PROCEEDINGS OF SCIENCE

Z_2 symmetry in Z_2+Higgs theory

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Using Monte Carlo methods, we study Z_2 symmetry in Z_2+Higgs theory. In 3 + 1 space-time dimensions, our simulation results suggest that the Z_2 symmetry is realized at large number of temporal lattice points (N_τ) in the Higgs symmetric phase. In order to see the dependence of Z_2 symmetry on the number of temporal lattice points we have also studied a simple temporal one dimensional model for a given spatial site. We show that the Z_2 symmetry is observed at the level of free energy at large N_τ limit. For this model, we also compute the density of states (DoS) for various N_τ values, where we show that the realization of the Z_2 symmetry happens at higher N_τ. Therefore, the realization of Z_2 symmetry may be due to dominance of the DoS.

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1. Introduction

At high temperatures hadrons melt into quark-gluon plasma (QGP). Theoretical studies in Quantum Chromodynamics (QCD) show that this melting proceeds via a transition known as confinement-deconfinement (CD) transition. In pure SU(N) gauge theories this CD transition is described by Z_N symmetry [1]. This Z_N symmetry is broken spontaneously at high temperatures but in presence of matter fields this symmetry is explicitly broken. Previous studies of Z_N symmetry in SU(N)+Higgs theories suggest that the Z_N symmetry is realized in the Higgs symmetric phase at large number of temporal lattice (N_τ) [2]. A much simpler model, coupled gauge spin theory found to have confinement and have similar phase diagram as above theory. Therefore, one can instead study this simpler theory to better understand the non-abelian models. Z_2+Higgs model [3, 4] is one such theory, where Z_2 symmetry is explicitly broken because of the gauge Higgs coupling term. The Polyakov loop (L), which is defined as the product of links along the temporal direction, found to reflect the Z_2 symmetry. Under Z_2 gauge transformations, the Polyakov loop transforms like magnetisation in such spin models. Therefore in this work, we compute Polyakov loop to study the Z_2 symmetry and the nature of the CD transition in Z_2+Higgs theory. We consider the theory on a $3+1$ dimensional lattice. Our numerical results show that this Z_2 symmetry is realized in the Higgs symmetric phase for large number of temporal lattice sites. Though the action does not have Z_2 symmetry but partition function averages exhibit Z_2 symmetry for large number of temporal sites. In order to see the Z_2 symmetry we have also considered a simple temporal one dimensional model for a given spatial sites. To simplify the problem we have considered a gauge choice in which all the gauge links are set to unity except the last one. The resulting free energy is found to have the Z_2 symmetry at large number of temporal lattice sites.
The proceedings is organized in the following way. In section 2, we have described the Z_2 symmetry in Z_2+Higgs theory in $3+1$ dimensions and the simulation results are shown in sub-section 2.1 for pure Z_2 gauge theory and with Higgs field. Section 3 presents the Z_2 symmetry in lower dimensions with simulation results. Finally the conclusions are given in section 4.

2. Z_2 symmetry in Z_2+Higgs theory in $3+1$ dimensions

The lattice action for the Z_2+Higgs theory in $3+1$ dimensional space ($N_s^3 \times N_\tau$) is given by,

$$S = -\beta_g \sum_P U_P - \kappa \sum_{n,\hat{\mu}} \Phi_{n+\hat{\mu}} U_{n,\hat{\mu}} \Phi_n. \tag{1}$$

Here the Higgs field Φ_n is defined on the lattice site n and $U_{n,\hat{\mu}}$ is the gauge link which connects site n and $n+\hat{\mu}$. The lattice site n has four components n_1, n_2, n_3, n_4 with $1 \leq n_1, n_2, n_3 \leq N_s$ and $1 \leq n_4 \leq N_\tau$. β_g is the the gauge coupling constant and κ is the strength of gauge Higgs interaction. The plaquette $U_P = U_{n,\hat{\mu}} U_{n+\hat{\mu},\hat{\nu}} U_{n+\hat{\nu},\hat{\mu}} U_{n,\hat{\nu}}$, is the path ordered product of links ($U_{n,\hat{\mu}}$) along an elementary square in $\mu-\nu$ plane. Here both the $U_{n,\hat{\mu}}$ and Φ_n take values ± 1. For this theory, under the Z_2 gauge transformations, the gauge links $U_{n,\hat{\mu}}$ transform as,

