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New methods to reconstruct $X_{\max}$ and the energy of gamma-ray air showers with high accuracy in large wide-field observatories

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A new method to reconstruct the slant depth of the maximum of the longitudinal profile ($X_{\max}$) of high-energy showers initiated by gamma-rays as well as their energy (E_0) are presented. The method was developed for gamma rays with energies ranging from a few hundred GeV to around 10 TeV. An estimator of $X_{\max}$ is obtained, event-by-event, from its correlation with the distribution of the particles' arrival time at the ground, or the signal at the ground for lower energies. An estimator of E_0 is obtained, event-by-event, using a parametrization that has as inputs the total measured energy at the ground, the amount of energy contained in a region near to the shower core and the estimated $X_{\max}$. Resolutions about 40 (20) g/cm2 and about 30(20)% for, respectively, $X_{\max}$ and E_0 at 1 (10) TeV energies are obtained, considering vertical showers. The obtained results are auspicious and can lead to the opening of new physics avenues for large wide field-of-view gamma-ray observatories.

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1. Introduction

Ground-based gamma-ray observatories sample the particles (mainly electrons and photons) arriving at the ground level. Using their time and position distributions can determine, with reasonable accuracy, the shower core position and the direction of the primary gamma-ray. The determination of the shower energy has, however, a large uncertainty. Indeed, energy resolutions of the order of several tens of percent are often quoted. For instance, in [1] using a Likelihood fit with Monte Carlo template distributions, the energy reconstruction resolution at 10 TeV is $\sim$ 50%, and the HAWC collaboration has recently reported an improved energy resolution of 40% at the same energy using a neural network analysis [2].

One of the main limiting factors in the reconstruction of the primary energy for ground arrays is the uncertainty of the position of the first interaction in the atmosphere and the shower development stage. In this work, we propose a new method to improve the determination of the primary energy by using the electromagnetic energy at the ground, S_{em}, and estimate the shower development stage using either S_{em} or at higher energies the curvature of the shower front. We also show that it is necessary to correct for anomalous shower development and that such can be achieved using the ratio of electromagnetic energy near the shower core (up to 20 meters) with respect to S_{em}.

All the present results were obtained using CORSIKA (version 7.5600) [3] to simulate vertical gamma-ray showers assuming an observatory at an altitude of 5200 m a.s.l. The primaries, with energies between 250 GeV and 15 TeV, were injected following an energy spectrum of E^{-1}, which guarantees high enough statistics over the whole simulated energy range. It was used as hadronic interaction model for low and high energy, FLUKA and QGSJET-II.04, respectively, although the choice of these models has little impact on the simulation of electromagnetic showers. The total energy of electromagnetic shower particles was recorded at the observation level and histogrammed in radial bins of 4 meters. This would mimic a calorimeter detector compact array, where the station unit covers an area of $\sim 12\,\mathrm{m}^2$. A full study including different zenith angles, detailed simulations of realistic detectors, a wider energy range, and its application to hadronic induced showers is out of the scope of this work. The main focus is the explanation of the newly proposed method to achieve an accurate energy reconstruction of gamma-ray air showers between several hundreds of GeV and a few TeV.

2. Energy distribution at the ground

The energy distribution at the ground as a function of the distance r to the core position and its cumulative function, $F(r)$, are shown in figure 1 (left) for an event with $S_{\mathrm{em}} = 96.5$ GeV, $E_0 = 1165.9$ GeV and $X_{\mathrm{max}} = 334\ \mathrm{g\,cm^{-2}}$.

The strategy followed in this article is different from one usually employed by shower array experiments [2, 4]. Instead of using the shower particle density at the ground at some optimized distance from the shower core, the aim is to characterize the shower development through two variables, that will be then used to predict, event by event, the calibration factor between the gamma-ray energy (E_0) and the electromagnetic energy arriving at the ground (S_{em}).

