \quad g = 4.5^{+0.7}_{-1.5}\ \text{GeV}^{-1} \\ \text{exp}\ \chi_{c2}(3930): \quad m - M_{av} &= 854 \pm 1\ \text{MeV}, \quad g = 2.65 \pm 0.12\ \text{GeV}^{-1}. \end{aligned} \tag{2}$$

  The masses are reasonably close, while the coupling from lattice QCD is larger than that measured experiment, but not inconsistent given the statistical and systematic uncertainties.

- **Broad 0^{++} resonance and relation to χ_{c0}(3860)**

[2]The presence of this state was not mentioned in Ref. [10], as such virtual bound states were not searched for.

The coupled $D\bar{D} - D_s\bar{D}_s$ scattering shows a broad peak in the $D\bar{D}$ channel featured in the left pane in Fig. 6). It is related to a resonance that couples mostly to $D\bar{D}$. We compare it to $\chi_{c0}(3860)$ discovered by Belle [12] that is a candidate for the conventional $\chi_{c0}(2P)$

$$\begin{aligned} \text{lat}: \ m - M_{av} &= 880^{+28}_{-20}\ \text{MeV}\,, \quad g = 1.35\ ^{+0.04}_{-0.08}\ \text{GeV} \\ \text{exp}\ \chi_{c0}(3860): m - M_{av} &= 793\ ^{+48}_{-35}\ \text{MeV}\,, \quad g = 2.5^{+1.2}_{-0.9}\ \text{GeV}\,. \end{aligned} \tag{3}$$

The mass and coupling are reasonably consistent with experiment, in particular, when considering the experimental errors and the systematic uncertainties in the lattice results.

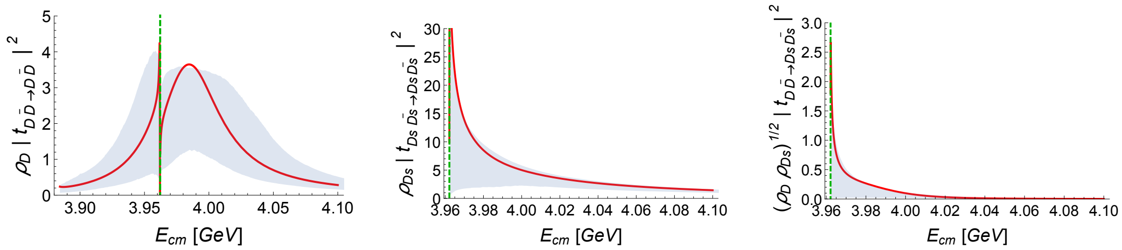


Figure 6: Coupled channel $D\bar{D}$, $D_s\bar{D}_s$ scattering in partial wave $l = 0$. The quantity $\sqrt{\rho_i\rho_j}|t_{ij}|^2 \propto p|\sigma|$ related to the number of events in experiment is shown ($\rho = 2p/E_{cm}$ and t is the scattering matrix).

- **Narrow 0^{++} state dominated by $\mathrm{D_s\bar{D}_s}$ slightly below threshold and relation to $X(3915)/\chi_{c0}(3930)$**

  The coupled $D\bar{D} - D_s\bar{D}_s$ scattering gives also an indication for a narrow 0^{++} resonance slightly below the $D_s\bar{D}_s$ threshold, which shows up as the sharp increase at the threshold in Fig. 6. It has a large coupling to $D_s\bar{D}_s$ and a very small coupling to $D\bar{D}$, as shown in Fig. 6 of [2]. The latter is responsible for its small decay rate to $D\bar{D}$ and the small total width. We compare the resulting resonance parameters with the experimental state $X(3915)/\chi_{c0}(3930)$ that shares similar features to the state we find

$$\begin{aligned} \text{lat}: \ m - 2m_{D_s} &= -0.2\ ^{+0.16}_{-4.9}\ \text{MeV}\,, \quad g = 0.10\ ^{+0.21}_{-0.03}\ \text{GeV} \\ \text{exp}\ X(3915)/\chi_{c0}(3930): \ m - 2m_{D_s} &= -14.3 \pm 1.8\ \text{MeV}\,, \quad g = 0.69 \pm 0.07\ \text{GeV}\,. \end{aligned} \tag{4}$$

