

Rare decays of K mesons

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Kaon mesons offer a unique opportunity to search for physics beyond the Standard Model, by measurements of flavour physics observables in rare decays, search for decays violating lepton universality, flavour and number, and search for production of low mass feebly interacting particles. The experimental status of rare and forbidden kaon decays is reviewed; in particular the latest results on the $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K \rightarrow \mu^+ \mu^-$ ultra-rare decays; the study of lepton universality with the $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay; the search for the lepton flavour and number violation decays $K \rightarrow \pi \mu e$; and the search for heavy neutral lepton production in $K^+ \rightarrow l^+ N$ ($l = e, \mu$) decays. Medium and long term prospects for experimental kaon physics are also discussed.

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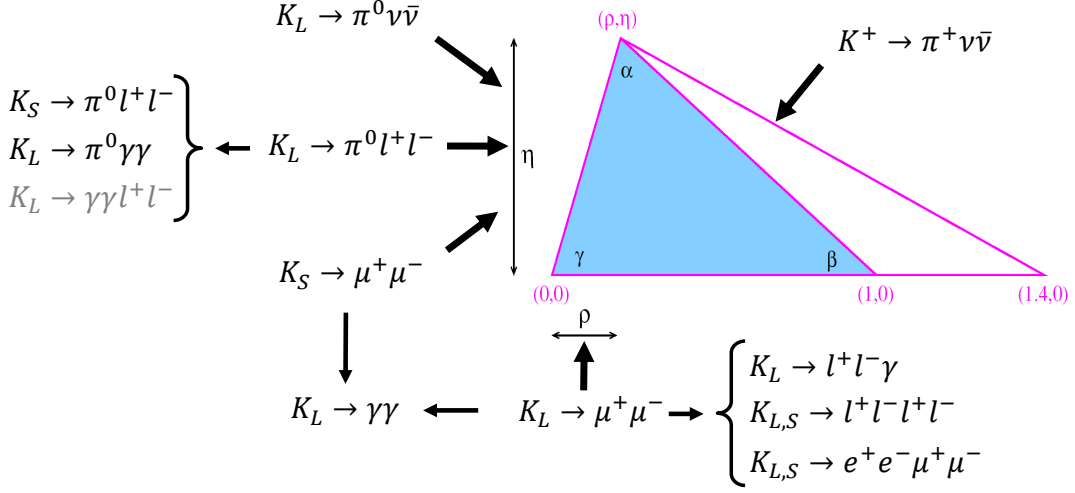


Figure 1: Relation between kaon rare decay modes and the parameters ρ and η of the unitarity triangle (UT). The direct link between decay modes and UT indicates short distance terms dependent on ρ or η contributing to the corresponding decay amplitudes. Decays not directly connected to UT are relevant to interpret the experimental results of the decay modes to which they are related.

1. Rare K decays and Flavour Physics

Kaon mesons offer a unique opportunity to investigate physics beyond the Standard Model (BSM) by the measurement of flavour physics observables complementary to B_{sd} , D , lepton physics and EDMs searches. The most sensitive flavour quantities to BSM physics in K sector are: the indirect CP-violation parameter ϵ_K and the $K^0 - \bar{K}^0$ mass difference ΔM_K ; the direct CP-violation parameter ϵ'/ϵ ; the branching ratios of the rare decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $K_{L,S} \rightarrow \mu^+ \mu^-$, $K_L \rightarrow \pi^0 e^+ e^-$ and $K_L \rightarrow \pi^0 \mu^+ \mu^-$. This section focuses on the latest developments of the experimental studies of rare kaon decays.

Figure 1 schematically represents the link between rare kaon decays and flavour physics. CP-violating short distance (SD) terms contribute to the decays $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $K_L \rightarrow \pi^0 e^+ e^-$, $K_L \rightarrow \pi^0 \mu^+ \mu^-$ and $K_S \rightarrow \mu^+ \mu^-$, making them dependent on the height η of the unitarity triangle. The SD part of the $K_L \rightarrow \mu^+ \mu^-$ decay is CP-conserving and the corresponding amplitude depends on the base ρ of the unitarity triangle. The amplitude of the rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ contains terms depending on η and ρ .

