Higher Curvature Supergravity

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We discuss higher curvature (super) gravity in four dimensions. We describe the spectrum of the theory which contains a massive spin-two ghost and we propose a way to turn it into a canonical massive spin-two field at the linearized level.

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Higher curvature theories are theories of the general form $R + \mathscr{R}^n$, where R is the Ricci scalar describing Einstein gravity and $\mathscr{R}^n$ denotes n^{th} power of the Weyl, Riemann and Ricci tensors and the curvature scalar [1, 2, 3, 4, 5, 6, 7, 8]. This class of models has also been explored in [9]. In particular, as has been noted in [1], the inclusion of quadratic curvature terms in the action makes the theory renormalizable. But there is a price to be paid namely the appearance of a massive ghost states. This ghost state originates from the square of the Riemann, the Ricci or the Weyl tensor. A particular class of such models, the $R + R^2$ supergravity has been studied [10] and it is connected to the inflationary dynamics of the Starobinsky model [11]. This models propagates the usual massless graviton and a massive spin-0 state, the "scalaron". The latter can be identified with the inflaton field. Therefore, in the $R + R^2$ theory, inflation is driven entirely by the gravity itself, without the need for an extra scalar and thus making the theory quite appealing. In addition, the theory can easily be embedded in $\mathscr{N} = 1$ supergravity, and its spectrum in has been determined in [13] and [14].

However, the $R + R^2$ theory should also include only the square of the Weyl tensor which is also quadratic in the curvature since terms quadratic in the Riemann (or Ricci) tensor are related to the square of the Weyl or the scalar curvature by the 4D Gauss-Bonnet topological term. The inclusion of the Weyl tensor now leas to a considerable problem: the theory now propagates a ghost spin-2 massive state [1]. It should be noted here that such states leads to the impossibility of maintain at one shot both unitarity and propagation of positive energy states. Indeed, using a $+i\varepsilon$ choice, opposite to the stanfdard one, one may achieve unitarity but in this case, negative energies are propagaiting forward in time.

Here we will describe the massive Weyl^2 theory, which contains also the standard Einstein-Hilbert action, and its supersymmetric extensions. The latter are the $\mathscr{N}$-extended Weyl^2 super-gravities. The inclusion of the Einstein-Hilbert action can be regarded as a mass deformation of the massless theory. In addition, the maximal $\mathscr{N} = 8$ of Poincare supergravity corresponds to $\mathscr{N} = 4$ Weyl supergravity [15].

An interesting limit is the massless limit $m \to 0$ of the $m^2 R + \text{Weyl}^2$ theory [16, 17, 18] of the conformal supergravity [19, 20, 21]. These theories are very interesting although they propagate ghost sates as in the massless limits, i.e. no Einstein-Hilbert term, they are simple examples of superconformal theories with four derivative terms.

Let us now discuss the linearized super-Weyl^2 theory. The super-Weyl tensor $\mathscr{W}_{\alpha\beta\gamma}$ is a chiral superfield, where $\alpha, \beta, \gamma = 1, 2$ are standard $SL(2, \mathbf{C})$ spinor indices. The spin of the highest component of the Weyl superfield $\mathscr{W}_{\alpha\beta\gamma}$ is a fermionic spin-3/2 field and therefore, the spin of the Weyl multiplet is is $(\frac{3}{2}, 0)$. It also contains the spin-two field $g_{\mu\nu}$ with five degrees of freedom and it satisfies

$$\mathscr{W}_{\alpha\beta\gamma} = \mathscr{W}_{(\alpha\beta\gamma)} \,, \; (\mathscr{W}_{\alpha\beta\gamma})^* = \overline{\mathscr{W}}_{\dot\alpha\dot\beta\dot\gamma} \,, \; \overline{\mathscr{D}}_{\dot\delta}\mathscr{W}_{\alpha\beta\gamma} = 0 \,, \tag{1}$$

Its bosonic content follows from

$$\mathscr{D}_\gamma \mathscr{W}_{\delta\varepsilon\alpha}| = \frac{1}{6}\left(\frac{i}{4}\varepsilon_{\gamma\delta}\mathscr{D}_{\varepsilon\dot\varepsilon}A_\alpha^{\dot\varepsilon} - \frac{1}{4}\varepsilon_\varepsilon^\beta \varepsilon_\gamma^\rho W_{\delta\beta\alpha\rho} \right) + (\delta\,\varepsilon\,\alpha) \,, \tag{2}$$

where $W_{\delta\beta\alpha\rho}$ and $A_\alpha^{\dot\varepsilon}$ are the Weyl tensor and the vector auxiliary of the $\mathscr{N} = 1$ supergravity (in spinorial notation). $(\delta\,\varepsilon\,\alpha)$ denotes the additional five terms from the symmetrization of the fermionic indices δ, ε and α [31].

