PROCEEDINGS OF SCIENCE

2021 Update on ε_K with lattice QCD inputs

Jeehun Kim,[a,1] **Yong-Chull Jang,**[b,1] **Sunkyu Lee,**[a,1] **Weonjong Lee,**[a,1,*] **Jaehoon Leem,**[c,1] **Chanju Park**[a,1] **and Sungwoo Park**[d,1]

[a] *Lattice Gauge Theory Research Center, CTP, and FPRD, Department of Physics and Astronomy, Seoul National University, Seoul 08826, South Korea*

[b] *Columbia University, Department of Physics, 538 West 120th Street, New York, NY 10027, USA*

[c] *School of Physics, Korea Institute for Advanced Study (KIAS), Seoul 02455, South Korea*

[d] *Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606, USA*

E-mail: wlee@snu.ac.kr

We present recent updates for ε_K determined directly from the standard model (SM) with lattice QCD inputs such as $\hat{B}_K$, $|V_{cb}|$, $|V_{us}|$, ξ_0, ξ_2, $\xi_{\rm LD}$, f_K, and m_c. We find that the standard model with exclusive $|V_{cb}|$ and other lattice QCD inputs describes only 66% of the experimental value of $|\varepsilon_K|$ and does not explain its remaining 34%, which leads to a strong tension in $|\varepsilon_K|$ at the $4.5\sigma \sim 3.7\sigma$ level between the SM theory and experiment. We also find that this tension disappears when we use the inclusive value of $|V_{cb}|$ obtained using the heavy quark expansion based on the QCD sum rule approach.

The 38th International Symposium on Lattice Field Theory, LATTICE2021 26th-30th July, 2021 Zoom/Gather@Massachusetts Institute of Technology

[1]The SWME collaboration

[*]Speaker

PoS(LATTICE2021)078

1. Introduction

This paper is an update of our previous papers [1–6]. Here, we present recent progress in determination of $|\varepsilon_K|$ with updated inputs from lattice QCD.

Here, we follow the color convention of our previous papers [1–6] in Tables 1–7. We use the red color for the new input data which is used to evaluate ε_K. We use the blue color for the new input data which is not used for some obvious reason.

2. Input parameter ξ_0

The absorptive part of long distance effects on ε_K is parametrized into ξ_0.

$$\xi_0 = \frac{\text{Im}\,A_0}{\text{Re}\,A_0}, \qquad \xi_2 = \frac{\text{Im}\,A_2}{\text{Re}\,A_2}, \qquad \text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \frac{\omega}{\sqrt{2}|\varepsilon_K|}(\xi_2 - \xi_0)\,. \tag{1}$$

There are two independent methods to determine ξ_0 in lattice QCD: the indirect and direct methods. The indirect method is to determine ξ_0 using Eq. (1) with lattice QCD results for ξ_2 combined with experimental results for ε'/ε, ε_K, and ω. The direct method is to determine ξ_0 directly using the lattice QCD results for $\text{Im}\,A_0$, combined with experimental results for $\text{Re}\,A_0$.

In Table 1 (a), we summarize experimental results for $\text{Re}\,A_0$ and $\text{Re}\,A_2$. In Table 1 (b), we summarize lattice results for $\text{Im}\,A_0$ and $\text{Im}\,A_2$ calculated by RBC-UKQCD. In Table 1 (c), we summarize results for ξ_0 which is obtained using results in Table 1 (a) and (b).

Here, we use results of the indirect method for ξ_0 to evaluate ε_K, since its systematic and statistical errors are much smaller than those of the direct method.

3. Input parameters: $|V_{cb}|$

In Table 2 (a) and (b), we present recent updates for exclusive $|V_{cb}|$ and inclusive $|V_{cb}|$ respectively. In Table 2 (a), we summarize results for exclusive $|V_{cb}|$ obtained by various groups: HFLAV, BELLE, BABAR, FNAL/MILC, LHCb, and FLAG. Results from LHCb comes from analysis on $B_s \to D_s^* \ell \bar{\nu}$ decays which are not available in the B-factories. Since the decays modes of B_s have poor statistics, the final results have overall uncertainty much larger than those of B by an order of magnitude. Hence, we drop out results of LHCb in this article without loss of fairness. The rest of results for exclusive $|V_{cb}|$ have comparable size of errors and are consistent with one another within 1.0σ statistical uncertainty. In addition, it is nice to observe all the results be consistent between the CLN and BGL analysis, after all the boisterous debates [2, 11].