The decays $K_L \rightarrow \pi^0 e^+ e^-$, $K_L \rightarrow \pi^0 \mu^+ \mu^-$ and $K_{L,S} \rightarrow \mu^+ \mu^-$ are long distance (LD) dominated and the extraction of their SD components from experimental data proceeds through the study of ancillary decay modes, listed in Figure 1 in association with the rare decays mentioned above. Long distance contributions are sub-dominant to the amplitudes of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay and negligible for the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay. These two decay modes are, therefore, the theoretically cleanest probe of the flavour structure of the kaon sector, but experimentally among the most challenging meson decays to investigate.

1.1 $K \rightarrow \pi\nu\bar{\nu}$ decays

The $K \rightarrow \pi\nu\bar{\nu}$ decays involve $s \rightarrow d$ quark transitions. They proceed through box and penguin diagrams, SD dominated by the leading contribution of the virtual t quark exchange. The quadratic GIM-mechanism and the $t \rightarrow d$ Cabibbo suppression make these processes among the rarest meson decays in the SM. Their decay amplitudes can be parametrized in terms of the precisely measured $K^+ \rightarrow \pi^0 e^+ \nu$ decay, allowing a theoretical computation free from hadronic uncertainties. The SM predictions of the $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ and $K_L \rightarrow \pi^0 \nu\bar{\nu}$ branching ratios are $(8.4 \pm 1.0) \times 10^{-11}$ and $(3.4 \pm 0.6) \times 10^{-11}$, respectively [1]. The theory uncertainty of the prediction is below 2%, and the rest of the uncertainty is parametrical, dominated by the experimental knowledge of the CKM matrix elements, in particular V_{cb} .

The extreme SM suppression makes these decays particularly sensitive to New Physics (NP). In model independent terms the $K \rightarrow \pi\nu\bar{\nu}$ decays probe NP at the highest mass scales [2]. Existing experimental constraints on NP affect the $K \rightarrow \pi\nu\bar{\nu}$ weakly, especially those on minimal flavour violating models. Model dependent scenarios predict deviation of the branching ratios from the SM as large as 30-40% and also correlations between the charged and neutral modes, depending on the model [3–10]. Therefore measurements of the $K \rightarrow \pi\nu\bar{\nu}$ branching ratios with $O(10\%)$ precision at least are needed to pin down possible BSM effects.

The experiments KOTO at JPARC [11] and NA62 at CERN [12] aim to study the $K_L \rightarrow \pi^0 \nu\bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu\bar{\nu}$, respectively. The two experiments are designed to detect the decay products of kaons produced by protons impinging on target. The main ring at JPARC provides the protons for KOTO, the SPS at CERN gives the protons to NA62. KOTO exploits a neutral beam of 1.4 GeV/ c peak momentum, made of neutron, photons and K_L . NA62 makes use of a positive beam of 75 GeV/ c momentum consisting of protons, π^+ and K^+ . Both KOTO and NA62 aim to tag $K \rightarrow \pi\nu\bar{\nu}$ decays by the measurement of the missing energy associated with the final state pion. This requires detector configurations that are as hermetic as possible are required, to reveal all the particles except neutrinos produced by kaons decaying in a hardware-defined fiducial volume. KOTO consists mainly of electromagnetic calorimeters to detect the π^0 mesons from K_L decays. NA62 employs tracking systems to reconstruct both the K^+ and π^+ , Cherenkov devices and calorimeters for particle identification and electromagnetic calorimeters for photon detection.

1.1.1 $K_L \rightarrow \pi^0 \nu\bar{\nu}$ at KOTO

The signal selection exploits the high resolution of the electromagnetic calorimeter to reconstruct the energy of the π^0 and the position of the K_L decay vertex. The rejection of K_L decays with multiple π^0 in final state exploits efficient calorimeters and of the capability to resolve overlapping pulses. Beam neutrons scattered off beam line elements and leaving signals in detectors are a significant source of background; their suppression proceeds through the analysis of the shape of the calorimetric clusters performed with neural network techniques. The ultimate source of background are decays of K^+ produced by K_L charge exchange interactions with material of the beam line.