$$U_{n,\hat{\mu}} \to V_n U_{n,\hat{\mu}} V_{n+\hat{\mu}}^{-1} \tag{2}$$

The matter fields (Φ_n), being in the fundamental representation, transform as, $\Phi_n \to V_n \Phi_n$. Here V_n and $V_{n+\hat{\mu}}$ are the elements of Z_2 gauge group and they can take values ± 1. The V_n's satisfy the following equation,

$$V(\vec{n}, n_4 = 1) = zV(\vec{n}, n_4 = N_\tau). \quad (3)$$

where $z \in Z_2$ with $z = \pm 1$. The pure gauge part of the action, in Eq. (1), is invariant under the Z_2 gauge transformations of the gauge links i.e Z_2 symmetry is always there for pure gauge theory which is just spontaneously broken at high temperature. The Polyakov loop, $L(\vec{n}) = \prod_{n_4=1}^{N_\tau} U_{(\vec{n},n_4),\hat{4}}$, is the order parameter of this theory and transforms non-trivially under Z_2 gauge transformations [5] i.e

$$L(\vec{n}) \longrightarrow zL(\vec{n}). \quad (4)$$

Now since the Higgs fields are periodic, they satisfy the boundary condition, $\Phi(\vec{n}, n_4 = 1) = \Phi(\vec{n}, n_4 = N_\tau)$. So the gauge transformed Higgs fields Φ_g satisfy the boundary condition, $\Phi_g(\vec{n}, n_4 = 1) = z\Phi_g(\vec{n}, n_4 = N_\tau)$. Since $z = \pm 1 \in Z_2$, so the gauge transformed Higgs fields (Φ_g) does not remain periodic when $z = -1$. Therefore, in the presence of Higgs fields (Φ_n) the Z_2 symmetry is broken explicitly. For $\kappa \neq 0$ case, under Z_2, $U \longrightarrow U_g$. But $\Phi \longrightarrow \Phi_g = V\Phi$ is not considered as Φ_g is not periodic. So $S(U, \Phi) \neq S(U_g, \Phi)$ and these pair of configurations will not contribute equally to the partition function. The change in the action due to Z_2 "rotation" of gauge links can be compensated by changing the Higgs field appropriately. This was numerically tested by updating the Higgs field using Monte Carlo steps after Z_2 rotating the gauge links. In the following, we study the Z_2 symmetry using Monte Carlo methods.

2.1 Simulation results

To study Z_2 symmetry, we use Monte Carlo methods, in this method the gauge links $U_{n,\hat{\mu}}$ and Higgs fields Φ_n are updated using Metropolis algorithm [6]. The nature of CD transition has been studied for both pure Z_2 gauge theory ($\kappa = 0$) and in presence of Higgs field ($\kappa = 0.13$) for $N_\tau = 8$. In Fig. 1 the plot of Polyakov loop vs β_g for $\kappa = 0$ clearly suggests that there is a range of β_g over which two separated states (green line) are present. This indicates that the CD transition is first order [7]. We also plot a phase diagram in $\beta_g - \kappa$ plane, the line in Fig. 2 separates Higgs

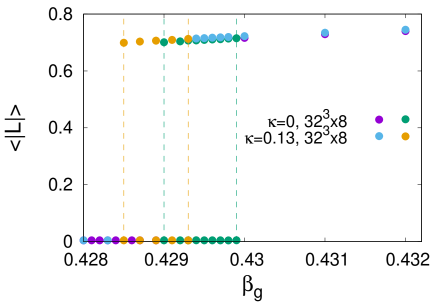

Figure 1: Average of L vs β_g for $N_\tau = 8$

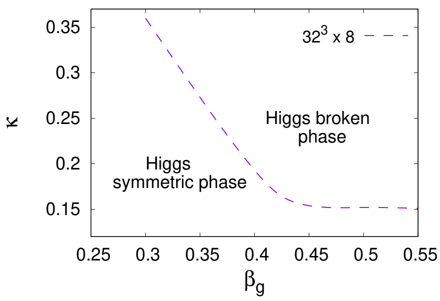

Figure 2: Phase diagram for $N_\tau = 8$

symmetric phase and broken phase. Higgs transition is first order for intermediate range of β_g and crossover for smaller and larger values of β_g [8, 9]. In the Higgs symmetric phase ($\kappa < \kappa_c$), it is the entropy i.e the distribution of the interaction term dominates over the action. In this phase there is a possibility for realization of Z_2 symmetry. In the Higgs broken phase ($\kappa > \kappa_c$), i.e large κ, the