In Table 2 (b), we present recent results for inclusive $|V_{cb}|$. The HFLAV group has reported the same results for inclusive $|V_{cb}|$ in 2021 as in 2017, while FLAG reported updated results.

4. Input parameters: Wolfenstein parameters

In Table 3 (a), we present the Wolfenstein parameters on the market. As explained in Ref. [2, 6], we use the results of angle-only-fit (AOF) in Table 3 (a) in order to avoid unwanted correlation between $(\varepsilon_K, |V_{cb}|)$, and $(\bar{\rho}, \bar{\eta})$. We determine λ from $|V_{us}|$ which is obtained from the $K_{\ell 2}$ and $K_{\ell 3}$ decays using lattice QCD inputs for form factors and decay constants. We determine the A parameter from $|V_{cb}|$.

parameter	method	value	Ref.	source
Re A_0	exp	$3.3201(18) \times 10^{-7}$ GeV	[7, 8]	NA
Re A_2	exp	$1.4787(31) \times 10^{-8}$ GeV	[7]	NA
ω	exp	$0.04454(12)$	[7]	NA
$\lvert\varepsilon_K\rvert$	exp	$2.228(11) \times 10^{-3}$	[9]	PDG-2021
Re $(\varepsilon'/\varepsilon)$	exp	$1.66(23) \times 10^{-3}$	[9]	PDG-2021

(a) Experimental results for ω, Re A_0 and Re A_2.

parameter	method	value (GeV)	Ref.	source
Im A_0	lattice	$-6.98(62)(144) \times 10^{-11}$	[10]	RBC-UK-2020 p4t1
Im A_2	lattice	$-8.34(103) \times 10^{-13}$	[10]	RBC-UK-2020 p31e90

(b) Results for Im A_0, and Im A_2 in lattice QCD.

parameter	method	value	ref	source
ξ_0	indirect	$-1.738(177) \times 10^{-4}$	[10]	SWME
ξ_0	direct	$-2.102(472) \times 10^{-4}$	[10]	SWME

(c) Results for ξ_0 obtained using the direct and indirect methods in lattice QCD.

Table 1: Results for ξ_0. The p4t1 is an abbreviation for Table 1 in page 4. The p31e90 is an abbreviation for Eq. (90) in page 31.

5. Input parameters: $\hat{B}_K$, ξ_{LD}, and others

In FLAG 2021 [17], they report lattice QCD results for $\hat{B}_K$ with $N_f = 2$, $N_f = 2+1$, and $N_f = 2+1+1$. Here, we use the results for $\hat{B}_K$ with $N_f = 2+1$, which is obtained by taking an average over the four data points from BMW 11, Laiho 11, RBC-UKQCD 14, and SWME 15 in Table 4 (a).

The dispersive long distance (LD) effect is defined as

$$\xi_{\text{LD}} = \frac{m'_{\text{LD}}}{\sqrt{2}\Delta M_K}\,, \qquad m'_{\text{LD}} = -\text{Im}\left[\mathcal{P}\sum_C \frac{\langle \overline{K}^0|H_{\text{w}}|C\rangle\langle C|H_{\text{w}}|K^0\rangle}{m_{K^0} - E_C}\right] \tag{2}$$

As explained in Refs. [2], there are two independent methods to estimate ξ_{LD}: one is the BGI estimate [27], and the other is the RBC-UKQCD estimate [28, 29]. The BGI method is to estimate the size of ξ_{LD} using chiral perturbation theory as follows,

