Disconnected contributions to the magnetic polarisability of the neutral pion

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The magnetic polarisability of the neutral pion has been calculated using the background field method in lattice QCD. These early results do not consider the effect of the disconnected loop contractions arising from the breaking of charge symmetry in a background magnetic field. Recent work in chiral perturbation theory has shown that these quark-self-annihilation contractions provide the leading loop-order contributions to the magnetic polarisability. A first investigation of these contractions in a background magnetic field is presented.

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1. Introduction

The response of the structure of the pion to an external electric or magnetic field is characterised by the electric (α_π) and magnetic (β_π) polarisabilities. These are experimentally measured using Compton scattering experiments [1–4] where the polarisabilities describe the scattering angular distribution [4–8].

Pion polarisabilities have been studied in a variety of theoretical frameworks; including chiral perturbation theory [8–10], the linear σ model [11] and dispersion sum rules [12–14]. Close agreement with experiment is possible using chiral perturbation theory [9]. The *ab initio* method for low energy QCD known as lattice QCD has also been to used to calculate electric and magnetic polarisabilities of the pion [15–21]. The lattice QCD simulations are typically performed with a number of non-physical constraints; chief among them is the neglect of disconnected quark contributions.

It is this feature which is considered herein for the first time. We compute the full neutral pion correlator, including disconnected contributions by using stochastic methods [22, 23] in order to calculate estimates of the $x - x$ or *all-to-all* quark propagator. The consideration of the full neutral pion and the connected pion allows disconnected terms to be isolated. This is in contrast to previous studies which have considered only the connected pion [15–19]

$$\chi_{\pi_0 u|d} = \left(\overline{q}_{u|d} \, \gamma_5 \, q_{u|d}\right).\tag{1}$$

The magnetic polarisability can be calculated using lattice QCD by inducing a uniform external magnetic field using the background field method. This external field changes the energy of the zero-momentum neutral pion according to the relativistic energy-field relation [15–18, 24, 25]

$$E_{\pi_0}^2 (B) = m_\pi^2 - 4 \pi \, m_\pi \, \beta_{\pi_0} \, B^2 + O\left(B^3\right).\tag{2}$$

The extraction of the magnetic polarisability term from this energy is nontrivial as the energy contribution of the magnetic polarisability term is small compared to the overall energy of the particle [17, 18, 26–29]. In fact, this is a requirement if the energy-field relation of Eq. (2) is to have small $O\left(B^3\right)$ contributions. As such specialised quark operators are used to provide enhanced coupling to the energy eigenstates of the neutral pion in an external magnetic field.

In this study a projection operator defined by the eigenmodes of the two-dimensional $SU(3) \times U(1)$ lattice Laplacian operator [30] is used to project the quark sink. This enables the pion ground state in an external magnetic field to be isolated, hence allowing for an accurate determination of the magnetic polarisability of the pion. The background-field-corrected (BFC) form [17] of the clover-fermion action [31, 32] is used in order to remove the spurious background-field dependent quark mass renormalisation associated with the Wilson term [16, 17].

This research is presented in the following manner. Section 2 describes the chief motivation behind considering disconnected Wick contractions. A brief summary of disconnected propagator techniques is presented in Section 3. The details of the lattice simulations performed in this work are presented in Section 4 and results in Section 5.
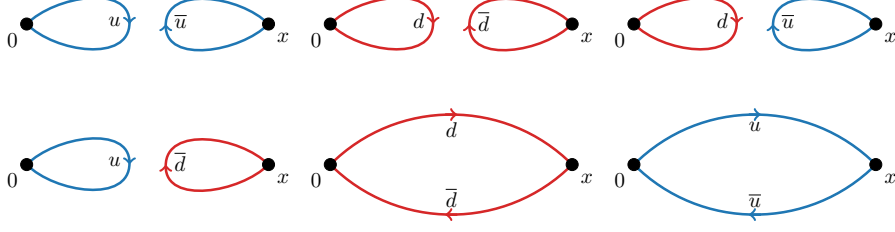
Figure 1: Full π_0 meson operator Wick contractions of Eq. (4).