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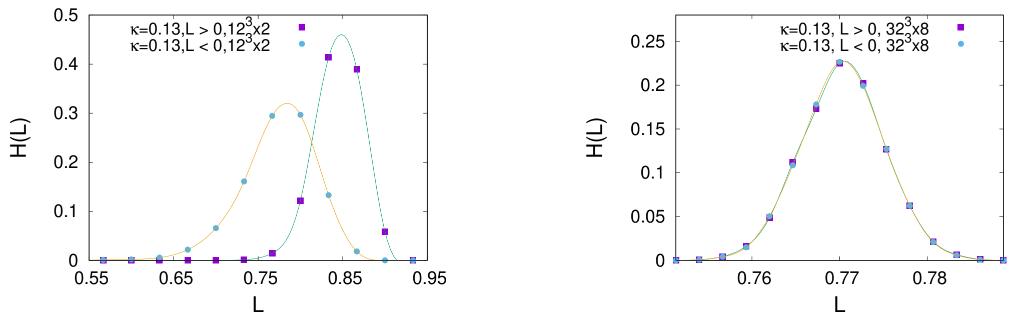
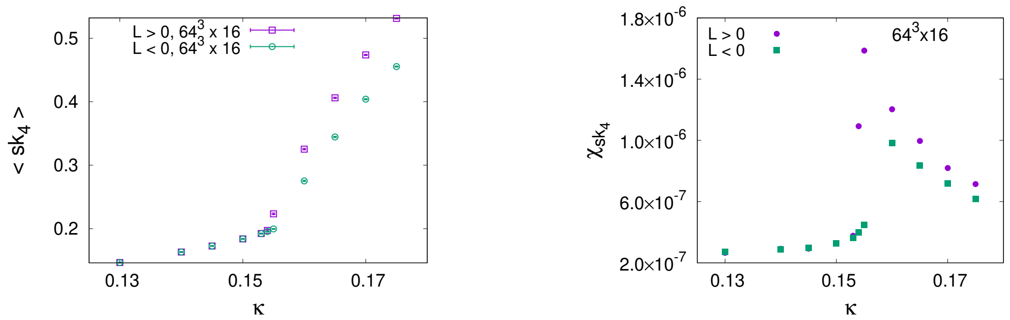
interaction term dominates over the entropy and Z_2 symmetry is badly broken in this phase. Our study is mostly focussed on studying the CD transition and Z_2 symmetry in the Higgs symmetric phase. To observe the effect of Φ field on the CD transition, in Fig. 1 we have shown how the order parameter behaves with β_g for $\kappa = 0.13$. We can see the CD transition is still first order as two states (yellow line) clearly appear here as well for a given range of β_g. It is clear from the results that in presence of Φ field ($\kappa = 0.13$) the range of β_g over which two states appear moves towards left of β_g i.e the critical β_g decreases.

In Fig. 3a-3b the histograms of the Polyakov loop $H(L)$ is studied numerically for $\kappa = 0.13$ to see the effect of N_τ on Z_2 symmetry. This study is done in the deconfined phase for the two Polyakov loop sectors at $N_\tau = 2, 8$. Here $L < 0$ data is Z_2 rotated to compare with $L > 0$ data. For $N_\tau = 2$, the histograms of the two Polyakov loop sectors do not agree with each other, which indicates that the Z_2 symmetry is explicitly broken here. But for $N_\tau = 8$, the two distributions corresponding to the two Polyakov loop sectors agree well which leads to realization of Z_2 symmetry at higher N_τ. The



(a) **(b)**

Figure 3: (a) $H(L)$ vs L in deconfined phase for $N_\tau = 2$, (b) $H(L)$ vs L in deconfined phase for $N_\tau = 8$.