The depth of the shower maximum, X_{max}, taken in this work as proxy to the shower stage and whose estimator will be discussed in the next section, is naturally one of these variables. The

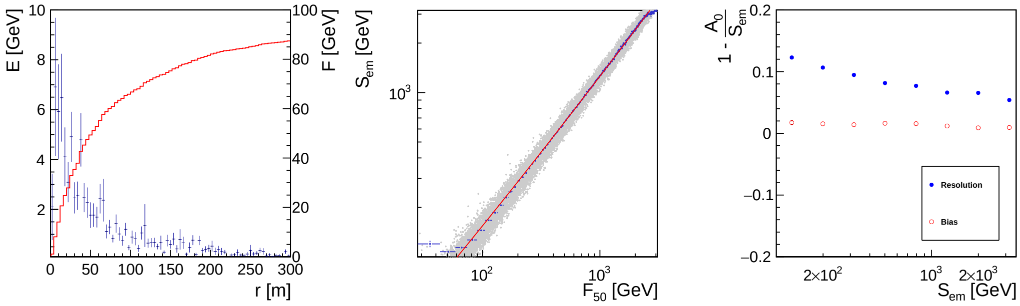

Figure 1: (left) The energy distribution at the ground for one event with $S_{\text{em}} = 96.5$ GeV, $E_0 = 1165.9$ GeV and $X_{\max} = 334$ g cm^{-2}. Also shown the respective cumulative function $F(r)$. (middle) Correlation of the electromagnetic energy deposited at the ground, S_{em}, with F_{50}, the energy deposited at the ground within a radius of 50 m from the core position (see text for the simulation details). (right) Bias and the relative resolution of the estimator A_0 of the energy deposited at the ground as a function of the energy deposited at the ground, S_{em} (see text for the simulation details).

other, f_{20}, is defined as the ratio between the energy at the ground collected at a distance less than 20 m from the shower core and the total energy at the ground. This parameter will be the main responsible for the improvement of the energy resolution reached in this article. For a given E_0 and $X_{\max}$, the development of the shower between the $X_{\max}$ region and the ground level will strongly determine f_{20}.

An estimator of S_{em}, designated as A_0, may be obtained using the correlation between S_{em} and $F_{r_0} \equiv F(r_0)$, being r_0 a reference distance. In any case, r_0 should be greater than 20 m to ensure a good correlation, and lower than some tens of meters to ensure a high number of events where the event footprint, with $r < r_0$, is fully contained within the compact array region of the observatory. For the purpose of this article, in the following $r_0 = 50$ m is used.

The correlation between S_{em} and F_{50} is shown in figure 1 and A_0 is parametrized as $A_0 = F_{50} + G\, F_{50}^{\delta}$, where G and δ are free positive parameters, with the best values $G = 1.63\,\text{GeV}^{0.28}$ and $\delta = 0.72$.

The obtained resolutions and bias[1] of A_0 are summarized in figure 1, as a function of S_{em}. As a reference a primary energy of 1 TeV and 10 TeV corresponds to a mean value of S_{em} of 115 GeV and 3 TeV, respectively (see figure 3 (middle)). Thus, resolutions of about 12% and 5% are found at primaries energies of 1 TeV and 10 TeV, respectively, while the bias is consistently in the order of a few percent.

The variable f_{20} is then defined as F_{20}/A_0. The choice of 20 m for the definition of this variable is a compromise which should be optimized for each specific experiment. Nevertheless, its value should be typically between 15 m and 30 m. Lower values will conflict with the possible experimental resolutions on the shower core, higher values will enter in the region where the cumulative function has a slower increase and also where, for events with the core nearer to the border of the compact region of the array, there will be no direct measurement of the cumulative

[1]In this work, the bias and resolutions of the estimator, $\hat{x}$, of variable, x, are taken fitting a gaussian function to the residuals, $(1 - (\hat{x}/x))$. The bias and the resolution corresponds to the mean and the sigma parameter of the fitted gaussian, respectively.