The analysis of the data taken from 2016 to 2018, has allowed KOTO to reach a single event sensitivity (SES) of 7.1×10^{-10} . From these data KOTO has reported four signal candidates [13], inconsistent with the 0.04 SM signal events expected, but also with the about 0.4 background events expected. A reanalysis of the data has shown that one signal candidate was actually the result of a

wrong application of the selection criteria against overlapping clusters. KOTO has then collected data in 2020 to measure the K^+ flux in the K_L beam. The analysis of these data has shown a K^+ flux significantly larger than that predicted by simulations, leading to a factor three increase of K^+ induced background. The corresponding expected background from data collected in 2016-17-18 is now set to 1.05 ± 0.28 events, mostly due to K^+ decays. KOTO plans to install dedicated detectors to suppress this background, aiming to reach the SM sensitivity of the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay in future data.

1.1.2 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ at NA62

The $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ signal selection takes advantage of the reconstruction of both K^+ and π^+ that allow a precise determination of the $\nu \bar{\nu}$ invariant mass to kinematically suppress backgrounds. Kaon decays with π^+ momentum lower than 45 GeV/ c are selected, to guarantee at least additional 25 GeV energy produced together with the π^+ , given the 75 GeV momentum of the K^+ beam. Muon and π^0 rejection factors larger than 10^8 complement kinematics to suppress tree level background processes like $K^+ \rightarrow \mu^+ \nu$ and $K^+ \rightarrow \pi^+ \pi^0$ decays. A precise timing of the charged particles with 100 ps resolution provides reduction of backgrounds due to random coincidences between K^+ and accidental π^+ mesons. The main background remaining after signal selection is due to π^+ mesons coming from K^+ mesons decaying upstream of the fiducial volume.

NA62 has recently analysed the data taken in 2018, reporting the observation of 17 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ candidates, with about 7.6 SM signal events and $5.28^{+0.99}_{-0.74}$ background events expected. While observation and expectation are statistically compatible, this is the first $> 3\sigma$ evidence of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay. Combining the 2018 result with the analysis of the data taken in 2016 [14] and 2017 [15], 20 signal candidates have been observed by NA62 with about 7 background events expected. This leads to a preliminary measured branching ratio of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ equal to $(11.0^{+4.0}_{-3.5})_{stat} \pm 0.3_{syst} \times 10^{-11}$.

Figure 2 summarizes the up-to-date experimental status of the $K \rightarrow \pi \nu \bar{\nu}$ decay. Here the latest NA62 result is compared with the previous measurement [16, 17] from the BNL E787/949 experiments and the SM prediction. The direct upper limit on the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ branching ratio set by KOTO from the analysis of the 2015 data [11] and the theoretical Grossman-Nir bound [18] are also shown.

1.2 $K^0 \rightarrow \mu^+ \mu^-$ decay

The branching ratio of the $K_L \rightarrow \mu^+ \mu^-$ decay is driven by the LD contribution, with a subdominant CP-conserving SD component [19]. The SM prediction exhibits large uncertainties due to the ambiguity of the sign of the interference between the LD and SD terms contributing to the decay amplitude. This decay has been studied experimentally and the measured branching ratio is $(6.84 \pm 0.11) \times 10^{-9}$ [20]. The uncertainty of the SM prediction prevents a clean theoretical interpretation of this result.

The $K_S \rightarrow \mu^+ \mu^-$ decay is CP-violating and theoretically cleaner than the corresponding K_L mode, but more suppressed, and the SM predicts the branching ratio to be $(5.0 \pm 1.5) \times 10^{-12}$ [21]. New physics can increase the branching ratio of this decay up to 10^{-11} [22].

Recent studies have shown that the effective branching ratio of a $\mu^+ \mu^-$ final state from a $K^0 - \bar{K}^0$ mixture is also sensitive to NP and can resolve the sign ambiguity of the $K_L \rightarrow \mu^+ \mu^-$ decay [23].