(a) **(b)**

Figure 4: (a) $\langle sk_4 \rangle$ vs κ for $\beta_g = 0.435$ on $64^3 \times 16$ lattice, (b) χ_{sk_4} vs κ for $\beta_g = 0.435$ on $64^3 \times 16$ lattice.

Z_2 symmetry also depends on the phase of Higgs. The thermal average of the temporal components of interaction, $sk_4 = \sum_n \Phi_n U_{n,\hat{4}} \Phi^\dagger_{n+\hat{4}}$ and the corresponding susceptibility $\chi_{sk_4} = \left\langle sk_4^2 \right\rangle - \langle sk_4 \rangle^2$ is studied in the deconfined phase at $\beta_g = 0.435$. The results for $(\langle sk_4 \rangle, \chi_{sk_4})$ are shown in Fig. 4a-4b for $N_\tau = 16$. In $(\langle sk_4 \rangle, \chi_{sk_4})$ along κ-axis on the left ($\kappa < 0.154$) it is Higgs symmetric phase and on the right ($\kappa > 0.154$) it is Higgs broken phase. In the Higgs symmetric phase, at higher N_τ, the

κ value at which Z_2 symmetry is observed increases i.e $\langle sk_4 \rangle$ and χ_{sk_4} for the two Polyakov loop sectors agrees for higher κ. But in the Higgs broken phase, the Z_2 symmetry can not be observed even at higher N_τ.

3. Z_2 symmetry in lower dimensions with simulation results

To understand the realization of Z_2 symmetry in this theory, we consider a simple temporal one dimensional model for a given spatial site. The gauge Higgs interaction action for this $0 + 1$ dimensional model is,

$$S_{1D} = -\kappa sk_4, \quad sk_4 = \sum_{n=1}^{N_\tau} \Phi_n U_n \Phi_{n+1} \tag{5}$$

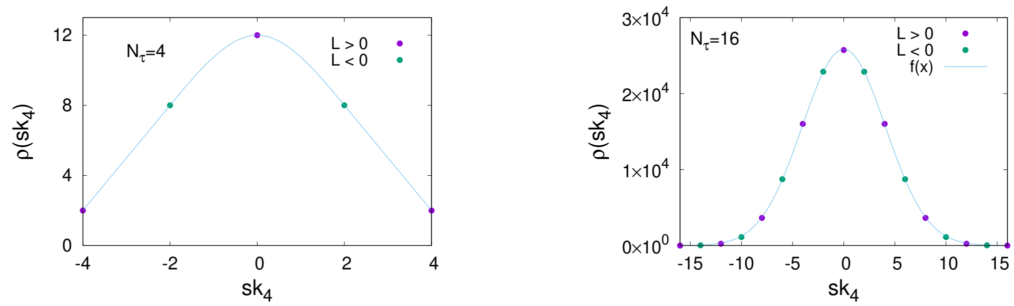
n denotes the temporal lattice site, i.e $1 \leq n \leq N_\tau$ and Φ_{N_τ} satisfies the periodic boundary condition $\Phi_{N_\tau+1} = \Phi_1$. The free energy $V(L, N_\tau)$ needs to be calculated analytically for this $0+1$ dimensional model to observe the Z_2 symmetry and its dependence on N_τ. To simplify the calculation we have considered a gauge choice in which all the gauge links are set to unity except the last one i.e $U_i = 1$ for $i = 1, 2, \ldots N_\tau - 1$ and $U_{N_\tau} = L$. With this gauge choice, for $L = 1$ this model behaves like an one dimensional Ising model. The Z_2 rotated part of it i.e $L = -1$ can be obtained by making the coupling between the fields Φ_{N_τ} and Φ_1 as anti-ferromagnetic. The exact partition functions for the two Polyakov loop sectors are given by,

$$\mathcal{Z}(L = 1) = \lambda_1^{N_\tau} + \lambda_2^{N_\tau}, \quad \mathcal{Z}(L = -1) = \lambda_1^{N_\tau} - \lambda_2^{N_\tau} \tag{6}$$