2. Pion Polarisability

The full neutral pion interpolating operator is

$$\chi_{\pi_0} = \frac{1}{\sqrt{2}} \left(\overline{q}_u \, \gamma_5 \, q_u - \overline{q}_d \, \gamma_5 \, q_d \right), \tag{3}$$

which produces the corresponding two-point correlation function

$$\begin{aligned}
\mathcal{G}(t, \vec{p}) = \frac{1}{2} \sum_{\vec{x}} e^{-i\vec{p} \cdot \vec{x}} \Bigg[\\
- \text{Tr}\big[\gamma_5 U^{ee}(x,x)\big] \text{Tr}\big[\gamma_5 U^{e'e'}(0,0)\big] - \text{Tr}\big[\gamma_5 D^{ee}(x,x)\big] \text{Tr}\big[\gamma_5 D^{e'e'}(0,0)\big] \\
+ \text{Tr}\big[\gamma_5 U^{ee}(x,x)\big] \text{Tr}\big[\gamma_5 D^{e'e'}(0,0)\big] + \text{Tr}\big[\gamma_5 D^{ee}(x,x)\big] \text{Tr}\big[\gamma_5 U^{e'e'}(0,0)\big] \\
+ \text{Tr}\big[\gamma_5 D^{ee'}(x,0)\gamma_5 D^{e'e}(0,x)\big] + \text{Tr}\big[\gamma_5 U^{ee'}(x,0)\gamma_5 U^{e'e}(0,x)\big] \Bigg],
\end{aligned}$$

where colour indices are made explicit by e, e' and U (D) represents an up (down) quark propagator. The disconnected quark propagators $S(x,x)$ of Eq. (4) require a source at each lattice point. The point-point $S(0,0)$, point-all $S(x,0)$ and all-to-all $S(x,x)$ propagators are represented in Figure 1. Due to their expense, the disconnected terms are typically neglected in lattice QCD calculations.

In order to connect the lattice results of Ref. [18] to the physical regime, He *et al.* [33] performed a chiral extrapolation using the exact leading-order nonanalytic terms of chiral perturbation theory. Electro-quenching effects were considered through the framework of partially quenched chiral perturbation theory where it was found that the electro-quenching has no effect on the magnetic polarisability at leading loop-order. It is shown in Ref. [33] that the loops that contribute to the leading term are the mixed-flavour disconnected loops of Figure 1. The results of Ref. [33] are separated into leading loop-order and components in Figure 2. Here the leading loop-order contribution provides a $\sim 15\%$ correction at the physical point. This is a substantial proportion which must be considered in future precision-era lattice QCD studies.

