PROCEEDINGS OF SCIENCE

Equation of State of dense QCD in external magnetic field

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In this proceeding we present our first results of the study of the QCD Equation of State at non-zero baryon density and in external magnetic field. We focused on the first three non-vanishing expansion coefficients of pressure in chemical potential and their dependence on magnetic field. The study is carried out within lattice simulations with $N_f = 2+1$ dynamical quarks with physical quark masses. To overcome the sign problem, the simulations are carried out at imaginary baryon chemical potential. Our results suggest that external magnetic field considerably enhances the expansion coefficients and modifies their dependence on temperature.

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1. Introduction

Equation of State (EoS) of Quantum Chromodynamics (QCD) plays a fundamental role both from theoretical and practical points of view. From the theoretical perspective EoS contains an important information about thermal QCD phase transition. On the other hand, from the practical perspective EoS is used for hydrodynamic simulations of heavy-ion collision experiments as well as in different astrophysical applications. In such applications quark-gluon matter is subject to various external conditions like high temperature, large baryon density, strong magnetic field etc. For this reason it is important to study how EoS is affected by these external conditions.

There are a lot of phenomenological papers devoted to the calculation of the EoS under different external conditions (see, for instance, [1–3]). Important information about EoS was obtained by means of lattice QCD simulations. At zero baryon density it was studied in papers [4–8]. Extension of lattice EoS studies to non-zero baryon chemical potential was conducted in papers [9–12]. EoS for QCD in external magnetic field was studied in [13–15]. The second-order fluctuations of the baryon number, electric charge and strangeness, which are related to the EoS, in external magnetic field were studied in paper [16]. Lattice results on the QCD phase diagram with nonzero magnetic field and baryon density can be found in [17].

In this Proceeding we present our first results of the study of the QCD EoS both at non-zero baryon density and in external magnetic field. The focus is mainly done on the expansion coefficients of pressure in chemical potential and their dependence on magnetic field. The study is carried out within lattice simulations with $N_f = 2 + 1$ dynamical quarks with physical quark masses. To overcome the sign problem, the simulations are carried out at imaginary baryon chemical potential.

2. Basic definitions

The basic quantity for the Equation of State is the pressure p, which can be expressed through the partition function as

$$p = \frac{T}{V} \ln \mathcal{Z}(T, \mu_B, \mu_Q, \mu_S, eB) \,, \tag{1}$$

where V, T are spatial volume and temperature, eB is external magnetic field, μ_B, μ_Q, μ_S are chemical potentials of the conserved baryonic, electric, and strangeness charges. The chemical potentials μ_B, μ_Q and μ_S are related to the chemical potentials of individual quarks μ_u, μ_d, μ_s as follows

$$\begin{aligned} \mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q \,, \\ \mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q \,, \\ \mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S \,. \end{aligned} \tag{2}$$

In our first exploratory study we consider a simple particular combination of chemical potentials:

$$\mu_u = \mu_d = \mu \,, \qquad \mu_s = 0 \,, \tag{3}$$

what implies $\mu_B = 3\mu$, $\mu_Q = 0$, $\mu_S = \mu$. For this parameterization and for sufficiently small chemical potential μ, the EoS can be expanded in powers of $\theta = \mu/T$. We restrict our consideration by the first four non-zero terms in this expansion

$$\frac{p}{T^4} = c_0 + c_2\theta^2 + c_4\theta^4 + c_6\theta^6 \ . \tag{4}$$

It is important to notice that the coefficients c_2, c_4, c_6 are related to the fluctuations and correlations of the conserved charges. In particular, c_2 can be represented in the following way

$$c_2 = \frac{1}{2}(9\chi_2^B + 6\chi_{11}^{BS} + \chi_2^S) \ , \tag{5}$$

where we used the designations

$$\chi_{ijk}^{BQS} = \frac{\partial^{i+j+k} p/T^4}{\partial(\mu_B/T)^i \partial(\mu_Q/T)^j \partial(\mu_S/T)^k} \ . \tag{6}$$

In this Proceeding we focus on the coefficients c_2, c_4, c_6 and their dependence on external magnetic field.

