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Inclusive $\Upsilon(1S) \to \eta^{(\prime)} + X$ Decays with Account of α_s Running in Effective $\eta^{(\prime)} g^* g$-Vertex

Ahmed Ali,[a] **Alexander Ya. Parkhomenko**[b,*] **and Alexander V. Rusov**[b]

[a] *Deutsches Elektronen-Synchrotron DESY, D-22607 Hamburg, Germany*

[b] *P. G. Demidov Yaroslavl State University, Sovietskaya 14, 150003 Yaroslavl, Russia*

E-mail: ahmed.ali@desy.de, parkh@uniyar.ac.ru

The η'-meson energy spectrum in the inclusive $\Upsilon(1S) \to \eta^{(\prime)} ggg \to \eta^{(\prime)} + X$ decay measured by the CLEO Collaboration in 2002 allowed one to constrain the lowest Gegenbauer coefficients B_2^q and B_2^g of the quark-antiquark and gluonic distribution amplitudes of the η'-meson entering the $\eta' g^* g$ effective vertex function (EVF). The fitting procedure of the CLEO data on the hard part of the η'-meson energy spectrum was based on the theoretical expression calculated in the leading-order perturbative QCD in the static-quark limit for the orthoquarkonium. The resulting constraints were combined with the existing ones on these coefficients from an analysis of the $\eta' - \gamma$ transition form factor. The updated measurements of the η'-meson energy spectrum by the CLEO Collaboration in 2006 are not in accord with the so-calculated theoretical η'-energy spectrum. As a first step, we incorporate the dependence of the strong coupling constant in the $\eta' g^* g$ EVF on the quark energy and determine the Gegenbauer coefficients. With these values we compute the η'- and η-energy spectra in the inclusive decays $\Upsilon(1S) \to \eta^{(\prime)} ggg \to \eta^{(\prime)} + X$, which can be tested against the high statistics Belle and Belle-II data from $\Upsilon(1S)$-decays.

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*Speaker

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Quantitative description of rare decays with the η- and η'-meson production, such as $B \to \eta^{(\prime)} K^{(*)}$, $B \to \eta^{(\prime)} X_s$, $\Upsilon(1S) \to \eta^{(\prime)} X$, $\Upsilon(1S) \to \eta^{(\prime)}\gamma$, requires understanding of the $\eta^{(\prime)} g^* g^{(*)}$ effective vertex function (EVF), $F_{\eta^{(\prime)} g^* g^{(*)}}(q_1^2, q_2^2, m^2_{\eta^{(\prime)}})$, also known as the $\eta^{(\prime)} - g$ transition form factor. At small energies, this vertex is determined by the axial anomaly contribution resulting from a local Lagrangian of the $\eta^{(\prime)}$-meson interaction with a gluon pair, while at large energies the internal structure of the meson manifests itself and gives an additional contribution to the $\eta^{(\prime)} g^* g^{(*)}$ EVF. For a highly energetic $\eta^{(\prime)}$-meson, its collinear degrees of freedom dominate the interaction while the transverse ones can be safely neglected. Concentrating on the η'-meson, the Fock-state decomposition of its wave-function can be presented in the form [1, 2]: $|\eta'\rangle = \sin\phi\,|\eta'_q\rangle + \cos\phi\,|\eta'_s\rangle + |\eta'_g\rangle$, with $|\eta'_q\rangle \sim |\bar{u}u + \bar{d}d\rangle/\sqrt{2}$, $|\eta'_s\rangle \sim |\bar{s}s\rangle$, and $|\eta'_g\rangle \sim |gg\rangle$. More precisely, the eigenfunction of the mixed $|\bar{q}q\rangle$ and $|gg\rangle$ state can be written as follows [1, 2]:

$$\Psi = C \begin{pmatrix} \phi^{(q)}_{\eta'}(x, Q^2) \\ \phi^{(g)}_{\eta'}(x, Q^2) \end{pmatrix}, \qquad C = \sqrt{2}\, f_q \sin\phi + f_s \cos\phi, \tag{1}$$

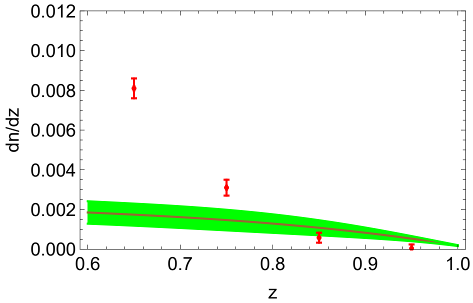
where f_q and f_s are decay constants and ϕ is the $\eta - \eta'$ mixing angle. Here, $\phi^{(q)}_{\eta'}(x, Q^2)$ and $\phi^{(g)}_{\eta'}(x, Q^2)$ are the quark-antiquark and gluonic Light-Cone Distribution Amplitudes (LCDAs), respectively, which are expressed in terms of the Gegenbauer polynomials taking into account the evolution under renormalization. The usual procedure is to truncate the Gegenbauer series after a few first terms, exemplified here as $\phi^{(q)}_{\eta'}(x, Q^2) = 6x\bar{x}\left[1 + 6(1 - 5x\bar{x})\, A_2(Q^2) + \ldots\right]$ and $\phi^{(g)}_{\eta'}(x, Q^2) = 5x^2\bar{x}^2\,(x - \bar{x})\, B_2(Q^2) + \ldots$, where x id the reduced energy of quark or gluon. This input enables one to calculate processes involving an energetic $\eta^{(\prime)}$-meson.