$$\xi_{\text{LD}} = -0.4(3) \times \frac{\xi_0}{\sqrt{2}} \tag{3}$$

The RBC-UKQCD method is to estimate the size of ξ_{LD} as follows,

$$\xi_{\text{LD}} = (0 \pm 1.6)\%. \tag{4}$$

channel	value	method	ref	source
ex-comb	39.13(59)	comb	[12]	HFLAV-2017
ex-comb	39.25(56)	CLN	[13] p115e223	HFLAV-2021
$B \to D^* \ell \bar{\nu}$	39.0(2)(6)(6)	CLN	[14] erratum p4	BELLE-2021
$B \to D^* \ell \bar{\nu}$	38.9(3)(7)(6)	BGL	[14] erratum p4	BELLE 2021
$B \to D^* \ell \bar{\nu}$	38.40(84)	CLN	[15] p5t2	BABAR-2019
$B \to D^* \ell \bar{\nu}$	38.36(90)	BGL	[15] p5t1	BABAR-2019
$B \to D^* \ell \bar{\nu}$	38.57(78)	BGL	[11] p27e5.22, p34e6.1	FNAL/MILC-2021
$B_s \to D_s^* \ell \bar{\nu}$	41.4(6)(9)(12)	CLN	[16] p15	LHCb-2020
$B_s \to D_s^* \ell \bar{\nu}$	42.3(8)(9)(12)	BGL	[16] p15	LHCb-2020
ex-comb	39.48(68)	comb	[17] p191	FLAG-2021

(a) Exclusive $|V_{cb}|$ in units of 10^{-3}.

channel	value	ref	source
kinetic scheme	42.19(78)	[12, 13]	HFLAV-2021
kinetic scheme	42.00(64)	[17] p192	FLAG-2021
1S scheme	41.98(45)	[12, 13]	HFLAV-2021

(b) Inclusive $|V_{cb}|$ in units of 10^{-3}.

Table 2: Results for (a) exclusive $|V_{cb}|$ and (b) inclusive $|V_{cb}|$. The same notation as in Table 1 is used.

WP	CKMfitter		UTfit		AOF	
λ	0.22475(25)	[18]	0.22500(100)	[19]	0.2249(5)	[17] p80
$\bar{\rho}$	0.1577(96)	[18]	0.148(13)	[19]	0.146(22)	[20]
$\bar{\eta}$	0.3493(95)	[18]	0.348(10)	[19]	0.333(16)	[20]

(a) Wolfenstein parameters

Input	Value	Ref.
η_{cc}	1.72(27)	[3]
η_{tt}	0.5765(65)	[21]
η_{ct}	0.496(47)	[22]

(b) η_{ij}

Table 3: (a) Wolfenstein parameters and (b) QCD corrections: η_{ij} with $i, j = c, t$.

Here, we use both methods to estimate the size of ξ_{LD}.

In Table 3 (b), we present higher order QCD corrections: η_{ij} with $i, j = t, c$. A new approach using $u - t$ unitarity instead of $c - t$ unitarity appeared in Ref. [30], which is supposed to have a better convergence with respect to the charm quark mass. But we have not incorporated this into our analysis yet, which we will do in near future.

In Table 4 (b), we present other input parameters needed to evaluate ε_K.

6. Quark mass

In Table 5, we present the charm quark mass $m_c(m_c)$ and top quark mass $m_t(m_t)$. From FLAG 2021 [17], we take the results for $m_c(m_c)$ with $N_f = 2+1$, since there is some inconsistency among

Collaboration	Ref.	$\hat{B}_K$
SWME 15	[23]	0.735(5)(36)
RBC/UKQCD 14	[24]	0.7499(24)(150)
Laiho 11	[25]	0.7628(38)(205)
BMW 11	[26]	0.7727(81)(84)
FLAG 2021	[17]	0.7625(97)

(a) $\hat{B}_K$

Input	Value	Ref.
G_F	$1.1663787(6) \times 10^{-5}$ GeV^{-2}	PDG-21 [9]
M_W	80.379(12) GeV	PDG-21 [9]
θ	$43.52(5)^\circ$	PDG-21 [9]
m_{K^0}	497.611(13) MeV	PDG-21 [9]
ΔM_K	$3.484(6) \times 10^{-12}$ MeV	PDG-21 [9]
F_K	155.7(3) MeV	FLAG-21 [17]

(b) Other parameters

Table 4: (a) Results for $\hat{B}_K$ and (b) other input parameters.