3. Lattice setup

In our study we consider the partition function for $N_f = 2 + 1$ QCD with chemical potentials μ_f $(f = u, d, s)$ coupled to quark number operators, $\mathcal{Z}(T, \mu_u, \mu_d, \mu_s, eB)$, in a setup $\mu_u = \mu_d = \mu$, $\mu_s = 0$. The path integral formulation of $\mathcal{Z}(T, \mu_B, eB)$, discretized using improved rooted staggered fermions and the standard exponentiated implementation of the chemical potentials, reads

$$\mathcal{Z} = \int \mathcal{D}U e^{-\mathcal{S}_{\mathrm{YM}}} \prod_{f=u,d,s} \det\left[M_{\mathrm{st}}^f(U, \mu_f)\right]^{1/4} \ , \tag{7}$$

where

$$\mathcal{S}_{\mathrm{YM}} = -\frac{\beta}{3} \sum_{i,\mu\neq\nu} \left(\frac{5}{6} W_{i;\mu\nu}^{1\times1} - \frac{1}{12} W_{i;\mu\nu}^{1\times2}\right) \tag{8}$$

is the tree-level Symanzik improved action ($W_{i;\mu\nu}^{n\times m}$ stands for the trace of the $n \times m$ rectangular parallel transport in the μ-ν plane and starting from site i), and the staggered fermion matrix is defined as

$$M_{\mathrm{st}}^f(U, \mu_f) = am_f\delta_{i,j} + \sum_{\nu=1}^{4} \frac{\eta_{i;\nu}}{2} \left[e^{a\mu_f\delta_{\nu,4}} u_{i;\,\nu}^f U_{i;\nu}^{(2)} \delta_{i,j-\hat\nu} - e^{-a\mu_f\delta_{\nu,4}} u_{i-\hat\nu;\,\nu}^{f*} U_{i-\hat\nu;\nu}^{(2)\dagger} \delta_{i,j+\hat\nu}\right] , \tag{9}$$

where $U_{i;\nu}^{(2)}$ are two-times stout-smeared links, with isotropic smearing parameter $\rho = 0.15$ [18] and $u_{i;\,\mu}^f$ is the Abelian field phase. The Abelian transporters corresponding to a uniform magnetic field B_z directed along $\hat z$ axis are chosen in the standard way leading to the quantization condition

$$\frac{e}{3}B_z = \frac{2\pi b}{a^2 N_x N_y} \ , \tag{10}$$

where b is an integer.

Bare parameters have been set so as to stay on a line of constant physics [4, 19], with equal light quark masses, $m_u = m_d = m_l$, a physical strange-to-light mass ratio, $m_s/m_l = 28.15$, and a physical pseudo-Goldstone pion mass, $m_\pi \simeq 135$ MeV.