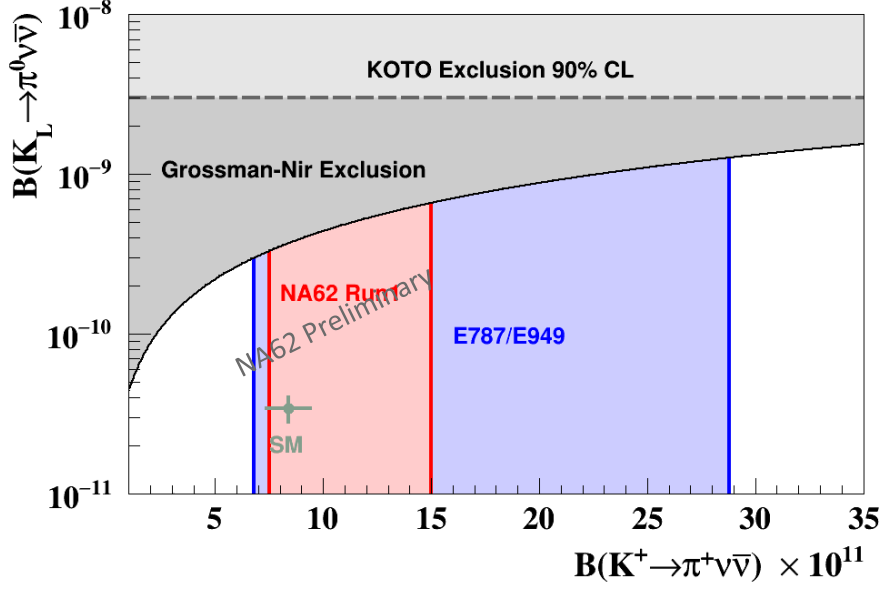


Figure 2: State of the art of the measurements of the branching ratios of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decays. The band labelled NA62 Run1 represents the latest preliminary result from NA62 using data from 2016-2017-2018, where the width of the band corresponds to the measurement uncertainty. The band labelled E787/E949 displays the previous BNL measurements of the branching ratio of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decays. The upper bound on the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ branching ratio comes from the analysis of 2015 data taken by the KOTO experiment. The SM prediction of the two branching ratios and the region forbidden by the Grossman-Nir limit are also shown.

The study of the $K_S \rightarrow \mu^+ \mu^-$ decay is the main goal of the kaon physics program of the LHCb experiment at CERN LHC. The high production rate of kaon mesons in proton-proton collisions at 13 TeV partly compensates the low acceptance of LHCb in detecting kaon decays due to the long lifetime. The trigger employed by LHCb in Run 1 and Run 2 further suppressed kaon decays, because optimized for particles expected from the decay of heavy flavour mesons. During the 2012 data taking, a trigger dedicated to $K_S \rightarrow \mu^+ \mu^-$ was implemented. The analysis of data from Run 1 and 2 has allowed LHCb to set the most stringent limit to the branching ratio of the $K_S \rightarrow \mu^+ \mu^-$ decay, corresponding to $BR(K_S \rightarrow \mu^+ \mu^-) < 2.1(2.4) \times 10^{-10}$ at 90%(95%) CL [24]. A fully dedicated trigger will allow LHCb to exploit a rich kaon physics program starting from Run 3. This program focuses on K_S physics, with the main purpose to reach the SM sensitivity of the $K_S \rightarrow \mu^+ \mu^-$ decay, and also to study rare kaon decays like $K_S \rightarrow \pi^0 \mu^+ \mu^-$ and $K_S \rightarrow \mu^+ \mu^- \mu^+ \mu^-$.

2. Rare K decays and explicit SM violation

New physics can manifest itself not necessarily in terms of high mass scale particles affecting the Yukawa sector of the SM. Kaon decays gives also the possibility to probe processes that explicitly violate the SM and are signature of physics BSM per se, like violation of lepton universality (LUV), lepton number and flavour (LNV/LFV).