The disconnected terms of Eq. (4) hence pose a particularly interesting question. Figure 3 illustrates how the mixed-flavour contractions can produce a π^+ dressing to the neutral pion. This effect is absent in the connected pion correlation functions investigated in previous studies [16, 18, 19]. This highlights the importance of including disconnected contributions to the neutral pion. In this study we will focus on contractions which produce a charged pion dressing as these are expected to be the dominant contribution at one-loop order [33].
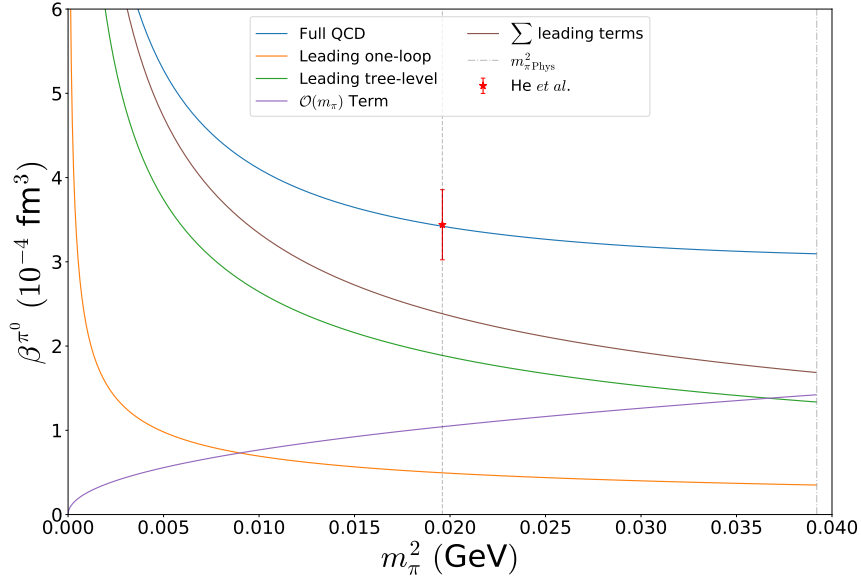
Figure 2: Leading order contributions to the magnetic polarisability of the pion as described by Eq. 20 of He *et al.* [33] for the range $0 \leq m_\pi^2 \leq 2\, m_{\pi\,\mathrm{Phys}}^2$ where reasonable convergence is expected. The uncertainty in the result of He *et al.* reflects the statistical uncertainty of the lattice results, the uncertainty in the low energy coefficients and a systematic uncertainty associated with two-loop diagrams in the electroquenched theory.
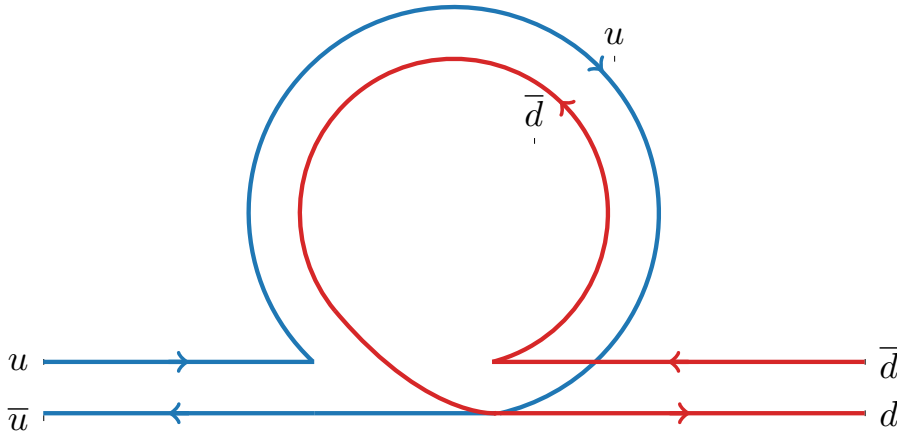

Figure 3: One-loop quark flow diagram for the neutral pion. Here we consider the Wick contraction $\mathrm{Tr}\big[\gamma_5 D^{ee}(x,x)\big]\mathrm{Tr}\big[\gamma_5 U^{e'e'}(0,0)\big]$ where the neutral pion is dressed by a π^+.

3. Propagator Methods

The disconnected propagators $S(x, x)$ of Eq. (4) require a different method to conventional point-to-all propagators; here we use stochastic estimation of the matrix inverse [22, 23].

For each correlator, we construct sets of random noise vectors $\{\eta\}$ with elements drawn from $\mathbb{Z}_4$ such that averaging over noise vectors gives

$$\left\langle \eta_{a\alpha}(x)\, \eta_{b\beta}^{\dagger}(y) \right\rangle = \delta_{xy}\, \delta_{ab}\, \delta_{\alpha\beta}, \tag{4}$$

where a, b are colour indices, α, β are spin indices and the space-time indices are x, y. Each noise vector has a corresponding solution vector

$$\chi = M^{-1}\, \eta, \tag{5}$$

where M is the fermion matrix. Hence the fermion propagator matrix element is stochastically estimated as

$$S_{ab;\alpha\beta}(x, y) \simeq \left\langle \chi_{a\alpha}(x)\, \eta_{b\beta}^{\dagger}(y) \right\rangle. \tag{6}$$

The source smearing methods of Ref. [34] are adopted when smearing disconnected quark propagators. The $SU(3) \times U(1)$ eigenmode projection technique of Ref. [30] is incorporated in the $S(x, x)$ propagator by applying it to the solution vector χ before the disconnected propagator calculation.

