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Uncertainties of the energy loss by inelastic interactions of muons with nuclei

Alexander Sandrock,[a,*] **Edgar Bugaev,**[b] **Rostislav Kokoulin**[a] **and Anatoly Petrukhin**[a]

[a] *National Research Nuclear University MEPhI, Kashirskoe shosse 31, 115409 Moscow, Russian Federation*

[b] *Institute for Nuclear Research of the Russian Academy of Sciences, Prospekt 60-letiya Oktyabrya 7a, 117312 Moscow, Russian Federation*

E-mail: alexander.sandrock@mail.ru, bugaev_e_v@mail.ru, RPKokoulin@mephi.ru, AAPetrukhin@mephi.ru

High-energy muons loose their energy by ionization, pair production, bremsstrahlung and inelastic interaction with nuclei. The process with the largest uncertainty is the inelastic interaction with nuclei. Since the energy loss is dominated by soft interactions with small momentum transfer, parton distribution functions are not applicable and phenomenological parametrizations have to be used. The parametrizations of the proton structure functions that are commonly used in muon transport simulation tools such as PROPOSAL, MUM, MUSIC or Geant4 were determined on the basis of the data available about 20 years ago. In this contribution, we refit several commonly used parametrizations to the data on deep inelastic scattering available today, including the precise combined data from the HERA experiments H1 and ZEUS, which have become available a few years ago. We compare the goodness of fit and calculate the uncertainty of the average energy loss from the uncertainties and correlations of the fit parameters.

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1. Introduction

Among the charged leptons of the Standard Model of Particle Physics, muons are distinguished by their comparatively long lifetime and slow energy loss. While electrons are stable, they quickly loose their energy in bremsstrahlung and ionization; tau-leptons loose their energy slower than muons of the same energy, but their short lifetime prevents them from travelling large distances at energies below the multi-PeV range. In underground laboratories, two sources of muons have to be considered: muons produced in extensive air showers in the atmosphere, and muons produced in neutrino interactions. This is especially important in the context of very large volume neutrino telescopes, such as IceCube, KM3Net and GVD; their large detection volume in the cubic kilometer range allows to investigate the behaviour of muons over distances of several hundred meters.

Energy is lost by ionization, electron-positron pair production, bremsstrahlung, and inelastic nuclear interaction. Ionization and pair production lead, roughly speaking, to numerous, but small energy losses, which quasi-continuously decrease the muon energy; bremsstrahlung and inelastic nuclear interaction, in contrast, lead also to comparatively large energy losses of the order of tenths of the muon energy, which are often called stochastic losses.

The largest uncertainties at high energies stem from the inelastic nuclear interaction energy losses [1]. This is mainly due to the fact that most of the interactions happen in the regime of very low momentum transfer Q^2 and small Bjorken scaling variable x, where perturbative QCD is not applicable, such that phenomenological models have to be used.

One of these phenomenological approaches is vector meson dominance, which describes photohadronic interactions via intermediate vector mesons such as ρ, ω, ϕ and the heavier members of the respective meson families ($\rho', \rho'', \ldots$). The most widespread in muon transport simulation codes is the parametrization by Bezrukov & Bugaev [2, 3], which was augmented in [4] with a hard component based on the color-dipole picture.

Another approach is Regge theory, the most widespread of which in muon transport codes is the ALLM parametrization [5, 6]. This approach is based on the analyticity of amplitudes as functions of complex variables. Other works in this direction, occasionally used in lepton transport calculations, include the Froissart-bound-inspired parametrization of [7].

A principal disadvantage of many parametrizations is the neglect of the behaviour of the structure functions in limiting kinematic regions. The first attempt to obtain a parametrization of the structure function in the whole kinematic region was done in [8].

As phenomenological models, these models cannot be calculated from first principles and contain a sizable number of free parameters that have to be determined by a fit to experimental data. In addition, at very high energies lepton transport calculations require the evaluation of nucleon structure functions at x, Q^2 outside the range of collider experiments. To address these problems and assess the uncertainty of the energy losses of muons via inelastic interaction, we refit the parametrizations for the nucleon structure functions using low-x DIS data and photoabsorption measurements, and calculate the average energy loss and its uncertainty for the respective parametrizations.