Collaboration	N_f	$m_c(m_c)$	Ref.
FLAG 2021	$2+1$	1.275(5)	[17]
FLAG 2021	$2+1+1$	1.278(13)	[17]

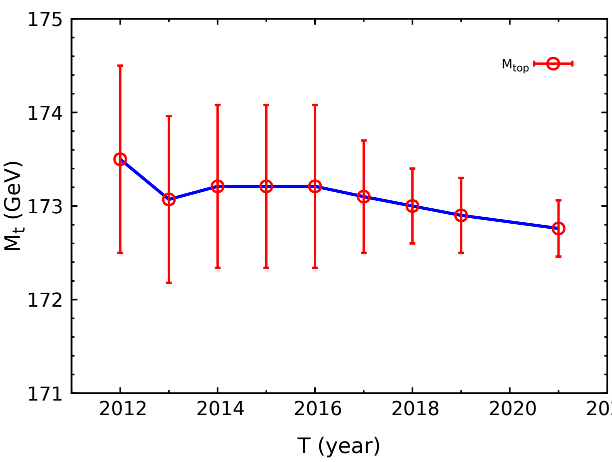
(a) $m_c(m_c)$ [GeV]

Collaboration	M_t	$m_t(m_t)$	Ref.
PDG 2019	172.9(4)	163.08(38)(17)	[31]
PDG 2021	172.76(30)	162.96(28)(17)	[9]

(b) $m_t(m_t)$ [GeV]

Table 5: Results for (a) charm quark mass and (b) top quark mass.

the lattice results of various groups with $N_f = 2+1+1$. For the top quark mass, we use the PDG 2021 results for the pole mass M_t to obtain $m_t(m_t)$.



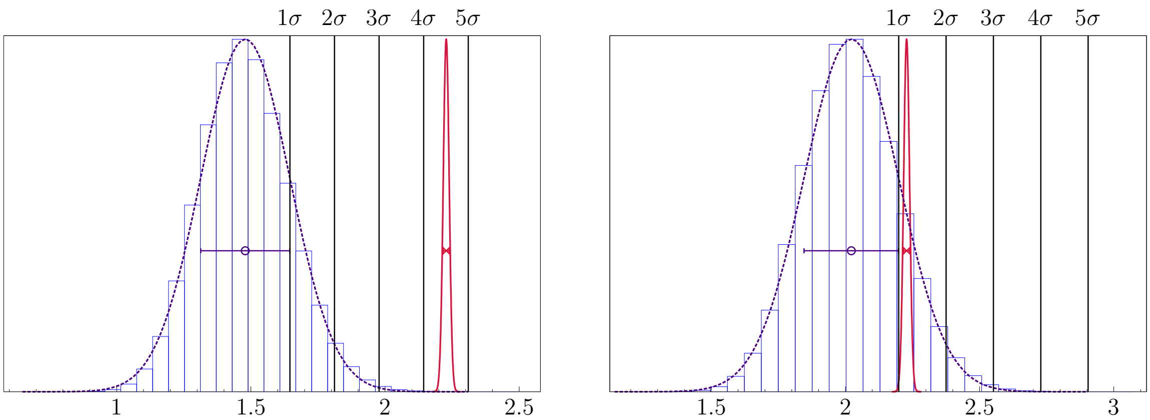
(a) History of M_t (top quark pole mass).

source	error (%)	memo
$\lvert V_{cb} \rvert$	44.7	Exclusive
$\bar{\eta}$	21.4	AOF
η_{ct}	18.1	$c-t$ Box
η_{cc}	7.7	$c-c$ Box
$\bar{\rho}$	3.2	$c-c$ Box
ξ_{LD}	1.9	RBC-UKQCD
$\hat{B}_K$	1.5	FLAG
⋮	⋮	⋮

(b) Error budget for $\lvert\varepsilon_K\rvert^{\text{SM}}$

Table 6: (a) M_t history (b) error budget.

In Table 6 Fig. (a), we present the time evolution of top pole mass M_t. Here we find that the average value drifts downward a little bit and the error shrinks fast as time goes on, since LHC has been accumulating high statistics on M_t. The data for 2020 is dropped out intentionally to reflect on the absence of Lattice 2020 due to COVID-19, even though it is available.