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It turns out that the supersymmetric action of the massive Weyl gravity is

$$\mathscr{L}_{\Phi,\mathscr{W}} = \int d^2\Theta \, 2\mathscr{E}\mathscr{R} + 4\int d^2\Theta \, 2\mathscr{E}\,\tau\,\mathscr{W}^2 + c.c., \tag{3}$$

where we have denoted as τ the complex coupling

$$\tau = \frac{1}{g^2} + i\alpha. \tag{4}$$

The bosonic sector of (3) turns out to be

$$e^{-1}\mathscr{L}_{\mathscr{W}} = \frac{1}{2}R + \frac{1}{3}A_\mu A^\mu - \frac{1}{3}u\bar{u} + \frac{\alpha}{2}\left(\frac{1}{2}R^2_{HP} - \frac{2}{3}F_{\mu\nu}F_{\rho\sigma}\varepsilon^{\mu\nu\rho\sigma}\right) + \frac{1}{2g^2}\left(W^{\mu\nu\rho\sigma}W_{\mu\nu\rho\sigma} - \frac{4}{3}F^{\mu\nu}F_{\mu\nu}\right), \tag{5}$$

where $R^2_{HP} = W_{\mu\nu\kappa\lambda}\varepsilon^{\kappa\lambda\rho\sigma}W_{\rho\sigma}{}^{\mu\nu}$ is the topological Hirzebruch-Pontryagin term, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of the "auxiliary" A_μ and u is the auxiliary scalar. Note however, that the vector "auxiliary" A_μ is dynamical in Weyl supergravity.

It is now straightforward to determine the spectrum of the massive $\mathscr{N} = 1$ Weyl supergravity. It contains the following states:

(i) The standard massless spin-2 graviton multiplet (h, q_R) with helicity h and q_R $U(1)_R$ charges, with $n_B + n_F = 4$ dof:

$$g_{\mathscr{N}=1}: \; (+2,0) + (+\frac{3}{2},+\frac{1}{2}), \tag{6}$$

together with its CPT conjugate

$$(-\frac{3}{2},-\frac{1}{2}) + (-2,0). \tag{7}$$

(ii) A massive spin-two supermultiplet in the non-standard sector. This is the massive $\mathscr{N} = 1$ super-Weyl multiplet [33] with $n_B + n_F = 16$ dof:

$$w_{\mathscr{N}=1}: \; \text{Spin}(2) + \underline{2} \times \text{Spin}(3/2) + \text{Spin}(1). \tag{8}$$

The massive states in $\mathscr{N} = 1$ supergravity are arranged in $USp(2)$ representations.

Therefore, the massive super-(Weyl)^2 gravity theory contains a total of $n_B + n_F = 20$ dof.

Similarly, the spectrum of the all existing $\mathscr{N}$-extended Weyl^2 supergravities ($\mathscr{N} = 1,2,3,4$) in 4D can be determined. The physical spectrum of the various Weyl-supergravity theories is summarized in the following table. The top helicities of each multiplet together with its CPT conjugate are denoted in the first column [16, 17]:

	$\mathcal{N}=4$	$\mathcal{N}=3$	$\mathcal{N}=2$	$\mathcal{N}=1$
$h_{\max}=2$	2			
$h_{\max}^{\mathrm{CPT}}=0$	2			
$h_{\max}=3/2$	4			
$h_{\max}^{\mathrm{CPT}}=1/2$	4			
$h_{\max}=2$		2		
$h_{\max}^{\mathrm{CPT}}=-1/2$		2		
$h_{\max}=3/2$		3		
$h_{\max}^{\mathrm{CPT}}=0$		3		
$h_{\max}=1$		1		
$h_{\max}^{\mathrm{CPT}}=1/2$		1		
$h_{\max}=2$			2	
$h_{\max}^{\mathrm{CPT}}=-1$			2	
$h_{\max}=3/2$			2	
$h_{\max}^{\mathrm{CPT}}=-1/2$			2	
$h_{\max}=1$			1	
$h_{\max}^{\mathrm{CPT}}=0$			1	
$h_{\max}=2$				2
$h_{\max}^{\mathrm{CPT}}=-3/2$				2
$h_{\max}=3/2$				1
$h_{\max}^{\mathrm{CPT}}=-1$				1
$h_{\max}=1$				1
$h_{\max}^{\mathrm{CPT}}=-1/2$				1

$$(9)$$

The bosonic massive Weyl^2 gravity theory in four dimensions is described by has the following action

$$S = \int d^4x \sqrt{-g}\left(\frac{a}{2}W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma} + \kappa^2 R\right), \tag{10}$$

where $W_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - g_{\mu[\sigma}R_{\rho]\nu} - g_{\mu[\rho}R_{\sigma]\nu} - R/3\,g_{\mu[\rho}g_{\sigma]\nu}$ is the Weyl tensor. The Weyl^2-term is conformal invariant since under the conformal transformation

$$g_{\mu\nu} \to \widehat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \tag{11}$$

the Weyl tensor is inert

$$\widehat{W}^{\mu}{}_{\nu\rho\sigma} = W^{\mu}{}_{\nu\rho\sigma}. \tag{12}$$

Contrary, the Einstein-Hilbert term transforms under conformal transformations, as

$$\widehat{R} = \Omega^{-2}R - 6\Omega^{-3}g^{\mu\nu}\nabla_\mu\nabla_\nu\Omega. \tag{13}$$

and thus, it can be regarded as a mass term in the theory.

The eom can be written as

$$B_{\mu\nu} + \frac{2\kappa^2}{a}G_{\mu\nu} = 0, \tag{14}$$

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where $G_{\mu\nu}$ is the Einstein tensor and the Bach tensor $B_{\mu\nu}$ is given by

$$B_{\mu\nu} = \nabla^\rho \nabla_\sigma W^\sigma_{\mu\rho\nu} + \frac{1}{2} R^{\rho\sigma} W_{\rho\mu\sigma\nu}. \tag{15}$$

The Bach tensor transforms covarinatly under conformal transformations

$$\widehat{B}_{\mu\nu} = \Omega^{-2} B_{\mu\nu}. \tag{16}$$

It is also symmetric and traceless due to conformal invariance as well as divergence-free due to diff-invariance

$$B^\mu_\mu = 0, \quad \nabla^\mu B_{\mu\nu} = 0. \tag{17}$$

To find the spectrum of the theory one should determine the poles in the propagators. It turns out that we have [1]

(i) A helicity-± 2 massless standard graviton $g_{\mu\nu}$.

(ii) A massive spin-two particle $w_{\mu\nu}$ with mass κ^2/a. It is due to the Weyl^2 term and it is is a ghost for $a > 0$ or a tachyon for $a < 0$. This is the non-standard sector of the theory.

Therefore, the massive (Weyl)^2 gravity theory contains seven propagating dof.

An alternative way to identify the spectrum of the bosonic massive Weyl theory is to express it as a bimetric gravity theory with two spin-2 fields $g_{\mu\nu}$ and $w_{\mu\nu}$. Consider for example the action

$$S = \int_{\mathscr{M}} d^4x \sqrt{-g} \left(M_P^2 R + c_1 W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma} + c_2 R^2 \right). \tag{18}$$

Now, let us introduce a second symmetric tensor field $w_{\mu\nu}$. It can be shown that the two-derivative action

$$S = \int_{\mathscr{M}} d^4x \sqrt{-g} \left(M_P^2 R(g) + \frac{2}{g_W} G_{\mu\nu}(g) w^{\mu\nu} - (w^{\mu\nu} w_{\mu\nu} - aw^2) \right). \tag{19}$$

where $G_{\mu\nu} = R_{\mu\nu} - 1/2R\, g_{\mu\nu}$ is the Einstein tensor for the metric $g_{\mu\nu}$ and the last term describes a mass term for $w_{\mu\nu}$ is classically equivalent to (18). Indeed, using the equation of motion for $w_{\mu\nu}$