where $\lambda_1 = e^\kappa + e^{-\kappa}$ and $\lambda_2 = e^\kappa - e^{-\kappa}$. The free energies corresponding to the partition function in the large N_τ limit are given by,

$$V(L = 1) = V(L = -1) = -TN_\tau \log(\lambda_1). \tag{7}$$

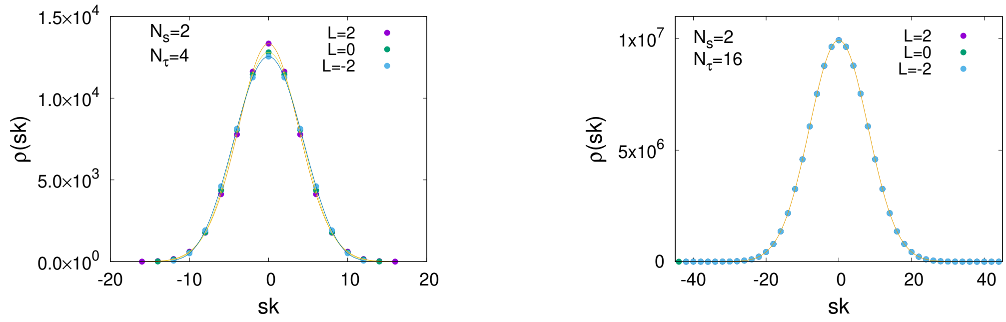
It is clear that the free energies for the two Polyakov loop sectors are equal at large N_τ limit in this $0 + 1$ dimensional model. This clearly indicates that the Z_2 symmetry realization happens at large N_τ limit even in the presence of Φ. So the realization of the Z_2 symmetry can be better



(a) (b)

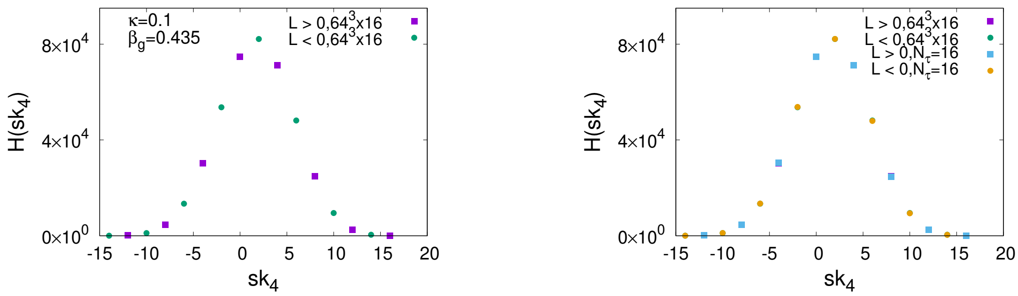
Figure 5: (a) $\rho(sk_4)$ for $\kappa = 0$ in 0+1 dimensions, (b) $\rho(sk_4)$ for $\kappa = 0$ in 0+1 dimensions.

explained from the DoS as shown in Fig. 5a-5b. For smaller N_τ ($N_\tau = 4$), the DoS or $\rho(sk_4)$ for the two Polyakov loop sectors are not described by a single function, which indicates that there is no Z_2 symmetry. But for large N_τ ($N_\tau = 16$), the distribution of sk_4 for the two Polyakov loop sectors are well described by a single gaussian function f(x) whose peak is at $sk_4 = 0$ and $\sqrt{N_\tau}$ as standard deviation. So the peak height and the distribution of sk_4 around this peak dominates the thermodynamics at large N_τ limit where Z_2 symmetry is observed even in the presence of Higgs fields. To see the effects of nearest neighbour interaction along the spatial directions, a $1+1$ dimensional model is considered for two given spatial sites ($N_s = 2$). Here sk is the total gauge Higgs interaction action for this model. In this model the possible values of Polyakov loop L is $0, \pm 2$. The distribution of sk i.e $\rho(sk)$ is studied for $N_\tau = 4, 16$ in Fig. 6a-6b. For $N_\tau = 4$, it is clear that there is no Z_2 symmetry as the distribution $\rho(sk)$ for different Polyakov loop sectors do not agree with each other. But for $N_\tau = 16$, the distribution $\rho(sk)$ is independent of L i.e the realization of Z_2 symmetry at large N_τ. So it is clear from this $1+1$ dimensional study that the interaction along the spatial directions does not affect the $0+1$ dimensional results of Z_2 symmetry realization.



