

# Instructive Review of Novel SFT with Non-interacting constituents “objects”, and attempt Generalization to p-adic theory

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We have constructed a new formalism for describing a situation with **several (dual) strings** present at a time, a **string field theory**, by means of a constituent / a strings from objects picture similar to, but importantly different from the “bits” by Charles Thorn[1]. Our “objects” (essentially the bits) represent rather a making a lattice in the **light cone variables** on the string. The remarkable feature and simplicity of our formalism is, that the “objects” do **NOT interact**, basically just run or sit trivially fixed. **Scattering is a fake** in our formalism.

This opens also up for hoping for generalizations inspired by hadrons with their partons all having Bjorken variable  $x = 0$ , and thus infinitely many constituents. The p-adic string is an example.

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## 1. Introduction

We have worked since long on a formalism for a string field theory[2] with the possibility of describing an arbitrary number of strings, in other words a string field theory, which we called a Novel SFT.[3] Our “Novel SFT” has same spirit as the before ours developed string from bits reformulation of string theory by Charles Thorn [1]. In Charles Thorn’s theory the string parametrized by the parameter along the string usually called  $\sigma$  is split into string-bits by discretizing the parameter  $\sigma$ . I.e. each string bit corresponds to a very small interval in the parameter  $\sigma$ . The crucial difference between our string and that of Charles Thorn is that we before going to the splitting into bits consider the wellknown conformal gauge choice formalism and use the splitting of the fields on the string into right mover and left mover components. Then the point is that **we** make discretization of the variable  $\tau + \sigma$  and  $\tau - \sigma$  for the left and the right moving components respectively.

The real crux of the matter is that we for each of the two “movers” only need **one** variable,  $\tau + \sigma$  or  $\tau - \sigma$  respectively, whereas say Thorn still has both the discretized variable  $\sigma$  and the “time” variable  $\tau$ . So while he has to have some time development with  $\tau$ , we basically got rid of the “time”, since we only need the variable, which we discretize, i.e. one or the other of the two  $\tau + \sigma$  or  $\tau - \sigma$ . In this light it should not be so surprising that a main characteristic of our formalism is that the time-development for our string-bits, which we since long called “objects” (which is good to distinguish them from Charles Thorn’s “string bits”) that they have only a trivial development as time goes on. This feature is then a very interesting feature and we want to extend and generalize that feature as being of interest in itself.

We can announce this feature, while thinking of our “objects” (analogous of the bits) as constituents of the strings; the strings being then thought upon as composed of such “objects” by: **Novel SFT with Non-interacting constituents “objects”, p-adic String Generalization**

Here when then also alluded to our formalism may bring hopefully some understanding of the relation of the usual string theory to the p-adic string theories[5, 4]. Well, they should probably then not be called string theories, because although the p-adic formalism leads to scattering amplitudes analogous to Veneziano models, they are according to the attempt to understand them in this article precisely of a different kind of structure than the true string: being a long thin structure. Indeed they rather share the clumpy structure of the p-adic numbers.

### 1.1 Major Achievement Anti-suggests putting it into objects:

The most important gain of (super)string theory over ordinary quantum field theories as a theory of everything is:

**QFT= Quantum field theories ONLY become non-divergent BY RENOMALIZATION; while the (even made a string field theory) STRING theory gives FINITE NUMBERS FROM THE START - Veneziano models.**

**The Wonderful Finiteness of (Super)String-theory gets Spoiled, if Replaced by a Quantum Field Theory of “bits” (unless the bits are not normal particles)**

- **Thorn** In Thorn’s the bit (= constituents) do interact, but in quite **different way from ordinary** quantum field theory particles.