$$\frac{\delta S}{\delta w_{\mu\nu}} \quad \Rightarrow \quad w_{\mu\nu} = \frac{1}{g_W} \left(R_{\mu\nu}(g) - \frac{1}{6} g_{\mu\nu} R \right), \tag{20}$$

in the action (19), it turns out that the theory is classically equivalent to (18) since

$$W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma} = GB + 2(R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2), \tag{21}$$

where GB is the Gauss-Bonnet term. In fact we find that

$$c_1 = \frac{1}{2g_W^2}, \quad c_2 = \frac{a-1}{4a-1} \frac{1}{3g_W^2}. \tag{22}$$

Therefore, for $a = 1$ the scalar mode associated to the R^2 term is absent and in this case, (19) describes massive Weyl gravity.

Eq.(19) describes a massless spin-2 $g_{\mu\nu}$ together with massive spin-2 $w_{\mu\nu}$ (for $\alpha = 1$). Note that the kinetic term for the massive spin-2 $w_{\mu\nu}$ is implicit in the in mixed term $G_{\mu\nu}(g)w^{\mu\nu}$. In order to explicitly see this, let us consider fluctuations $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ around Minkowski spacetime (metric $\eta_{\mu\nu}$). Then we find that to quadratic order we have

$$\mathscr{S}_2 = \int d^4x \sqrt{-g} \left\{ -\frac{M_P^2}{2} h^{\mu\nu} \mathscr{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} + \frac{1}{g_W} w^{\mu\nu} \mathscr{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} - \left(w^{\mu\nu} w_{\mu\nu} - w^2 \right) \right\}, \tag{23}$$

where $g_W = M_W/M_P$ and $\mathscr{E}_{\mu\nu}^{\alpha\beta}$ is as usual the tensor

$$\mathscr{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} = -\frac{1}{2} \left\{ \Box h_{\mu\nu} - 2\partial_{(\mu} \partial_{\alpha} h_{\nu)}^{\alpha} + \partial_{\mu}\partial_{\nu} h - \eta_{\mu\nu} \left(\Box h - \partial_{\alpha}\partial_{\beta} h^{\alpha\beta} \right) \right\}. \tag{24}$$

Let us now define a new field

$$\overline{h}_{\mu\nu} = h_{\mu\nu} - \frac{2}{g_W M_P^2} w_{\mu\nu}, \tag{25}$$

in tersm of which, the quadratic action (23) turns out to be written as

$$\mathscr{S}_2 = \int d^4x \sqrt{-g} \left\{ -\frac{M_P^2}{2} \overline{h}^{\mu\nu} \mathscr{E}_{\mu\nu}^{\alpha\beta} \overline{h}_{\alpha\beta} + \frac{1}{g_W^2 M_P^2} w^{\mu\nu} \mathscr{E}_{\mu\nu}^{\alpha\beta} w_{\alpha\beta} - \left(w^{\mu\nu} w_{\mu\nu} - w^2 \right) \right\}. \tag{26}$$

The field $\overline{h}_{\mu\nu}$ is clearly the massless graviton and $w_{\mu\nu}$ is a second massive spin-2 tensor field. Note however that the wrong sign (compared to $\overline{h}_{\mu\nu}$) of the kinetic term of $w_{\mu\nu}$ indicating that the second spin-2 tensor mode is actually a ghost. The bimetric gravity theory (19) may be turned into a ghost-free theory by the inclusion of an infinite number of terms. This procedure has been shown in [34] to be equivalent to adding an infinite number of higher derivative terms.