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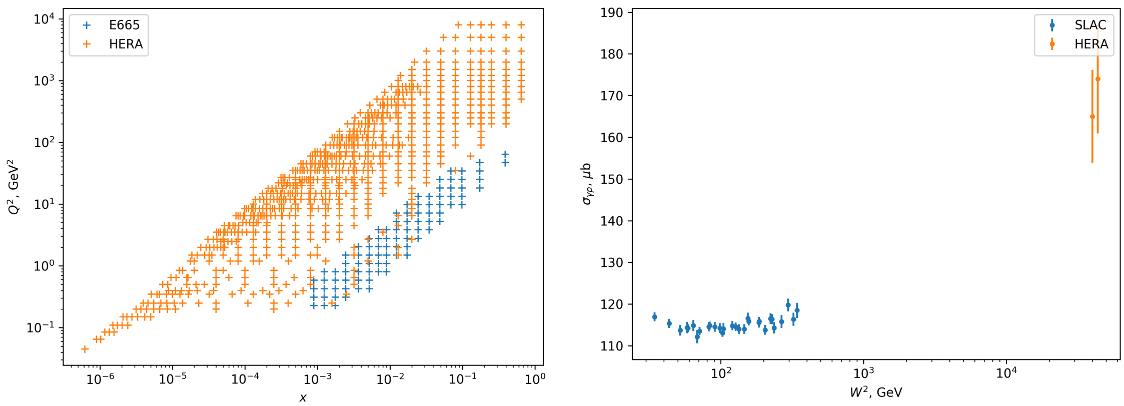

Figure 1: (x, Q^2) and W^2 regions covered by the experimental data used in this study.

2. Methods and used datasets

2.1 Datasets

To fit the parametrizations of the nucleon structure functions we use the precise combined ep scattering data from the H1 and ZEUS experiments at HERA [9] and the fixed target measurements at low x and Q^2 from the experiment E665 [10] and the photoproduction measurements from SLAC [11], H1 [12] and ZEUS [13]. The (x, Q^2) and W^2 regions covered by these datasets are shown in figure 1.

As the parametrizations investigated in this contribution mainly describe only the structure function F_2, but not the longitudinal structure function separately, we use the corresponding values extracted in [14] instead of the reduced cross sections from [9].

2.2 Uncertainty calculation

The average energy loss is defined by the integral $\langle -dE/dX \rangle = (N_\mathrm{A}/A) \int_{y_\mathrm{min}}^{y_\mathrm{max}} Ey(d\sigma/dy)dy$, where $y = (E - E')/E$ is the inelasticity, N_A the Avogadro constant, A the mass number of the material traversed, $d\sigma/dy$ the singly-differential cross section of the energy loss process under consideration, and $y_\mathrm{min} \gtrsim 0$, $y_\mathrm{max} \lesssim 1$ are the physical limits for this process. In the case of inelastic interaction with nuclei, the singly-differential cross section is typically expressed by the integral over the doubly-differential cross section

$$y\frac{\partial\sigma}{\partial y \partial Q^2} = \frac{4\pi\alpha^2}{Q^4}\left\{\left[1 - y - \frac{Mxy}{2E}\right]F_2 + \left(1 - \frac{2\mu^2}{Q^2}\right)\frac{y^2}{2}(1 + 4M^2x^2/Q^2)(F_2 - F_L)\right\} \tag{1}$$

with the Bjorken scaling variable $x = Q^2/(2MEy)$, the nucleon mass M, the lepton mass μ, and the structure functions F_2, F_L. The double integral over y, Q^2 can in general not be taken analytically and has to be calculated numerically.

The uncertainty of a quantity $f(X)$, given the covariance matrix Σ, can be calculated via

$$\sigma_f = \sqrt{\sum_{ik} \frac{\partial f}{\partial X_i} \Sigma_{ik} \frac{\partial f}{\partial X_k}}. \tag{2}$$

The gradient of the energy loss with respect to the fit parameters can be calculated using Leibniz' integral rule by exchanging in the energy loss integral the structure functions by the gradient of F_2, F_L, since the integral limits are indepedent of the fit parameters. The gradient of the structure functions can be determined using automatic differentiation; in this case the library [15] was used.