(a) Exclusive $|V_{cb}|$ (FNAL/MILC 2021, BGL) **(b)** Inclusive $|V_{cb}|$ (HFLAV 2021, 1S scheme)

Figure 1: $|\varepsilon_K|$ with (a) exclusive $|V_{cb}|$ (left) and (b) inclusive $|V_{cb}|$ (right) in units of 1.0×10^{-3}.

7. Results for ε_K

In Fig. 1, we show results for $|\varepsilon_K|$ evaluated directly from the standard model (SM) with lattice QCD inputs given in the previous sections. In Fig. 1 (a), the blue curve represents the theoretical evaluation of $|\varepsilon_K|$ obtained using the FLAG-2021 results for $\hat{B}_K$, AOF for Wolfenstein parameters, the [FNAL/MILC 2021, BGL] results for exclusive $|V_{cb}|$, results for ξ_0 with the indirect method, and the RBC-UKQCD estimate for $\xi_{\rm LD}$. The red curve in Fig. 1 represents the experimental results for $|\varepsilon_K|$. In Fig. 1 (b), the blue curve represents the same as in Fig. 1 (a) except for using the 1S scheme results for the inclusive $|V_{cb}|$.

Our results for $|\varepsilon_K|^{\rm SM}$ and $\Delta\varepsilon_K$ are summarized in Table 7. Here, the superscript $^{\rm SM}$ represents the theoretical expectation value of $|\varepsilon_K|$ obtained directly from the SM. The superscript $^{\rm Exp}$ represents the experimental value of $|\varepsilon_K| = 2.228(11) \times 10^{-3}$. Results in Table 7 (a) are obtained using the RBC-UKQCD estimate for $\xi_{\rm LD}$, and those in Table 7 (b) are obtained using the BGI estimate for $\xi_{\rm LD}$. In Table 7 (a), we find that the theoretical expectation values of $|\varepsilon_K|^{\rm SM}$ with lattice QCD inputs (with exclusive $|V_{cb}|$) has $4.54\sigma \sim 3.68\sigma$ tension with the experimental value of $|\varepsilon_K|^{\rm Exp}$, while there is no tension with inclusive $|V_{cb}|$ (obtained using heavy quark expansion and QCD sum rules).

In Fig. 2 (a), we show the time evolution of $\Delta\varepsilon_K$ starting from 2012 to 2022. In 2012, $\Delta\varepsilon_K$ was 2.5σ, but now it is 4.5σ with exclusive $|V_{cb}|$ (FNAL/MILC 2021, BGL). In Fig. 2 (b), we show the time evolution of the average $\Delta\varepsilon_K$ and the error $\sigma_{\Delta\varepsilon_K}$ during the period of 2012–2022.

At present, we find that the largest error ($\approx 45\%$) in $|\varepsilon_K|^{\rm SM}$ comes from $|V_{cb}|$.[1] Hence, it is essential to reduce the error in $|V_{cb}|$ significantly. To achieve this goal, there is an on-going project to extract exclusive $|V_{cb}|$ using the Oktay-Kronfeld (OK) action for the heavy quarks to calculate the form factors for $\bar{B} \to D^{(*)}\ell\bar{\nu}$ decays [32–37].

A large portion of interesting results for $|\varepsilon_K|^{\rm SM}$ and $\Delta\varepsilon_K$ could not be presented in Table 7 and in Fig. 2 due to lack of space: for example, results for $|\varepsilon_K|^{\rm SM}$ obtained using exclusive $|V_{cb}|$ (FLAG 2021), results for $|\varepsilon_K|^{\rm SM}$ obtained using ξ_0 determined by the direct method, and so on. We plan to report them collectively in Ref. [38].

[1]Refer to Table 6 (b) for more details.