Another way to remove the ghost stae is the following. Let us add to the action (19) a term of the form

$$c w^{\mu\nu} \mathscr{E}_{\mu\nu}^{ab} w_{ab} \tag{27}$$

with

$$c < -\frac{2}{g_w^2 M_P^2} \tag{28}$$

This term clearly removes the ghost, and, thus the theory will contain a graviton and a massive spin-2 particle with canonical kinetic term. The action in this case reads

$$S = \int d^4x \sqrt{-g} \left(M_P^2 R + \frac{2}{g_w} G_{\mu\nu} w^{\mu\nu} - \left(w^{\mu\nu} w_{\mu\nu} - w^2 \right) + c w^{\mu\nu} \mathscr{E}_{\mu\nu}^{ab} w_{ab} \right) \tag{29}$$

Notice that the derivatives and the metric that appear in the definition of $\mathscr{E}$ are now covariant derivatives and the full metric $g_{\mu\nu}$ in the previous action. The new EOM read

$$\frac{1}{g_w} G_{\mu\nu} - w_{\mu\nu} + w g_{\mu\nu} - \frac{c}{2} \left[\Box w_{\mu\nu} - 2\nabla_{(\mu} \nabla^a w_{\nu)a} + \nabla_{\mu}\nabla_{\nu} w - g_{\mu\nu} \left(\Box w - \nabla_a \nabla_b w^{ab} \right) \right] = 0 \tag{30}$$

or

$$\frac{1}{g_w} G_{\mu\nu} = \left[\delta_{\mu}^a \delta_{\nu}^b - g_{\mu\nu} g^{ab} + \frac{c}{2} \left(\Box \delta_{\mu}^a \delta_{\nu}^b - \nabla_{\mu}\nabla^a \delta_{\nu}^b - \nabla_{\nu}\nabla^a \delta_{\mu}^b + \nabla_{\mu}\nabla_{\nu} g^{ab} - g_{\mu\nu} g^{ab} \Box + g_{\mu\nu} \nabla^a \nabla^b \right) \right] w_{ab} \tag{31}$$

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and

$$M_P^2 G_{\mu\nu} = \frac{1}{g_w}\left[\Box w_{\mu\nu} - \nabla_\sigma\nabla_\mu w_\nu^\sigma - \nabla_\sigma\nabla_\nu w_\mu^\sigma + R_{\mu\nu}w - Rw_{\mu\nu} + \nabla_{(\mu}\nabla_{\nu)}w \right.$$

$$\left. - 2\left(G_\mu^a w_{\nu a} + G_\nu^a w_{\mu a}\right) + g_{\mu\nu}\left(G_{ab}w^{ab} - \frac{g_w}{2}\left(w^{ab}w_{ab} - w^2\right) + \nabla_a\nabla_b w^{ab} - \Box w\right) \right]$$

$$+ 2\left(w_{a(\mu}w_{\nu)}^a - ww_{\mu\nu}\right) - 2cw_{\kappa\lambda}\left[\frac{\delta}{\delta g^{\mu\nu}}\left(g^{\kappa\sigma}g^{\lambda\tau}\mathscr{E}_{\sigma\tau}^{ab}\right)\right]w_{ab} + \frac{c}{2}g_{\mu\nu}w^{\kappa\lambda}\mathscr{E}_{\kappa\lambda}^{ab}w_{ab} \quad (32)$$

Flat space-time $g_{\mu\nu} = \eta_{\mu\nu}$ along with $w_{\mu\nu} = 0$ remains a solution to (31) and (32). Linearizing (31) around this background yields

$$\frac{1}{g_w}\delta G_{\mu\nu} = \left[\delta_\mu^a\delta_\nu^b - \eta_{\mu\nu}\eta^{ab} + \frac{c}{2}\left(\Box\delta_\mu^a\delta_\nu^b - \partial_\mu\partial^a\delta_\nu^b - \partial_\nu\partial^b\delta_\mu^a + \partial_\mu\partial_\nu\eta^{ab} - \eta_{\mu\nu}\eta^{ab}\Box + \eta_{\mu\nu}\partial^a\partial^b\right)\right]w_{ab} \quad (33)$$

with

$$\delta G_{\mu\nu} = \frac{1}{2}\left(\partial_\mu\partial^\lambda h_{\nu\lambda} + \partial_\nu\partial^\lambda h_{\mu\lambda} - \Box h_{\mu\nu} - \partial_\mu\partial_\nu h - \eta_{\mu\nu}\partial^\kappa\partial^\lambda h_{\kappa\lambda} + \eta_{\mu\nu}\Box h\right) \quad (34)$$