4. Simulation details

We make use of the $2 + 1$ flavour dynamical QCD gauge configurations provided through the ILDG [35] by the PACS-CS [36] collaboration. The ensemble under consideration here has a pion mass m_π of 0.296 GeV and a lattice spacing of 0.0907(13) fm. The lattice volume is $L^3 \times T = 32^3 \times 64$ and 150 configurations were used in this preliminary study. The Background-Field-Corrected (BFC) clover fermion action of Ref. [17] is used to remove spurious lattice artefacts that are introduced by the Wilson term . The BFC action uses a non-perturbatively improved clover coefficient for the QCD portion of the clover term and a tree-level coefficient for the portion deriving from the background field in order to remove the additive-mass renormalisation induced by the Wilson term [16, 17].

All-to-all propagators are fully diluted in time, spin and colour indices in order to reduce the statistical variance [37]. In order to improve the signal-to-noise properties of the disconnected quark propagators, we use 2^3 spatial dilution in a manner similar to Refs. [38] and [39]. $\mathbb{Z}_4$ (complex $\mathbb{Z}_2$) noise is chosen due to it's superior noise reduction properties [40–42]. The disconnected quark propagators is computed only for $t \in [22, 36]$ where we expect ground state dominance for the connected pion correlator [30].

We implement a background magnetic field as described in Ref. [17] using the background field method [27, 43, 44]. The field strength is governed by k_d as $|qe B| = (2\pi k)/(N_x N_y a^2)$ where the down quark has the smallest charge magnitude q_d. As the background magnetic field was not included at configuration generation, these configurations are *electro-quenched*; the background field exists only for the valence quarks of the hadron. However, at leading loop order in the chiral expansion, there are no electroquenching errors [33, 45].
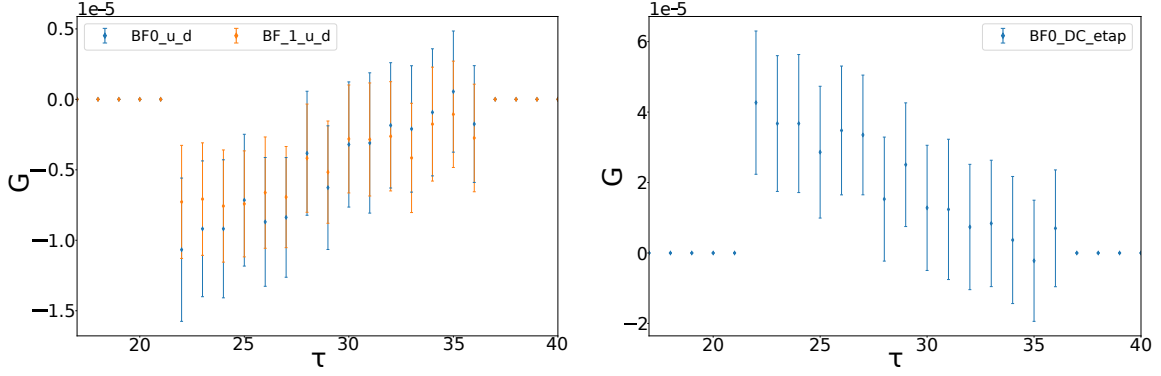
Figure 4: Left: Disconnected $\text{Tr}\left[\gamma_5 D^{ee}(x,x)\right]\text{Tr}\left[\gamma_5 U^{e'e'}(0,0)\right]$ term with no external background field and with an external background field with $k_d = -1$. **Right:** Coherent sum of disconnected terms as appropriate for the η' for zero-background field.

5. Results

Due to the inherent challenges in calculating the magnetic polarisability, particularly with stochastic propagators, we consider the change in the disconnected portion of the correlator as a function of field strength rather than the full magnetic polarisability. We compute correlation functions of both the connected pion and the full neutral pion as in Eqs. (1) and (3) respectively. Additionally we consider the single flavour operator with disconnected contributions. This enables the separation of the connected and disconnected portions of the correlation function.