3. Models for the structure functions

3.1 ALLM parametrization

The ALLM parametrization of the structure function F_2 was developed in [5] and has 23 free parameters, that were determined by a fit to fixed-target data available at the time; later the fit was repeated in [6] using early HERA data. Recently, the fit was repeated with the combined HERA data in [14], but the best-fit obtained there is mathematically ill-defined in the photoabsorption limit, because the evolution variable $t = \ln[\ln((Q^2 + Q_0^2)/\Lambda^2)/\ln(Q_0^2/\Lambda^2)]$ appears with a negative exponent. We therefore restrict the allowed range of fit parameters to avoid these unphysical solutions.

Using this parametrization, we obtain a best-fit $\chi^2/ndf = 1.01$. It is noteworthy that a simultaneously good description of the photoabsorption data points and the deep inelastic scattering measurements could not be achieved. The uncertainty derived from the covariance matrix of the parameters is of the order of about a percent. At high energies, the average energy loss is close to the values obtained from the ALLM97 fit (cf. Figure 2).

3.2 Bezrukov & Bugaev and Bugaev & Shlepin

The parametrization of Bezrukov & Bugaev was developed in [2, 3] in the context of the generalized vector meson dominance model. The calculations carried out in those articles were based on a large number of intermediate vector mesons; the commonly used parametrization with two effective masses m_1, m_2 is a simple approximate formula which deviates from the numerical calculations with an accuracy of $\lesssim 5\%$ for $Q^2 < 10\,\mathrm{GeV}^2$ and $10\,\mathrm{GeV} < \nu < 1\,\mathrm{PeV}$. This was then approximately integrated over Q^2 to obtain a closed analytic expression for $d\sigma/dy$, with an accuracy of a few percent compared to a numerical integration of the approximate formula.

Later in [4] the hard component was calculated on the basis of the color dipole model of [16], unitarized assuming a Gaussian form of the opacity function.

The best fit of this parametrization has a poor $\chi^2/ndf \sim 6 \gg 1$, but this is not surprising in view of the approximations which lead from the actual calculations to the simple formulae that allowed to express the differential cross section $d\sigma/dv$ in closed form. The average energy loss of the refit rises slower with energy than the original parametrization and is similar to the other parametrizations until muon energies of tens of TeV.

3.3 Petrukhin & Timashkov

In [8] a description of the structure function was developed on the basis of vector meson dominance, Regge theory and the perturbative DGLAP and BFKL equations by finding suitable interpolating functions and taking into account the behaviour in the limiting kinematical regions of quasielastic scattering, photoproduction and deep-inelastic scattering.

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Table 1: Goodness-of-fit values for the considered parametrizations

Parametrization	Number of free parameters	χ^2/ndf
ALLM	23	1.01
BB, BS	12	6
PT	5	8
BDH	9	1.10

The parametrization succeeded in obtaining a description of the data available at the time with a deviation of not more than 10–15%. The best fit of this parametrization on the data selection considered here has a poor $\chi^2/ndf \sim 8$; nevertheless the average energy loss is similar to the result calculated from the ALLM parametrization until muon energies of a few hundred TeV. The refit favors a less strong rise of the energy loss with energy, similar to the other parametrizations. The parametrization shows at lower energies a similar behaviour as the other cross sections.

3.4 BDH parametrization

In [7], a parametrization of F_2 was developed that leads to a saturated Froissart-bound, i.e. an asymptotic behavior of the cross section $\propto \ln^2 W$; as a boundary condition, the photoabsorption cross section parametrization of [17] was used, which reduces the number of free parameters from twelve to nine. Using the behaviour in limiting kinematical regions to develop parametrizations for structure functions was advocated earlier in [8]. We repeat their fit on the E665 and combined HERA data, using instead of the Block & Halzen parametrization the $\mathrm{HPR_1R_2}$ photoabsorption parametrization by [18], which has a smaller uncertainty due to the assumption of hadron universality and the fit to pp, $\bar{p}p$ and other hadronic cross section data in addition to the photoabsorption data points. Instead of the valence quark component from the CTEQ6L PDF fit added in [7] to describe the data at not very small x, we used the Regge component from the recent tensor pomeron fit of [19]. The photoabsorption data points are not included in the fit, because they are determined *a priori* by the $\mathrm{HPR_1R_2}$ parametrization.