The linearized action is then given by

$$S^{(2)} = \int d^4x\left[-\frac{M_P^2}{2}\bar{h}^{\mu\nu}\mathscr{E}_{\mu\nu}^{ab}\bar{h}_{ab} - \frac{1}{\bar{g}_w^2 M_P^2}w^{\mu\nu}\mathscr{E}_{\mu\nu}^{ab}w_{ab} - \left(w^{\mu\nu}w_{\mu\nu} - w^2\right)\right], \quad (35)$$

where

$$-\frac{1}{\bar{g}_w^2 M_P^2} = c + \frac{2}{g_w^2 M_P^2} \quad (36)$$

We can decompose $\bar{h}_{\mu\nu}$ and $w_{\mu\nu}$ as

$$\bar{h}_{\mu\nu} = \bar{h}_{\mu\nu}^\perp + \partial_\mu A_\nu^\perp + \partial_\nu A_\mu^\perp + \left(\partial_\mu\partial_\nu - \frac{1}{4}\eta_{\mu\nu}\Box\right)B + \frac{1}{4}\eta_{\mu\nu}\bar{h} \quad (37)$$

$$w_{\mu\nu} = w_{\mu\nu}^\perp + \partial_\mu C_\nu^\perp + \partial_\nu C_\mu^\perp + \left(\partial_\mu\partial_\nu - \frac{1}{4}\eta_{\mu\nu}\Box\right)D + \frac{1}{4}\eta_{\mu\nu}w \quad (38)$$

where $\bar{h}_{\mu\nu}^\perp$ and $w_{\mu\nu}^\perp$ are transverse traceless and $C_\nu^\perp$ is transverse. Then the action (35) becomes

$$S^{(2)} = \int d^4x\left[\frac{M_P^2}{4}\left(\bar{h}^{\perp\mu\nu}\Box\bar{h}_{\mu\nu}^\perp - \frac{3}{8}\left(\bar{h} - \Box B\right)\Box\left(\bar{h} - \Box B\right)\right)\right.$$

$$+ \frac{1}{2\bar{g}_w^2 M_P^2}\left(w^{\perp\mu\nu}\Box w_{\mu\nu}^\perp - \frac{3}{8}\left(w - \Box D\right)\Box\left(w - \Box D\right)\right)$$

$$\left. + 2C_\mu^\perp\Box C^{\perp\mu} - \frac{3}{4}\left(w - \Box D\right)\left(w + \Box D\right) - w_{\mu\nu}^\perp w^{\perp\mu\nu}\right] \quad (39)$$

We introduce the following scalars

$$\phi = \bar{h} - \Box B \quad , \quad S = w - \Box D \quad , \quad S' = w + \Box D \quad (40)$$

so that (39) turns out to be

$$S^{(2)} = \int d^4x\left[\frac{M_P^2}{4}\left(\bar{h}^{\perp\mu\nu}\Box\bar{h}_{\mu\nu}^\perp - \frac{3}{8}\phi\Box\phi\right)\right.$$

$$\left. + \frac{1}{2\bar{g}_w^2 M_P^2}\left(w^{\perp\mu\nu}\Box w_{\mu\nu}^\perp - \frac{3}{8}S\Box S\right) + 2C_\mu^\perp\Box C^{\perp\mu} - \frac{3}{4}SS' - w_{\mu\nu}^\perp w^{\perp\mu\nu}\right] \quad (41)$$

There is no kinetic term for the field S' and by using its EOM

$$S' = -\frac{1}{2\bar{g}_w^2 M_P^2} \Box S \qquad (42)$$

the action becomes

$$S^{(2)} = \int d^4x \left[\frac{M_P^2}{4} \left(\bar{h}^{\perp\mu\nu} \Box \bar{h}_{\mu\nu}^\perp - \frac{3}{8} \phi \Box \phi \right) + \frac{1}{2\bar{g}_w^2 M_P^2} \left(w^{\perp\mu\nu} \Box w_{\mu\nu}^\perp + \frac{3}{8} S \Box S \right) + 2C_\mu^\perp \Box C^{\perp\mu} - w_{\mu\nu}^\perp w^{\perp\mu\nu} \right] \qquad (43)$$

Note that ϕ is the ghost of standard General Relativity. The propagating degrees of freedom are therefore, a massless spin-2 field $\bar{h}_{\mu\nu}^\perp$, a massive spin-2 field $w_{\mu\nu}^\perp$, a massless spin-1 field $C_\mu^\perp$ and a scalar S.

Acknowledgements:

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