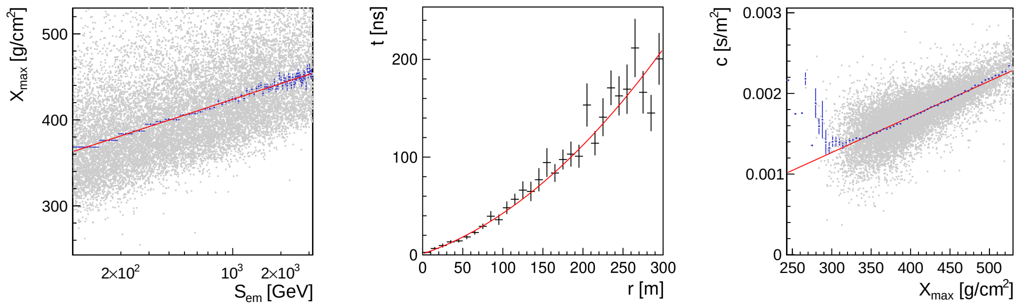

Figure 2: (left) Correlation of the slant depth of the maximum of the air shower longitudinal profile, $X_{\max}$, with the energy deposited at the ground, S_{em} (see text for the simulation details). (middle) Arrival time as a function of the distance to the core for an event with $X_{\max} = 339\,\mathrm{g\,cm^{-2}}$ and $E_0 = 1.3\,\mathrm{TeV}$. The line correspond to a quadratic fit as explained in the text with its $\chi^2/\mathrm{ndf} = 0.93$ (see text for the simulation details). (right) Correlation of the c curvature parameter with the slant depth of the maximum of the longitudinal profile, $X_{\max}$ (see text for details).

function.

3. $X_{\max}$ reconstruction and resolution

A first order estimation of $X_{\max}$ may be obtained observing that the mean value of $X_{\max}$ increases with the increase of the electromagnetic energy arriving at the ground (S_{em}), reflecting the increase of the shower size with the primary energy. This correlation is demonstrated in figure 2 where $X_{\max}$ is represented as a function of S_{em}. It is then possible to parametrize $X_{\max}$ as a function of S_{em} as $X^0_{\max} = B_0 + \gamma_0 \log(S_{\mathrm{em}}/\mathrm{GeV})$, with B_0 and γ_0 parameters tuned to describe the mean behaviour. The best achieved parameterization is shown in figure 2 as a red filled curve (corresponding to $B_0 = 237.1\,\mathrm{g\,cm^{-2}}$ and $\gamma_0 = 62.3\,\mathrm{g\,cm^{-2}}$).

A more precise estimate of $X_{\max}$ may be obtained exploring the fact that the shower front at the ground is a curved surface [5]. Ideally, if the shower particles were produced in a single point located along the shower axis, for instance at the $X_{\max}$, this surface will be spherical, assuming that all the particles travel approximately with the speed of light. The arrival time in each surface station would then change accordingly as a function of the distance to the shower core and with the primary particle direction. Using a simple geometrical fit, the $X_{\max}$ position would be reconstructed straightforwardly, with an accuracy that would mainly depend on the time resolution of the stations.

In reality, the geometry is more complex, but nevertheless, it is possible to establish a clear correlation between $X_{\max}$ and the arrival time distribution of particles at the ground. As an example in figure 2 this correlation is shown for an event with $X_{\max} = 339\,\mathrm{g\,cm^{-2}}$ and $E_0 = 1.3\,\mathrm{TeV}$. It was found that most of the events can be described by a quadratic polynomial of the form $t = a + b\,r + c\,r^2$. In fact, the application of the above equation to the time profiles as a function of the distance to the shower core leads to a well behaved χ^2/ndf distribution with the distribution maximum peaking at about 1.2.

The parameter of the quadratic term of the polynomial, c, is strongly correlated with $X_{\max}$ (figure 2 (right)). The parameter b is nearly independent of $X_{\max}$, and a is associated with the event