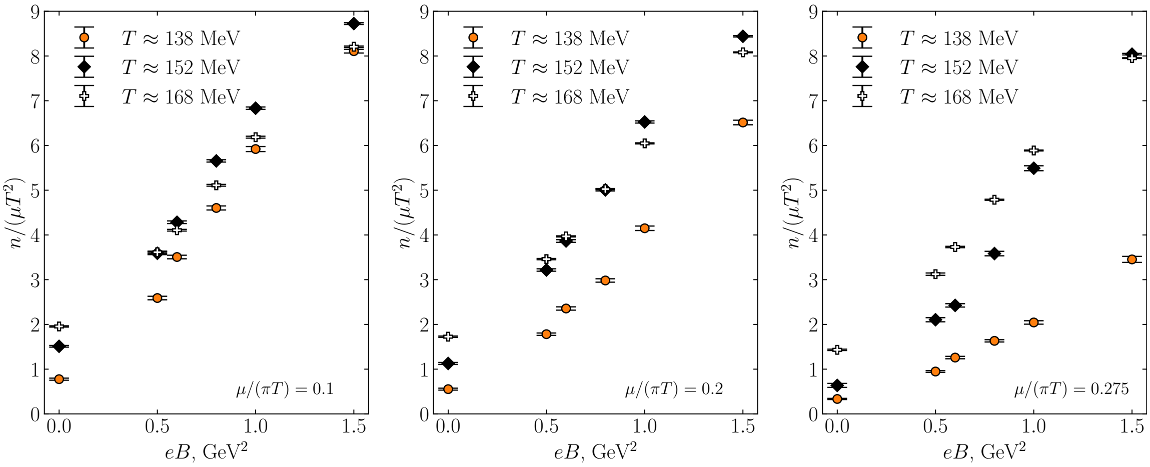

Figure 1: The ratio $n/\mu T^2$ as a function of magnetic field for various values of the chemical potential μ and temperature.

The simulations were carried out on the lattice 6×24^3 for $eB = 0,\ 0.5,\ 0.6,\ 0.8,\ 1.0,\ 1.5$ GeV^2 for a set of temperatures and chemical potentials μ. We used $O(100)$ statistically independent configurations for each set of lattice parameters used in our study.

One cannot measure directly the partition function and pressure in lattice simulations. Instead of it we measured the quark number density n and determined the coefficients c_2, c_4, c_6 of the expansion:

$$\frac{n}{T^3} = \frac{\partial p/T^4}{\partial \theta} = 2c_2\theta + 4c_4\theta^3 + 6c_6\theta^5. \tag{11}$$

4. The results of the calculations

In Fig. 1 we show the ratio $n/\mu T^2$ as a function of magnetic field for various values of the chemical potential μ and temperature. Fitting the data for the density n by formula (11) we determine the coefficients c_2, c_4, c_6.

In Fig. 2 we plot the coefficient c_2 as a function of temperature for various magnetic fields. One sees that magnetic field considerably enhances the value of the coefficient c_2, i.e. the fluctuations. Notice also that the phase transitions in QCD manifest themselves as an inflection point of the c_2. However, at sufficiently strong magnetic field this inflection point turns into a peak which shifts to the lower temperatures at larger magnetic fields. We believe that the behaviour of the peak position can be associated with the decrease of the critical temperature with magnetic field [20]. The height of the peak also increases what implies that magnetic field enhances fluctuations at the QCD phase transition point. These properties of the c_2 coefficient are in agreement with those observed in paper [16]. Notice, that in [17] we also observed a change of dense QCD properties at similar values of magnetic field $eB = eB^{\text{fl}} \sim 0.6$ GeV^2, in particular, the dependence of the width of the chiral thermal phase transition on the value of chemical potential changed direction at eB^{fl}.

In Fig. 3 and Fig. 4 we plot the coefficients c_4 and c_6 as a function of temperature for various values of magnetic field. The last leads to significant enhancement of these coefficients, similarly to the case of c_2. Moreover, at sufficiently large magnetic field the coefficients c_4 and c_6 change

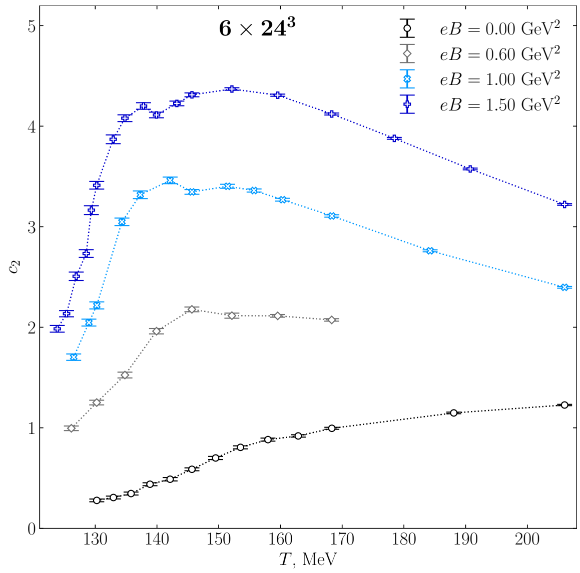

Figure 2: The coefficient c_2 as a function of temperature for various magnetic fields on the lattice 6×24^3.