Tests of lepton universality arises from the study of the rare decays $K^+ \rightarrow \pi^+\mu^+\mu^-(e^+e^-)$ and from the precise measurement of the observable R_K , defined as the ratio of the decay amplitudes of the $K^+ \rightarrow e^+\nu$ and $K^+ \rightarrow \mu^+\nu$ decays. Indications of LNV/LFV processes can come from the search for decays forbidden within the SM like $K^+ \rightarrow \pi\mu e$, $K_L \rightarrow (\pi^0)\mu e$, $K^+ \rightarrow \pi^-\mu^+\mu^+(e^+e^-)$.

Kaon mesons can also test NP models predicting feebly interacting low-mass particles, like the search for the production of heavy neutral leptons (HNL) in $K^+ \rightarrow e^+N, \mu^+N$ decays or dark scalars in $K^+ \rightarrow \pi^+X$ decays.

2.1 LUV with $K^+ \rightarrow \pi^+l^+l^-$

The amplitudes of the $K^+ \rightarrow \pi^+l^+l^-$ decays ($l = e, \mu$) are LD dominated and their branching ratio are about 10^{-7} . The differential decay amplitude of these decays with respect to the di-lepton invariant mass depends on two form factor parameters, termed a and b , that lepton universality predicts independent from the flavour of the leptons. Differences between a and b of the electron and muon channels can be correlated to LUV effects explaining possible observed lepton anomalies in B physics [25, 26].

The NA62 experiment has recently reported a new measurement of the form factors of the $K^+ \rightarrow \pi^+\mu^+\mu^-$ decay using data collected in 2017 and 2018. The very precise kinematic resolution of the NA62 experiment allowed an almost background-free selection of about 30×10^3 $K^+ \rightarrow \pi^+\mu^+\mu^-$ decays. The decay $K^+ \rightarrow \pi^+\pi^+\pi^-$ was used for normalization and the parameters a and b were extracted from a fit to the spectrum of the square of the di-lepton invariant mass, normalized to the square of the kaon mass. The relative precision achieved on a and b is 2% and 8%, significantly better than previous measurements (Figure 3). The results are also consistent within uncertainty with previous measurements of the same parameters performed both in the muon and electron channels [27–30]. The NA62 experiment is expected to increase significantly the precision of this test in the next years, addressing also the measurement of the $K^+ \rightarrow \pi^+e^+e^-$ form factors, thanks to the planned data taking periods at the restart of the CERN acceleration complex, and to the implementation of a dedicated trigger.

2.2 LFV/LNV with $K^+ \rightarrow \pi\mu e$ decay

Several NP models predicting violation of lepton number or flavour conservation allow for the decays $K^+ \rightarrow \pi^\pm\mu^\mp e^+$ to occur up to levels that can be tested by present experiments [31–34].

The NA62 experiment has searched for the signature of $K^+ \rightarrow \pi^+\mu^-e^+$ and $K^+ \rightarrow \pi^-\mu^+e^+$ decays, analyzing the data collected in 2017 and 2018. The experimental signature was three tracks consistent with closed kinematics of the K^+ decay. The background suppression exploited both the precise kinematic reconstruction and the high performing particle identification system of NA62. The signal region is defined in terms of the invariant mass of the three charged particles around the nominal K^+ mass, with $1.4 \text{ MeV}/c^2$ resolution. About one background event was expected in signal region in both the decay modes, due to $K^+ \rightarrow \pi^+\pi^+\pi^-$ decays, with one pion mis-identified as electron and another one decayed in flight. After a blind analysis procedure 0 events were observed in the $K^+ \rightarrow \pi^-\mu^+e^+$ and 2 in the $K^+ \rightarrow \pi^+\mu^-e^+$ channel. Figure 4 (left panel), shows the $\pi\mu e$ invariant mass of the $K^+ \rightarrow \pi^-\mu^+e^+$ channel, with zero events observed in the signal region denoted by the narrow band at the mass of the K^+ . Using $K^+ \rightarrow \pi^+\pi^+\pi^-$ decays