The disconnected mixed-flavour $u - d$ contribution to the π_0 correlator shown in Figure 4 (left) enters with a positive sign in Eq. (4), reflecting the negative sign between the uu and dd terms in the full π_0 interpolating operator. This is in contrast to the iso-scalar η'

$$\chi_{\eta'_{is}} = \frac{1}{\sqrt{2}}\left(\overline{q}_u\,\gamma_5\,q_u + \overline{q}_d\,\gamma_5\,q_d\right),\tag{7}$$

where all the flavours of disconnected loops sum coherently and contribute with a sign opposite to the mixed flavour contributions to the pion. This acts to increase the negative slope of the correlator, hence increasing the mass of the η' [46]. This is evident in Figure 4 (right). The iso-scalar η' is considered here in order to determine the quality of our data in a *best-case* scenario where the disconnected terms sum.

There is also clearly some sensitivity to the external magnetic field in Figure 4 (left), hence encouraging the future investigation of correlated ratios of correlators at different field strengths.

6. Conclusion

In this proceedings, we have presented the first preliminary calculation exploring the effect of a background magnetic field to the disconnected loop contractions of the neutral pion. These disconnected loops are the next-leading terms in the magnetic polarisability of the neutral pion and represent an important step towards a complete *ab-initio* calculation of the magnetic polarisability of the neutral pion using lattice QCD and the background field method.

The results presented in this study highlight that these disconnected contributions are in reach of today's computing efforts and represent an important opportunity to enhance our understanding of the magnetic polarisability of the neutral pion.

Acknowledgments

References

[1] Y. Antipov et al., *Measurement of pi- meson polarizability in pion compton effect*, *Phys.Lett.B* **121** (1983) 445–448.

[2] COMPASS collaboration, C. Adolph et al., *Measurement of the charged-pion polarizability*, *Phys. Rev. Lett.* **114** (2015) 062002, [arXiv:1405.6377].

[3] L. V. Fil'kov and V. L. Kashevarov, *Dipole Polarizabilities of $\pi^{\pm}$-Mesons*, *Int. J. Mod. Phys. Conf. Ser.* **47** (2018) 1860092, [arXiv:1806.06852].

[4] M. Moinester and S. Scherer, *Compton Scattering off Pions and Electromagnetic Polarizabilities*, *Int. J. Mod. Phys.* **A34** (2019) 1930008, [arXiv:1905.05640].

[5] B. R. Holstein, *Pion polarizability and chiral symmetry*, *Comments Nucl.Part.Phys.* **19** (1990) 221–238.

[6] M. A. Moinester and V. Steiner, *Pion and kaon polarizabilities and radiative transitions*, hep-ex/9801008.

[7] S. Scherer, *Real and virtual Compton scattering at low-energies*, *Czech. J. Phys.* **49** (1999) 1307–1345, [nucl-th/9901056].

[8] J. Ahrens et al., *Measurement of the pi+ meson polarizabilities via the gamma p → gamma pi+ n reaction*, *Eur. Phys. J.* **A23** (2005) 113–127, [nucl-ex/0407011].

[9] J. Gasser, M. A. Ivanov and M. E. Sainio, *Revisiting gamma gamma —> pi+ pi- at low energies*, *Nucl. Phys.* **B745** (2006) 84–108, [hep-ph/0602234].

[10] U. Burgi, *Charged pion pair production and pion polarizabilities to two loops*, *Nucl. Phys.* **B479** (1996) 392–426, [hep-ph/9602429].

[11] V. Bernard, B. Hiller and W. Weise, *Pion Electromagnetic Polarizability and Chiral Models*, *Phys. Lett.* **B205** (1988) 16–21.

[12] L. V. Fil'kov and V. L. Kashevarov, *Compton scattering on the charged pion and the process gamma gamma —> pi0 pi0*, *Eur. Phys. J.* **A5** (1999) 285–292, [nucl-th/9810074].