(a) **(b)**

Figure 6: (a) $\rho(sk)$ for $\kappa = 0$ in 1+1 dimensions, (b) $\rho(sk)$ for $\kappa = 0$ in 1+1 dimensions.



(a) **(b)**

Figure 7: (a) $H(sk_4)$ for $\kappa = 0.1$, $\beta_g = 0.435$ for $3+1$ dimension, (b) $H(sk_4)$ fitted with $0+1$ density of states with a Boltzmann factor.

To see how well the $0+1$ dimensional DoS describe the $3+1$ dimensional simulation results, the histogram of sk_4 is computed in $3+1$ dimensions for $\kappa = 0.1$, $\beta_g = 0.435$ at $N_\tau = 16$ as shown in Fig. 7a. For this value of β_g and κ the system is found to be in the deconfined and Higgs symmetric phase. It is observed from the results that the histograms for both $L > 0$ and $L < 0$

fall on the same function, which indicates the presence of Z_2 symmetry at $N_\tau = 16$ even in the presence of Φ. To compare with $0 + 1$ dimensional DoS here only the upper envelope of $H(sk_4)$ is considered. It is interesting that the $3 + 1$ dimensional histogram $H(sk_4)$ can be fitted with $0 + 1$ dimensional DoS $\rho(sk_4)$ by using the formula $H(sk_4) \propto \exp(\kappa' sk_4)\rho(sk_4)$ as shown in Fig. 7b. Here $\exp(\kappa' sk_4)$ is a Boltzmann factor which needs to be included to fit the data. Also the κ' should be greater than $\kappa = 0.1$, because in $3 + 1$ dimensions sk_4 at a given spatial site have an interaction with sk_4 at nearest neighbours.

4. Conclusions

In this proceedings, we have studied Z_2 symmetry and CD transition in Z_2+Higgs theory. Our simulation results in $3 + 1$ dimensional model show that the Z_2 symmetry is broken explicitly in presence of matter fields and this symmetry is realized at large N_τ limit in the Higgs symmetric phase. Our $0 + 1$ dimensional results suggest that the density of states (DoS) dominate the thermodynamics at larger N_τ resulting in realization of Z_2 symmetry. The free energy calculation in one-dimension also suggests that the free energy difference between the two Polyakov loop sectors vanishes at large N_τ limit, which leads to realization of Z_2 symmetry due to dominace of entropy. We have also seen that the DoS of the $0 + 1$ dimensional model can reproduce the $3 + 1$ dimensional Monte Carlo results.

References

[1] G. 't Hooft, Nucl. Phys. B **138**, 1-25 (1978) doi:10.1016/0550-3213(78)90153-0

[2] M. Biswal, S. Digal and P. S. Saumia, Nucl. Phys. B **910**, 30-39 (2016) doi:10.1016/j.nuclphysb.2016.06.025 [arXiv:1511.08295 [hep-lat]].

[3] M. Creutz, doi:10.1103/PhysRevD.21.1006

[4] E. H. Fradkin and S. H. Shenker, Phys. Rev. D **19**, 3682-3697 (1979) doi:10.1103/PhysRevD.19.3682

[5] B. Svetitsky and L. G. Yaffe, Nucl. Phys. B **210**, 423-447 (1982) doi:10.1016/0550-3213(82)90172-9

[6] M. Creutz, L. Jacobs and C. Rebbi, Phys. Rev. Lett. **42**, 1390 (1979) doi:10.1103/PhysRevLett.42.1390

[7] R. Balian, J. M. Drouffe and C. Itzykson, Phys. Rev. D **10**, 3376 (1974) doi:10.1103/PhysRevD.10.3376

[8] G. A. Jongeward and J. D. Stack, Phys. Rev. D **21**, 3360 (1980) doi:10.1103/PhysRevD.21.3360

[9] M. Creutz, L. Jacobs and C. Rebbi, Phys. Rept. **95**, 201-282 (1983) doi:10.1016/0370-1573(83)90016-9