A process with energetic η'-meson production is the inclusive decay $\Upsilon(1S) \to \eta' X$, where X denotes any number of light hadrons, which was measured by the CLEO Collaboration [3] a while ago. Perturbative QCD analysis of this decay was undertaken in [4]. The underlying quark process for this decay is $\Upsilon(1S) \to ggg^*(g^* \to \eta' g) \to \eta' X$. The static limit for the heavy quark and antiquark in $\Upsilon(1S)$-meson was used in calculating the differential branching fraction [4]. The definition of the η'-meson energy distribution function is as follow [3, 4]:

$$\begin{aligned} \frac{dn}{dz} &= \frac{1}{\Gamma^{(0)}_{3g}} \frac{d\Gamma_{\eta' X}(z)}{dz} = \frac{1}{\Gamma^{(0)}_{3g}} \frac{1}{3!} \frac{1}{(2\pi)^8} \frac{1}{2M} \int \frac{d\mathbf{k}_1}{2\omega_1} \frac{d\mathbf{k}_2}{2\omega_2} \frac{d\mathbf{k}_3}{2\omega_3} \frac{d\mathbf{p}_{\eta'}}{2E_{\eta'}} \\ &\times\ \delta^{(4)}(\mathcal{P} - k_1 - k_2 - k_3 - p_{\eta'})\, \delta(z - 2E_{\eta'}/M)\, \frac{1}{3} \sum |\mathcal{M}[\Upsilon \to \eta' ggg]|^2 . \end{aligned} \tag{2}$$

The differential branching fraction is normalized to the three-gluon decay width, $\Gamma^{(0)}_{3g}$, calculated in the leading order: $\Gamma^{(0)}_{3g} = (16/9)\,(\pi^2 - 9)\, C_F\, B_F\, \alpha_s^3(\mu^2_\Upsilon)\, |\psi(0)|^2/M^2$, where $C_F = (N_c^2 - 1)/(2N_c)$, $B_F = (N_c^2 - 4)/(2N_c)$, and $\mu_\Upsilon \sim m_b$ is a typical hard scale of the process. Explicit form of $\mathcal{M}[\Upsilon \to \eta' ggg]$ and analytical expression for the amplitude squared are presented in [4].

As mentioned above, the CLEO Collaboration has measured the η'-meson energy spectrum in $\Upsilon(1S) \to \eta' X$ decay [3]. The fits of the Gegenbauer coefficients $B_2^{(q)}(\mu_0^2)$ and $B_2^{(g)}(\mu_0^2)$ entering the LCDAs, based on the last three experimental bins with $z \geq 0.7$ gives $\chi^2_{\rm min} \simeq 2.4$ [4]. The updated CLEO measurements using the 1.2 fb^{-1} of data [5] has a significantly higher $\chi^2_{\rm min}$, indicating the inadequacy of the current theoretical framework [4]. Hence, improvements in the



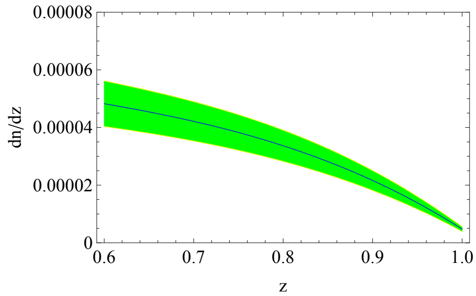


Figure 1: Energy spectra of η'- and η-mesons in the decays $\Upsilon(1S) \to \eta' + X$ and $\Upsilon(1S) \to \eta + X$. The red bars in the left plot represent the CLEO data [5].

existing theoretical calculations are necessary, which are being undertaken in the computation of $F_{\eta^{(\prime)} g}(p^2) = F_{\eta^{(\prime)} g^* g}(p^2, 0, m^2_{\eta^{(\prime)}})$ EVF. In particular, this EVF is now rederived under the assumption that the strong coupling α_s is dependent on the energy x of the parton in the meson. An improved expression for dn/dz will be presented in a forthcoming paper. Preliminary numerical results for the energy spectrum in the $\Upsilon(1S) \to \eta' + X$ and $\Upsilon(1S) \to \eta + X$ decays are presented in Fig. 1 and compared with the CLEO data [5] shown as red bars in the left plot. There is a reasonable agreement of the last three z-bins with theoretical estimations within the uncertainties

With the higher statistics data from the BaBar and Belle Collaborations, and much higher statistics at Belle II, the η- and η'-energy spectra can be measured precisely in the inclusive $\Upsilon(nS)$-decays ($n = 1, 2, 3$), and compared with the CLEO data and theoretical predictions.

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