[13] L. V. Fil'kov and V. L. Kashevarov, *Dipole and quadrupole polarizabilities of the pion*, in *Proceedings, 6th International Workshop on the Physics of Excited Nucleons (NSTAR 2007): Bonn, Germany, September 5-8, 2007*, 2008. arXiv:0802.0965.

[14] R. Garcia-Martin and B. Moussallam, *MO analysis of the high statistics Belle results on $\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0$ with chiral constraints*, *Eur. Phys. J.* **C70** (2010) 155–175, [arXiv:1006.5373].

[15] E. V. Luschevskaya, O. E. Solovjeva and O. V. Teryaev, *Magnetic polarizability of pion*, *Phys. Lett.* **B761** (2016) 393–398, [arXiv:1511.09316].

[16] G. S. Bali, B. B. Brandt, G. Endrodi and B. Glaessle, *Meson masses in electromagnetic fields with Wilson fermions*, *Phys. Rev.* **D97** (2018) 034505, [arXiv:1707.05600].

[17] R. Bignell, W. Kamleh and D. Leinweber, *Pion in a uniform background magnetic field with clover fermions*, *Phys. Rev. D* **100** (Dec, 2019) 114518, [arXiv:1910.14244].

[18] R. Bignell, W. Kamleh and D. Leinweber, *Pion magnetic polarisability using the background field method*, *Phys. Lett. B* **811** (2020) 135853, [arXiv:2005.10453].

[19] H. T. Ding, S. T. Li, A. Tomiya, X. D. Wang and Y. Zhang, *Chiral properties of (2+1)-flavor QCD in strong magnetic fields at zero temperature*, *Phys. Rev. D* **104** (2021) 014505, [arXiv:2008.00493].

[20] H. Niyazi, A. Alexandru, F. X. Lee and M. Lujan, *Charged pion electric polarizability from lattice QCD*, *Phys. Rev. D* **104** (2021) 014510, [arXiv:2105.06906].

[21] W. Wilcox and F. X. Lee, *Towards charged hadron polarizabilities from four-point functions in lattice QCD*, *Phys. Rev. D* **104** (2021) 034506, [arXiv:2106.02557].

[22] S.-J. Dong and K.-F. Liu, *Stochastic estimation with Z(2) noise*, *Phys. Lett. B* **328** (1994) 130–136, [hep-lat/9308015].

[23] J. Foley, K. Jimmy Juge, A. O'Cais, M. Peardon, S. M. Ryan and J.-I. Skullerud, *Practical all-to-all propagators for lattice QCD*, *Comput. Phys. Commun.* **172** (2005) 145–162, [hep-lat/0505023].

[24] J.-W. Lee and B. C. Tiburzi, *Reconciling the lattice background field method with nonrelativistic QED: Spinor case*, *Phys. Rev.* **D90** (2014) 074036, [arXiv:1407.8159].

[25] C. Itzykson and J. B. Zuber, *Quantum Field Theory.* International Series in Pure and Applied Physics. McGraw-Hill , New York, 1980.

[26] E. Chang, W. Detmold, K. Orginos, A. Parreno, M. J. Savage, B. C. Tiburzi et al., *Magnetic structure of light nuclei from lattice QCD*, *Phys. Rev.* **D92** (2015) 114502, [arXiv:1506.05518].

[27] M. Burkardt, D. B. Leinweber and X.-m. Jin, *Background field formalism in quantum systems*, *Phys. Lett.* **B385** (1996) 52–56, [hep-ph/9604450].

[28] B. C. Tiburzi and S. O. Vayl, *Method to extract charged hadron properties from lattice QCD in magnetic fields*, *Phys. Rev.* **D87** (2013) 054507, [arXiv:1210.4464].

[29] T. Primer, W. Kamleh, D. Leinweber and M. Burkardt, *Magnetic properties of the nucleon in a uniform background field*, *Phys. Rev.* **D89** (2014) 034508, [arXiv:1307.1509].

[30] R. Bignell, W. Kamleh and D. Leinweber, *Magnetic polarisability of the nucleon using a Laplacian mode projection*, *Phys. Rev. D* **101** (2020) 094502, [arXiv:2002.07915].