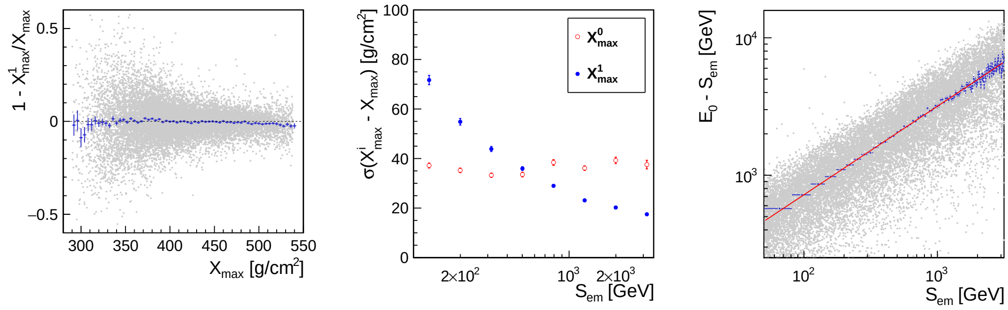

Figure 3: (left) Normalized deviation of the estimator X^1_{max} from the real X_{max}, as a function of X_{max} (see text for the simulation details). (middle) Resolution of the X_{max} estimators X^0_{max} and X^1_{max} as a function of S_{em} (see text for the simulation details). (right) Correlation of the energy deposited in the atmosphere (primary energy minus the electromagnetic energy at the ground), $E_0 - S_{\text{em}}$ with the energy deposited at the ground S_{em}. The line corresponds to the best parametrization (see text for details).

initial time, T_0, usually set to zero when the shower front reaches the shower core position. The dependence of c with X_{max} can be understood if one assumes that most of the particles produced in a shower come from X_{max} and the shower particles propagate as spherical front. This is of course an approximation but figure 2 supports it and it helps to build some intuition. Hence, it is possible to parametrize X_{max} as a function of c using $X^1_{\text{max}} = B_1 + \gamma_1\, c$. The parameters B_1 and γ_1 are tuned to describe the profile shown in figure 2 (right). The best achieved parametrization is shown by the red curve, with $B_1 = 11.2\,\text{g}\,\text{cm}^{-2}$ and $\gamma_1 = 2.28 \times 10^9\,\text{g}\,\text{s}^{-1}$.

The estimator X^1_{max} does not show any relevant bias even for low X_{max}, as shown in figure 3. Nevertheless, in a few cases, particularly at lower energies where the number of particles arriving at the ground is small, the fit may converge to c values leading to non-physical values of X_{max}. In practice, whenever the estimation of X_{max} from the fit indicates values lower than 300 g cm^{-2}, the first order estimation X^0_{max} is used. The obtained resolutions as a function of S_{em}, both for X^0_{max} and X^1_{max}, are summarized in figure 3. Resolutions of about 40 g/cm^2 and 20 g/cm^2 were found for primaries energies of 1 TeV and 10 TeV, respectively. The two resolutions are similar in the region $A_0^{\text{crX}} \approx 400 - 600$ GeV. To be on the safe side, avoiding possible tail effects, we set $A_0^{\text{crX}} = 600$ GeV. Therefore, the estimator of X_{max}, designated as X^R_{max} is defined as being X^1_{max} if $A_0 > A_0^{\text{crX}}$ and $X^1_{\text{max}} > 300\,\text{g}\,\text{cm}^{-2}$ otherwise it is simply X^0_{max}.

4. Energy reconstruction and resolution

In electromagnetic showers, the production of muons, either via the photo-production of mesons or by the direct creation of muon pairs is quite small [6] and can thus be neglected in the global accounting of the shower energy. On the other hand, the logarithm of the electromagnetic energy deposited in the atmosphere ($E_0 - S_{\text{em}}$) is linearly correlated with the logarithm of the energy deposited at the Earth surface (S_{em}), as shown in figure 3 (right).

It is then possible to parametrize E_0 as a function of S_{em} as:

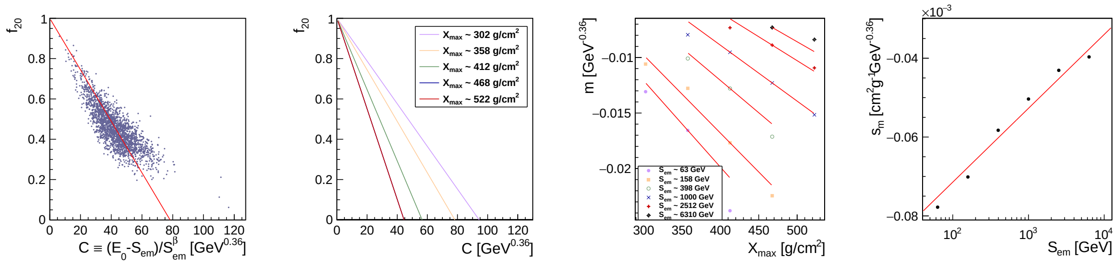

Figure 4: (left) Correlation of the variable f_{20}, as defined in the text, with the calibration coefficient C, for events with $S_{\rm em} \sim 200$ GeV and $X_{\max} \sim 350\,{\rm g/cm^2}$ (see text for the simulation details). (middle, left) Calibration lines (f_{20}, C) for several ranges of $X_{\max}$ and with $S_{\rm em} \sim 200$ GeV. (middle, right) Correlation of the slope of the calibration lines represented in 4 with $X_{\max}$, for different $S_{\rm em}$ bins (see caption). (right) Correlation of the slope s_m of the m calibration lines represented in fig 4 as a function of $S_{\rm em}$.

$$E_0^{(1)} = S_{\rm em} + C\,(S_{\rm em})^{\beta}, \tag{1}$$

where C and β are free positive parameters. This parametrization ensures, by construction, that $E_0^{(1)}$ is always greater then $S_{\rm em}$. The best values found for C and β are 37.2 GeV$^{0.36}$ and 0.64, respectively. The result is shown as a red curve in figure 3.

Using the above parametrization and A_0 (see section 2) as the estimator of $S_{\rm em}$, it is possible to make a first energy reconstruction considering an *ideal* detector[2]. An energy resolution of about 40% is obtained at 1 TeV.

The coefficient C simply is a constant, in this first calibration attempt. However, C can be shown to be correlated with f_{20}, $X_{\max}$ and $S_{\rm em}$. Indeed, it is shown in figure 4 (left) a remarkable correlation between f_{20} and $C = (E_0 - S_{\rm em})/(S_{\rm em})^{\beta}$, for events with $S_{\rm em} \in [100; 250]$ GeV and $X_{\max} \in [330; 385]\,{\rm g\,cm^{-2}}$. The line in the figure is the best linear parameterization imposing that for $C = 0$ (no energy deposited in the atmosphere), $f_{20} = 1$, i.e., all the deposited at the ground is at a distance lower than 20 m from the shower core position).

The set of the correlation lines (f_{20}, C) for several $X_{\max}$ ranges and $S_{\rm em} \sim 200$ GeV , are shown in figure 4 (middle, left). There is a linear monotonous decrease of the slope m of these lines with the increase of $X_{\max}$. In figure 4 (middle, right) the obtained m are represented as a function of $X_{\max}$ for different bins of $S_{\rm em}$ together with the best linear parametrization for each $S_{\rm em}$ bin.

The extrapolation of these lines for $X_{\max} = 0$ points to a non-physical small positive value of m ($b_m \sim 0.011\,{\rm GeV^{-0.36}}$), which means that this linear model is no longer valid for $X_{\max} < 200\,{\rm g\,cm^{-2}}$,which is far below the relevant $X_{\max}$ region for this article[3]. So we will keep the linear approximation using $b_m = 0.011\,{\rm GeV^{-0.36}}$ for all $S_{\rm em}$.

Finally, the slope s_m of the lines represented in figure 4 (right) are shown in figure 4 as a function of $\log(S_{\rm em})$. A linear correlation is found between s_m and $\log(S_{\rm em})$. As such, one can write $f_{20} =$

[2]By *ideal* detector it is assumed a detector able to accurately collects all the energy of electromagnetic particles reaching the station.

[3]For the energies considered in this article, most shower events have $X_{\max}$ values around $\sim 400 \pm 100\,{\rm g\,cm^{-2}}$. A shower with $X_{\max} \gtrsim 500\,{\rm g\,cm^{-2}}$ would have its $X_{\max}$ buried in the ground while a shower with $X_{\max} \lesssim 200\,{\rm g\,cm^{-2}}$ would only reach the ground if it strongly fluctuates.