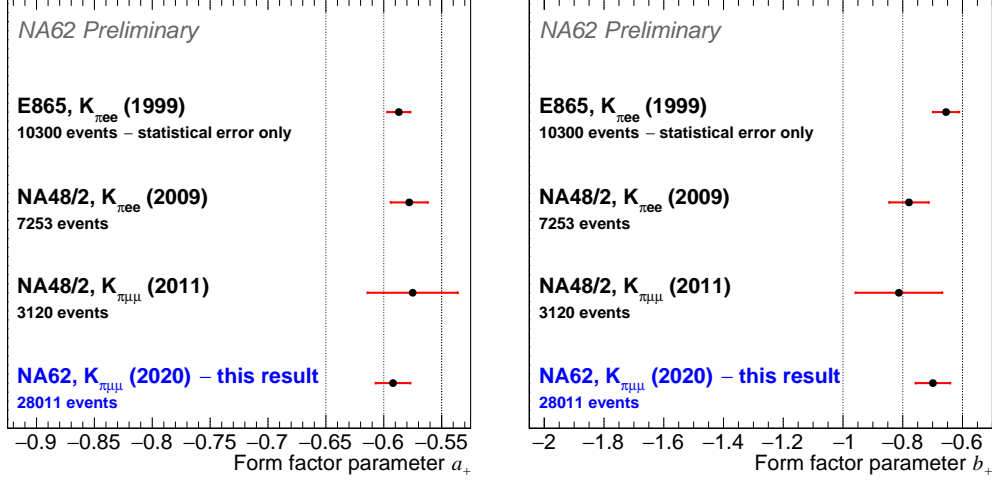


Figure 3: Measurement of the form factor parameters a (left panel) and b (right panel) of the $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay. The latest preliminary results of NA62 are shown together with previous measurements [27–30].

for normalization, the observed events lead to the upper limits at 90% CL on the branching ratios, $BR(K^+ \rightarrow \pi^- \mu^+ e^+) < 4.2 \times 10^{-11}$ and $BR(K^+ \rightarrow \pi^+ \mu^- e^+) < 6.6 \times 10^{-11}$, about an order of magnitude lower than the previous limits [35].

2.3 HNL production in $K^+ \rightarrow l^+ N$ decay

Extensions of the SM able to generate non-zero masses of SM neutrinos via seesaw mechanism predict the existence of heavy neutral leptons with masses in the MeV-GeV range [36, 37]. These particles are expected to couple feebly with SM fields and can be produced in $K^+ \rightarrow l^+ N$ decays.

The NA62 experiment has performed a search of HNL both in the electron and muon channels, using data collected in 2017 and 2018. The signal signature of one K^+ track in initial state and one track in final state, with no additional energy detected, perfectly suited the design of the experimental apparatus. The analysis technique exploited a mass peak search over the squared missing mass defined as the square of the difference of the K^+ and lepton 4-momenta. The search was performed in regions of the missing mass off the $K^+ \rightarrow \mu^+ \nu$ and $K^+ \rightarrow e^+ \nu$ peaks. The main background to $K^+ \rightarrow \mu^+ N$ came from the resolution and radiative tails of the $K^+ \rightarrow \mu^+ \nu$ missing mass. The background to the electronic channel was mainly due to $K^+ \rightarrow \mu^+ \nu$ with the μ^+ decaying in flight. The result of the peak search is shown in Figure 4 (right panel), where the 90% CL upper limit to the lepton-HNL coupling constant squared $|U_{l4}|^2$ as a function of the HNL mass hypothesis is shown. The limits to $|U_{l4}|^2$ in the electron channel [38] exclude up to about 350 MeV/ c^2 the values favoured by the constraint from Big Bang Nucleosynthesis (BBN). The limits to $|U_{l4}|^2$ in the muon channel significantly improve over the past searches above 300 MeV/ c^2 .

3. Future Prospects

KOTO aims to reach the single event sensitivity for the SM $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay by about 2025. The NA62 experiment expects to measure the branching ratio of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ with 10%