[31] K. G. Wilson, *Confinement of Quarks*, *Phys. Rev.* **D10** (1974) 2445–2459.

[32] B. Sheikholeslami and R. Wohlert, *Improved Continuum Limit Lattice Action for QCD with Wilson Fermions*, *Nucl. Phys.* **B259** (1985) 572.

[33] F. He, D. B. Leinweber, A. W. Thomas and P. Wang, *Chiral extrapolation of the magnetic polarizability of the neutral pion*, *Phys. Rev. D* **102** (2020) 114509, [arXiv:2010.01580].

[34] A. L. Kiratidis, W. Kamleh, D. B. Leinweber and B. J. Owen, *Lattice baryon spectroscopy with multi-particle interpolators*, *Phys. Rev. D* **91** (2015) 094509, [arXiv:1501.07667].

[35] M. G. Beckett, B. Joo, C. M. Maynard, D. Pleiter, O. Tatebe and T. Yoshie, *Building the International Lattice Data Grid*, *Comput. Phys. Commun.* **182** (2011) 1208–1214, [arXiv:0910.1692].

[36] PACS-CS collaboration, S. Aoki et al., *2+1 Flavor Lattice QCD toward the Physical Point*, *Phys. Rev.* **D79** (2009) 034503, [arXiv:0807.1661].

[37] TRINLAT collaboration, A. Ó Cais, K. J. Juge, M. J. Peardon, S. M. Ryan and J.-I. Skullerud, *Improving algorithms to compute all elements of the lattice quark propagator*, *Nucl. Phys. B Proc. Suppl.* **140** (2005) 844–849, [hep-lat/0409069].

[38] Y. Akahoshi, S. Aoki, T. Aoyama, T. Doi, T. Miyamoto and K. Sasaki, **I** = **2$\pi\pi$** *potential in the HAL QCD method with all-to-all propagators*, *PTEP* **2019** (2019) 083B02, [arXiv:1904.09549].

[39] Y. Akahoshi, S. Aoki, T. Aoyama, T. Doi, T. Miyamoto and K. Sasaki, *The HAL QCD potential in $I = 1$ $\pi\pi$ system with the ρ meson bound state*, *PTEP* **2020** (2020) 073B07, [arXiv:2004.01356].

[40] HADRON SPECTRUM collaboration, J. Bulava, R. Edwards and C. Morningstar, *Stochastic All-to-All Propagators for Baryon Correlators*, *PoS* **LATTICE2008** (2008) 124, [arXiv:0810.1469].

[41] J. Foley, C. H. Wong, J. Bulava, K. J. Juge, D. Lenkner, C. Morningstar et al., *A novel method for evaluating correlation functions in lattice hadron spectroscopy*, *PoS* **Lattice2010** (2014) 098, [arXiv:1011.0481].

[42] C. Morningstar, J. Bulava, J. Foley, K. J. Juge, D. Lenkner, M. Peardon et al., *Improved stochastic estimation of quark propagation with Laplacian Heaviside smearing in lattice QCD*, *Phys. Rev. D* **83** (2011) 114505, [arXiv:1104.3870].

[43] J. Smit and J. C. Vink, *Remnants of the Index Theorem on the Lattice*, *Nucl. Phys.* **B286** (1987) 485–508.

[44] Z. Davoudi and W. Detmold, *Implementation of general background electromagnetic fields on a periodic hypercubic lattice*, *Phys. Rev. D* **92** (2015) 074506, [arXiv:1507.01908].

[45] J. Hu, F.-J. Jiang and B. C. Tiburzi, *Pion Polarizabilities and Volume Effects in Lattice QCD*, *Phys. Rev. D* **77** (2008) 014502, [arXiv:0709.1955].

[46] J. J. Dudek, R. G. Edwards, B. Joo, M. J. Peardon, D. G. Richards and C. E. Thomas, *Isoscalar meson spectroscopy from lattice QCD*, *Phys. Rev. D* **83** (2011) 111502, [arXiv:1102.4299].