The best-fit with this parametrization has $\chi^2/ndf = 1.10$. At high energies, the average energy loss is close to the values obtained from the original fit by Block *et al.* (cf. Figure 2). The uncertainty calculated from the covariance matrix is about 3–5%.

4. Conclusions

We have reconsidered the widely used parametrizations of the proton structure functions by [2–5, 7, 8] in the light of the precise combined HERA data published a few years ago and other deep inelastic scattering data at low x and low Q^2 and photoabsorption data with the aim of investigating the uncertainty of these parametrizations. The ALLM and BDH parametrizations are able to describe the data with a χ^2/ndf of 1.0 and 1.1, respectively, while a refit of the Bezrukov & Bugaev parametrization with a hard component as in Bugaev & Shlepin leads to a poor $\chi^2/ndf \sim 6$, and

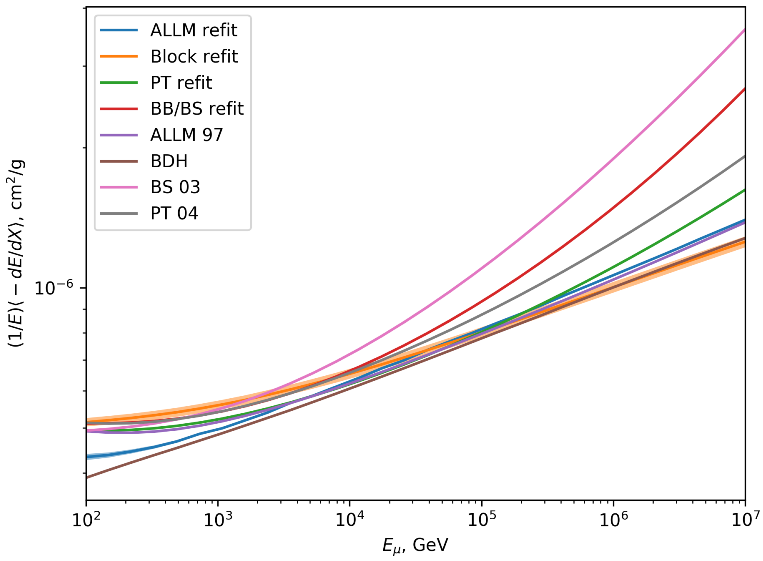

Figure 2: Average energy loss on protons according to parametrizations from the literature and to our refits. The uncertainty calculated from our refits is shown by the shadowed bands.

a refit of the Petrukhin-Timashkov parametrization, designed to describe the whole kinematical region leads to $\chi^2/ndf \sim 8$.

The best description of the given data selection was achieved with the ALLM parametrization, however the number of free parameters is very large and the physical significance of all parameters is difficult to ascertain; the lowest number of parameters is found in the Petrukhin-Timashkov parametrization, but the fit to new experimental data is not as good as on previous data (see figure 3 in [8]).

Nevertheless, the average energy loss due to inelastic nuclear interaction according to the different models is similar at lower energies. For energies up to the TeV range, the predictions agree within about 10–15%. Serious disagreements arise at higher energies due to the different assumptions the models are built upon. The energy loss predictions of the ALLM and BDH refits are close to their published counterparts in the high-energy region, while the refits of the other parametrizations favours a distinctly lower rise of the energy loss with increasing muon energy. Despite the rather poor goodness of fit of the latter, they correctly describe the general behaviour needed for applications in muon propagation.

It is noteworthy, that the calculated uncertainty for a given model is smaller than the differences between the models in the high-energy region. Therefore further work is necessary from the experimental as well as from the theoretical side. From the theorist side, models are required, which describe the existing data with good accuracy and clear theoretical fundament. First steps in this direction have been done already (see for example [8, 19–21]). From the experimental side, new data are required, in particular at higher energies. The planned construction of larger electron-hadron colliders as well as of larger neutrino telescopes will give an important contribution to the solution of this problem.

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