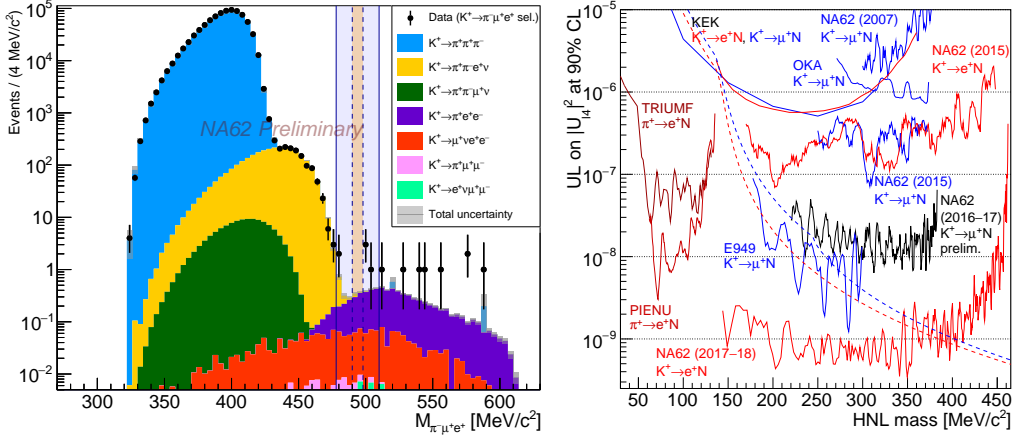


Figure 4: Left: invariant mass of $\pi\mu e$ of $K^+ \rightarrow \pi^- \mu^+ e^+$ decays for data and monte carlo simulations of the background modes. Simulations are normalized to the measured K^+ flux. The narrow band at the mass of the K^+ is the signal region, the light gray bands around it are the control regions used to validate on data the expected background. Right: 90% CL upper limit on the square of the coupling constant of a lepton and the HNL, as a function of the HNL mass hypothesis. The NA62 final result of the $K^+ \rightarrow e^+ N$ (2017-18) search [38] and the preliminary result of the $K^+ \rightarrow \mu^+ N$ search are shown, together with a compilation of previous results. The dotted lines denote the lower bound of the region allowed by BBN for the electron and muon channel.

precision, using data to be collected in 2021–2024. LHCb plans to attain a sensitivity for a 10^{-11} $K_S \rightarrow \mu^+ \mu^-$ decay in the upcoming LHC data taking and to attack challenging decay modes like $K_S \rightarrow \pi^0 l^+ l^-$.

A possible KOTO step-2 upgrade is under study and requires construction of a planned extension of the J-PARC hadron hall, as well as of a completely new detector several times larger than the present detector. The goal of the upgrade is to measure the branching ratio of the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay with 20% precision. Following upon the success to study the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay with a high energy kaon beam, a comprehensive program to investigate rare decays of K^+ and K_L could be launched after 2025 at CERN, exploiting a high-intensity kaon beams facility from SPS. This program could proceed in multiple steps: first a charged kaon experiment to measure $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ branching ratio with 5% precision; then a neutral kaon experiment to study the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay [39] that, combined with KOTO step-2, will push the precision of the branching ratio measurement below 15%. The detectors of these charged and neutral kaon experiments at CERN could also be configured to allow the study of the CP-violating $K_L \rightarrow \pi^0 l^+ l^-$ decay. On a similar time scale LHCb will attain the sensitivity for the SM $K_S \rightarrow \mu^+ \mu^-$ decay and could precisely study the $K_S \rightarrow \pi^+ l^+ l^-$ decay, essential to pin down the SD contribution of the corresponding K_L decay. The ultimate goal of this program is to over-constrain the unitarity triangle using loop level observables from kaon physics only, that, by comparison with B physics, could give insights into the flavour structure of possible physics BSM.

The upcoming new data taking periods and future prospects of high intense kaon facility will also boost down of an order of magnitude at least the sensitivity to investigate lepton universality

conservation and to search for signatures that are explicit violation of the SM.

4. Conclusions

Rare kaon decays can probe a variety of signature of NP: NP manifesting itself in the flavour structure of the quarks, or appearing as violation of lepton universality and lepton flavour and number violation, or emerging as new low mass feebly interacting particles. Much progresses have been made recently in the measurement of the $K \rightarrow \pi\nu\bar{\nu}$ branching ratio, paving the way to become precision measurements in upcoming years to allow the study of physics at the shortest distance scales. The high intensity frontier is the future of kaon physics, that will complement and integrate the future programs of experimental particle physics in the quest for physics BSM.

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