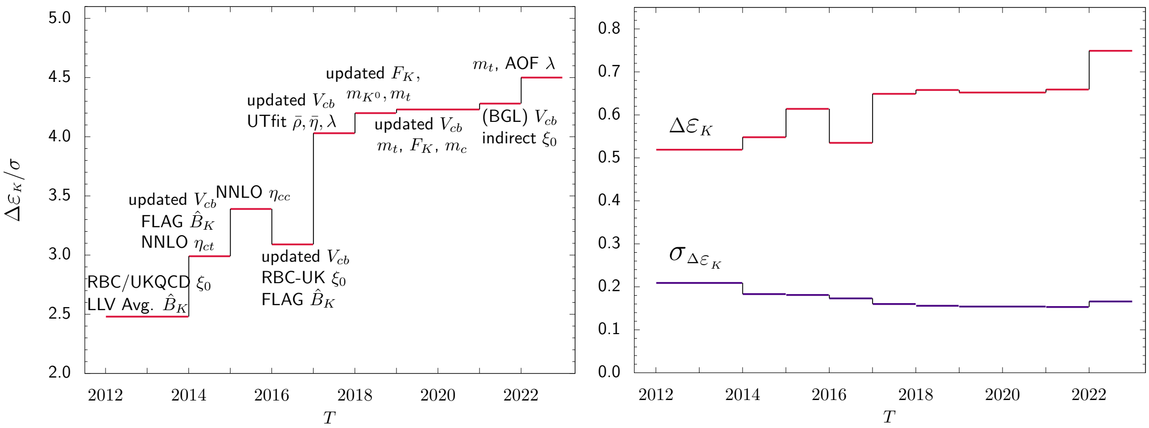
$\lvert V_{cb}\rvert$	method	reference	$\lvert\varepsilon_K\rvert^{\text{SM}}$	$\Delta\varepsilon_K$
exclusive	CLN	BELLE 2021	1.542 ± 0.181	3.79σ
exclusive	BGL	BELLE 2021	1.528 ± 0.190	3.68σ
exclusive	CLN	BABAR 2019	1.456 ± 0.170	4.54σ
exclusive	BGL	BABAR 2019	1.451 ± 0.176	4.42σ
exclusive	CLN	HFLAV 2021	1.577 ± 0.155	4.21σ
exclusive	BGL	FNAL/MILC 2021	1.479 ± 0.166	4.50σ
inclusive	kinetic	FLAG 2021	2.027 ± 0.195	1.03σ
inclusive	1S	HFLAV 2021	2.022 ± 0.176	1.17σ

(a) RBC-UKQCD estimate for ξ_{LD}

$\lvert V_{cb}\rvert$	method	reference	$\lvert\varepsilon_K\rvert^{\text{SM}}$	$\Delta\varepsilon_K$
exclusive	CLN	HFLAV 2021	1.625 ± 0.157	3.85σ
exclusive	BGL	FNAL/MILC 2021	1.527 ± 0.169	4.15σ

(b) BGI estimate for ξ_{LD}

Table 7: $\lvert\varepsilon_K\rvert$ in units of 1.0×10^{-3}, and $\Delta\varepsilon_K = \lvert\varepsilon_K\rvert^{\text{Exp}} - \lvert\varepsilon_K\rvert^{\text{SM}}$.



(a) Time evolution of $\Delta\varepsilon_K/\sigma$ **(b)** Time evolution of the average and error of $\Delta\varepsilon_K$

Figure 2: Time history of (a) $\Delta\varepsilon_K/\sigma$, and (b) $\Delta\varepsilon_K$ and $\sigma_{\Delta\varepsilon_K}$.

Acknowledgments

We thank Jon Bailey, Stephen Sharpe, and Rajan Gupta for helpful discussion. The research of W. Lee is supported by the Mid-Career Research Program (Grant No. NRF-2019R1A2C2085685) of the NRF grant funded by the Korean government (MOE). This work was supported by Seoul National University Research Grant in 2019. W. Lee would like to acknowledge the support from the KISTI supercomputing center through the strategic support program for the supercomputing application research (No. KSC-2018-CHA-0043, KSC-2020-CHA-0001). Computations were carried out in part on the DAVID cluster at Seoul National University.

References

[1] **LANL-SWME** Collaboration, J. Kim, S. Lee, W. Lee, Y.-C. Jang, J. Leem, and S. Park, *2019 update of ε_K with lattice QCD inputs*, *PoS* **LATTICE2019** (2019) 029, [1912.03024].