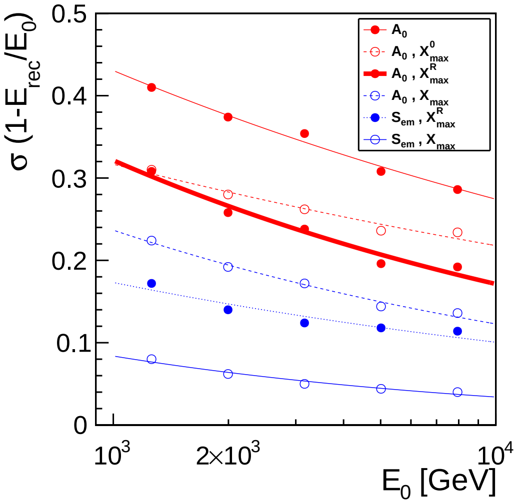

Figure 5: Energy resolution as a function of the primary energy. The red lines correspond to constant C coefficients (thin) and $C(X_{\max}, f_{20})$ coefficients, using $X^R_{\max}$ (thick) or $X^0_{\max}$ (dashed); resolutions obtained with simulated values are also shown for comparison (blue lines) : simulated value of $X_{\max}$ (dashed), simulated value of S_{em} (pointed) and both simulated values (thin).

$1 + m(X_{\max}, S_{\text{em}})\, C(f_{20}, X_{\max}, S_{\text{em}})$, and, $m(X_{\max}, S_{\text{em}}) = b_m + [s_{m0} + s_{m1} \log(S_{\text{em}}/\text{GeV})]\, X_{\max}$. The best achieved parametrization with the above equation is shown in figure 4, with parameters $s_{m0} = -1.1 \times 10^{-4}\,\text{GeV}^{-0.36}\,\text{g}^{-1}\text{cm}^2$ and $s_{m1} = 1.87 \times 10^{-5}\,\text{GeV}^{-0.36}\,\text{g}^{-1}\text{cm}^2$.

Finally we get as our best estimator for the primary energy,

$$E_0^{(2)} = S_{\text{em}} + C(f_{20}, X_{\max}, S_{\text{em}})\,(S_{\text{em}})^{\beta}, \tag{2}$$

with $C(f_{20}, X_{\max}, S_{\text{em}}) = \frac{1-f_{20}}{-(b_m+[s_{m0}+s_{m1}\log(S_{\text{em}})]\,X_{\max})}$.

Using in the above parametrization, A_0 (see section 2) as the estimator of S_{em} and $X^R_{\max}$ (see section 3) as the estimator of $X_{\max}$, an energy resolution below 30 % and 20 % were obtained at 1 TeV and 10 TeV, respectively.

Using instead the real $X_{\max}$ and S_{em}, these energy resolutions improve to about 8% and 4% . These results may be considered as the *ultimate* resolutions. The difference between the estimated resolutions and the *ultimate* resolutions are driven, in the TeV region, mainly by the resolution of the S_{em} estimator. Indeed, a resolution of 12% on the parameter A_0, the S_{em} estimator, even considering the simulated $X_{\max}$ value, would translate into a resolution on $E_0^{(2)}$ of 22% .

In figure 5 it is shown (full red thick line) the estimated energy ($E_0^{(2)}$) resolution as a function of the primary energy. For comparison, the equivalent resolutions obtained applying the constant C calibration ($E_0^{(1)}$), full red thin line, defined by equation 1, or using systematically the $X^0_{\max}$ estimator (dashed red thin line) are also shown. It is also shown the resolutions that would be obtained using the simulated value of $X_{\max}$ (dashed blue line), or the simulated value of S_{em} (pointed blue line) or finally using both simulated values (blue thin line).

5. Summary

In this article, new and innovative methods for the determination of the total electromagnetic energy at the ground, of the slant depth of the maximum of the longitudinal profile and, of the primary energy were proposed. In particular, as much as we know, it is the first time that the steepness in the core region of the cumulative function of the energy arriving at the ground is used as a determinant factor to obtain the energy calibration constants.

The results obtained, representing such a considerable improvement concerning the presently quoted energy resolutions of the existing or planned Wide Field of View Gamma-Ray Observatories, clearly will encourage detailed simulations and studies on the applicability of the proposed methods. More details on the analysis including a discussion on the impact of the expected experimental resolutions can be found in [7].

Acknowledgements

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