  The $\chi_{c0}(3930)$ with $J^{PC} = 0^{++}$ was recently discovered in $D\bar{D}$ decay by LHCb [6], while the $X(3915)$ with $J^{PC} = 0^{++}$ or 2^{++} was found in $J/\psi\omega$ decay by Belle [13] and BaBar [14–16]: both states are now listed as the same state in the PDG [3]. It lies high above the $D\bar{D}$ threshold, so one would naturally expect a broad width if it was a conventional charmonium, given the large phase space available to $D\bar{D}$ decay. Its narrow experimental width $\Gamma = 18.8 \pm 3.5$ MeV can only be explained if its decay to $D\bar{D}$ is suppressed by some mechanism.

  Our lattice results suggest that the $X(3915)/\chi_{c0}(3930)$ charmonium-like state is likely not a conventional charmonium $\bar{c}c$, but owes its existence to a large interaction in the $D_s\bar{D}_s$ channel near threshold. It lies just below this threshold on the lattice and in experiment. Our results indicate that it has a very small coupling to $D\bar{D}$, which explains why its width is small and its decay to $D\bar{D}$ is suppressed in experiment.

  A state dominated by $\bar{c}c\bar{s}s$ in this energy region was found also in phenomenological studies of molecular [17] and diquark-antidiquark [18–20] states.

- **1^{--} state related to $\psi(3770)$**

  The $\psi(3770)$ related to conventional $\bar{c}c$ state 1^3D_1 appears in experiment just above the $D\bar{D}$ threshold. In our lattice simulation with $m_{u/d} > m_{u/d}^{exp}$ and $m_c \gtrsim m_c^{exp}$, we find it as a bound state in the $D\bar{D}$ scattering amplitude for partial wave $l = 1$ slightly below threshold. The resulting resonance parameters show nice agreement with experiment[3]:

$$\begin{aligned} \text{lat}: \; m - M_{av} &= 707 \pm 7 \text{ MeV}, \qquad \sqrt{6\pi}g = 18.9\,^{+0.8}_{-0.7} \\ \psi(3770) \;\; \text{exp}: \; m - M_{av} &= 704.25 \pm 0.35 \text{ MeV}, \;\; \sqrt{6\pi}g = 18.7 \pm 0.9\,. \end{aligned} \tag{5}$$

  We note that $\psi(3770)$ is found as a resonance for our lighter charm quark mass in [1] and the corresponding coupling g has a very similar value.

- **3^{--} resonance related to $\psi_3(3842)$**

  The $D\bar{D}$ scattering with partial wave $l = 3$ shows a resonance pole that corresponds to the charmonium with $J^{PC} = 3^{--}$:

$$\begin{aligned} \text{lat}: \; m - M_{av} &= 754\,^{+4}_{-7} \text{ MeV} \\ \psi_3(3842) \;\; \text{exp}: \; m - M_{av} &= 773.9 \pm 0.2\,, \;\; \Gamma = 2.8 \pm 0.6\,. \end{aligned} \tag{6}$$

  We are not able to determine its width since there is no $D\bar{D}$ lattice energy level within the energy region of this narrow resonance. The candidate for conventional charmonium with $J^{PC} = 3^{--}$ was discovered recently by LHCb [21] and the mass is close to our lattice value.

4. Conclusions

Charmonium-like states with isospin zero are studied by simulating one-channel and coupled-channel scattering on the lattice. We find all conventional charmonium resonances and bound states with $J^{PC} = 0^{++}$, 1^{--}, 2^{++}, 3^{--} up to the $D_s\bar{D}_s$ threshold. In addition, the results suggest the existence of unconventional states just below $D_s\bar{D}_s$ and $D\bar{D}$ thresholds, where the first is likely related to $X(3915)/\chi_{c0}(3930)$, while the second state has not been discovered yet. The study makes several simplifying assumptions necessary for a first investigation of this coupled-channel system.

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[3]The coupling g is defined in (1), while the value of $\sqrt{6\pi}g$ is more commonly listed for vector resonances. The coupling for the bound state was extracted using $p^3 \cot\delta_1/E_{cm} = (m^2 - E_{cm}^2)/g^2$.

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