[2] J. A. Bailey *et al.*, *Updated evaluation of ϵ_K in the standard model with lattice QCD inputs*, *Phys. Rev.* **D98** (2018) 094505, [1808.09657].

[3] J. A. Bailey, Y.-C. Jang, W. Lee, and S. Park, *Standard Model evaluation of ε_K using lattice QCD inputs for $\hat{B}_K$ and V_{cb}*, *Phys. Rev.* **D92** (2015) 034510, [1503.05388].

[4] J. A. Bailey *et al.*, *2018 Update on ε_K with lattice QCD inputs*, *PoS* **LATTICE2018** (2018) 284, [1810.09761].

[5] Y.-C. Jang, W. Lee, S. Lee, and J. Leem, *Update on ε_K with lattice QCD inputs*, *EPJ Web Conf.* **175** (2018) 14015, [1710.06614].

[6] J. A. Bailey, Y.-C. Jang, W. Lee, and S. Park, *Determination of ε_K using lattice QCD inputs*, *PoS* **LATTICE2015** (2015) 348, [1511.00969].

[7] T. Blum *et al.*, *$K \to \pi\pi$ $\Delta I = 3/2$ decay amplitude in the continuum limit*, *Phys. Rev.* **D91** (2015) 074502, [1502.00263].

[8] Z. Bai *et al.*, *Standard Model Prediction for Direct CP Violation in $K \to \pi\pi$ Decay*, *Phys. Rev. Lett.* **115** (2015) 212001, [1505.07863].

[9] **Particle Data Group** Collaboration, P. Zyla *et al.*, *Review of Particle Physics*, *PTEP* **2020** (2020), no. 8 083C01.

[10] **RBC, UKQCD** Collaboration, R. Abbott *et al.*, *Direct CP violation and the $\Delta I = 1/2$ rule in $K \to \pi\pi$ decay from the standard model*, *Phys. Rev. D* **102** (2020), no. 5 054509, [2004.09440].

[11] **Fermilab Lattice, MILC** Collaboration, A. Bazavov *et al.*, *Semileptonic form factors for $B \to D^*\ell\nu$ at nonzero recoil from 2 + 1-flavor lattice QCD*, 2105.14019.

[12] Y. Amhis *et al.*, *Averages of b-hadron, c-hadron, and τ-lepton properties as of summer 2016*, *Eur. Phys. J.* **C77** (2017) 895, [1612.07233].

[13] **HFLAV** Collaboration, Y. S. Amhis *et al.*, *Averages of b-hadron, c-hadron, and τ-lepton properties as of 2018*, *Eur. Phys. J. C* **81** (2021), no. 3 226, [1909.12524].

[14] **Belle** Collaboration, E. Waheed *et al.*, *Measurement of the CKM matrix element $|V_{cb}|$ from $B^0 \to D^{*-}\ell^+\nu_\ell$ at Belle*, *Phys. Rev. D* **100** (2019), no. 5 052007, [1809.03290]. [Erratum: Phys.Rev.D 103, 079901 (2021)].

[15] **BaBar** Collaboration, J. P. Lees *et al.*, *Extraction of form Factors from a Four-Dimensional Angular Analysis of $\overline{B} \to D^*\ell^-\overline{\nu}_\ell$*, *Phys. Rev. Lett.* **123** (2019), no. 9 091801, [1903.10002].

[16] **LHCb** Collaboration, R. Aaij *et al.*, *Measurement of* $|V_{cb}|$ *with* $B_s^0 \to D_s^{(*)-}\mu^+\nu_\mu$ *decays*, *Phys. Rev. D* **101** (2020), no. 7 072004, [2001.03225].

[17] Y. Aoki *et al.*, *FLAG Review 2021*, 2111.09849.

[18] J. Charles *et al.*, *CP violation and the CKM matrix: Assessing the impact of the asymmetric B factories*, *Eur.Phys.J.* **C41** (2005) 1–131, [hep-ph/0406184]. updated results and plots available at: http://ckmfitter.in2p3.fr.