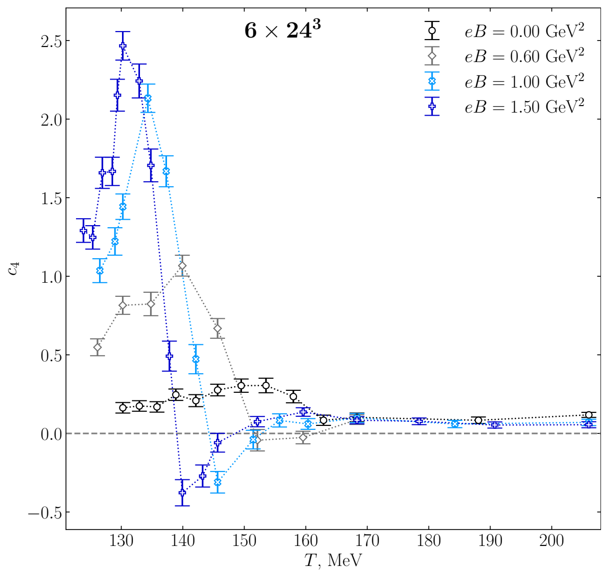

Figure 3: The coefficient c_4 as a function of temperature for various magnetic fields on the lattice 6×24^3.

their behaviour with temperature. They start to flip sign at some temperature. Notice, that these results were obtained at fixed $N_t = 6$ and they might be affected by discretization effects.

5. Discussion and conclusion

In this Proceeding we presented our first results on the study of the QCD EoS at non-zero baryon density and in external magnetic field. We focused on the three non-vanishing expansion coefficients of pressure in chemical potential and their dependence on magnetic field. The study is carried out within lattice simulations with $N_f = 2 + 1$ dynamical quarks with physical quark

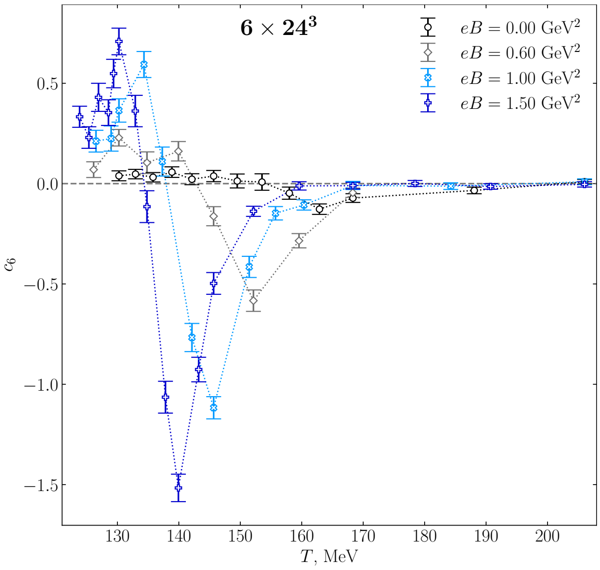

Figure 4: The coefficient c_6 as a function of temperature for various magnetic fields on the lattice 6×24^3.

masses. To overcome the sign problem, the simulations are carried out at imaginary baryon chemical potential. In our study we found that external magnetic field considerably enhances the expansion coefficients and modifies their temperature dependence. We observe, that at large magnetic fields the coefficient c_2 exhibits a peak in temperature dependence and coefficients c_4 and c_6 change sign at some temperature.

Our results might contain systematic uncertainties. We are going to reduce them in the forthcoming study.

Despite the systematic uncertainty in the results, we believe that our main conclusion remains to be true. The expansion coefficients of the EoS strongly depend on the magnetic field. This dependence might be explained by asymmetry between parallel and perpendicular to the external magnetic field directions [21], which effectively reduces dimension of the system under study.

Acknowledgments

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