[19] M. Bona *et al.*, *The Unitarity Triangle Fit in the Standard Model and Hadronic Parameters from Lattice QCD: A Reappraisal after the Measurements of Delta m(s) and BR(B —> tau nu(tau))*, *JHEP* **10** (2006) 081, [hep-ph/0606167]. Standard Model fit results: Summer 2016 (ICHEP 2016): http://www.utfit.org.

[20] G. Martinelli *et al.*, "Private communication with UTfit." http://www.utfit.org/UTfit/, 2017.

[21] A. J. Buras and D. Guadagnoli *Phys.Rev.* **D78** (2008) 033005, [0805.3887].

[22] J. Brod and M. Gorbahn, *ϵ_K at Next-to-Next-to-Leading Order: The Charm-Top-Quark Contribution*, *Phys.Rev.* **D82** (2010) 094026, [1007.0684].

[23] B. J. Choi *et al.*, *Kaon BSM B-parameters using improved staggered fermions from $N_f = 2 + 1$ unquenched QCD*, *Phys. Rev.* **D93** (2016) 014511, [1509.00592].

[24] T. Blum *et al.*, *Domain wall QCD with physical quark masses*, *Phys. Rev.* **D93** (2016) 074505, [1411.7017].

[25] J. Laiho and R. S. Van de Water, *Pseudoscalar decay constants, light-quark masses, and B_K from mixed-action lattice QCD*, *PoS* **LATTICE2011** (2011) 293, [1112.4861].

[26] S. Durr *et al.*, *Precision computation of the kaon bag parameter*, *Phys. Lett.* **B705** (2011) 477–481, [1106.3230].

[27] A. J. Buras, D. Guadagnoli, and G. Isidori, *On ϵ_K beyond lowest order in the Operator Product Expansion*, *Phys.Lett.* **B688** (2010) 309–313, [1002.3612].

[28] N. Christ *et al.*, *Long distance contribution to the KL-KS mass difference*, *Phys.Rev.* **D88** (2013) 014508, [1212.5931].

[29] N. Christ *et al.*, *Calculating the $K_L - K_S$ mass difference and ϵ_K to sub-percent accuracy*, *PoS* **LATTICE2013** (2014) 397, [1402.2577].

[30] J. Brod, M. Gorbahn, and E. Stamou, *Standard-Model Prediction of ϵ_K with Manifest Quark-Mixing Unitarity*, *Phys. Rev. Lett.* **125** (2020), no. 17 171803, [1911.06822].

[31] M. Tanabashi *et al.*, *Review of Particle Physics*, *Phys. Rev.* **D98** (2018) 030001. http://pdg.lbl.gov/2019/.

[32] B. J. Choi *et al.*, *Leptonic decays of $B_{(s)}$ and $D_{(s)}$ using the OK action*, *PoS* **LATTICE2019** (2019) 050.

[33] S. Jwa *et al.*, *Semileptonic decays $B_{(s)} \to D^{(*)}_{(s)} \ell\nu$ form factors using the OK action*, *PoS* **LATTICE2019** (2019) 056.

[34] T. Bhattacharya *et al.*, *Update on $B \to D^*\ell\nu$ form factor at zero-recoil using the Oktay-Kronfeld action*, *PoS* **LATTICE2018** (2018) 283, [1812.07675].

[35] J. A. Bailey *et al.*, *Calculation of $\bar{B} \to D^*\ell\bar{\nu}$ form factor at zero recoil using the Oktay-Kronfeld action*, *EPJ Web Conf.* **175** (2018) 13012, [1711.01786].

[36] J. Bailey, Y.-C. Jang, W. Lee, and J. Leem, *Improvement of heavy-heavy current for calculation of $\bar{B} \to D^{(*)}\ell\bar{\nu}$ form factors using Oktay-Kronfeld heavy quarks*, *EPJ Web Conf.* **175** (2018) 14010, [1711.01777].

[37] **LANL-SWME** Collaboration, J. A. Bailey, Y.-C. Jang, S. Lee, W. Lee, and J. Leem, *Improvement of heavy-heavy and heavy-light currents with the Oktay-Kronfeld action*, 2001.05590.

[38] **SWME** Collaboration, J. Bailey, J. Kim, S. Lee, W. Lee, Y.-C. Jang, J. Leem, S. Park, *et al.*, "in preparation."