Charles Thorn: For  $X(\mathcal{B}, \mathcal{T})$

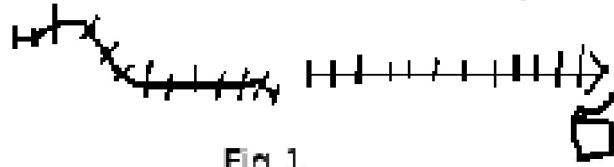


Fig 1

Ninomiya and I:

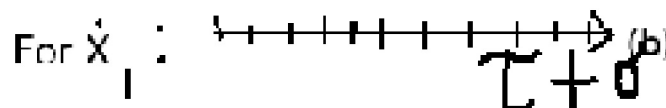
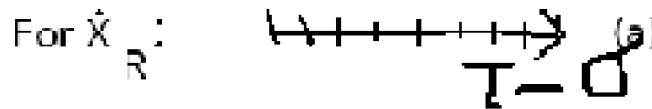


Fig 2 (a) and (b)

- **Nielsen-Ninomiya** : In ours it turns out that the “objects” (which is the name we use for our bits) **do NOT interact**. So the perturbation with the usual divergencies does not come up.

**1.2 Overview Ignoring Technical Details:**

The reader should get in mind that the connection from our “objects” to getting the more physically understandable string is of a new type and needs explanation:

At first have in mind that the state of the whole universe - i.e. the structure corresponding to the usual second quantized state in e.g. usual quantum field theory - is a state in which we have “objects” each object having degrees of freedom like that of a scalar particle. (For a string theory with closed strings there shall be two types of “objects” “right-mover type” and “left-mover-type”). These “objects” are the ones that **do not interact**, so we basically think of two types of free particles in this description of the analogon of the second quantized theory.

Now the rather new idea is that one shall now imagine selecting one right-mover type and one left mover type of these particles, which we call “objects”. Then we can combine such a pair and construct from that a position, which is the sum of the positions of the objects in the pair, and a momentum which is the sum of the momenta of the two objects in the pair.

This mathematically constructed **sum of the objects in the pair** shall now be identified by essentially a **string bit** like Charles Thorns (**at one moment** of time at first). That is to say that corresponding to such a pair we can - sometimes - attach an “infinitesimal” bit of a string, which then has its connection to the “object” formalism of ours by having the momentum for example equal to the sum of the “objects” in the pair.

Now we assume that the objects sit in what we call “cyclically ordered chains”, which first of all means that they are organized as sitting very closely along a topologically speaking circle.

That means you can imagine going around such a circle / a “cyclically ordered chain” visiting one object after the other one and finally return to the starting one again.

This means that these “objects” sit in a one-dimensional structure, the “cyclically ordered chain”.

But now when you attach the string bits - the small pieces of string - to **pairs** of “objects” - then one easily sees that the pairs will correspondingly form a **two-dimensional** sheet. This two-dimensional sheet is now meant in our picture to be identified with the **time track** of the string(s). That is to say that when we combine to make string bits all the pairs we can form from consisting of one “object” in the right-mover- cyclically ordered chain and one from the left mover one gets not only all the bits around a closed string at one moment of time (think of  $\tau$  or  $t$  as you like) but rather one gets the shape and momentum density say of the string at all different moments of time, too. That is to say that by having the two cyclically ordered chains carrying the information for a closed string we have not only the information of the string at one moment but get by combining into pairs of “objects” in different combinations also the behavior of the string as time develops!

This is very remarkable since it means that the whole **time-development** of the string is **already** basically **built into** the formalism in terms of **the objects**. One could say this with the words that we have solved the string time development by writing it into the “objects”. But that would be giving the “object” formulation too much honor, because it is wellknown since very long that the string dynamics is solvable by writing it into right and left mover  $X_R^\mu(\sigma, \tau)$ , and  $X_L^\mu(\sigma, \tau)$  (see (1.1)). However, we think that this **solving** string theory, is more new and more is achieved by our “object” formulation, when you apply this formulation to a situation with several cyclically ordered chains and also several strings resulting from combining objects in pairs.

The truly remarkable observation is that **even when strings scatter on each other as they do in string theory, you can claim that what goes on in terms of the objects is basically nothing**, the objects do not interact even in the (string-)scattering situation!

#### **Notice two objects to Each bit of String in Charles Thorn Sense**

This “TWO OBJECTS at each point of the string” is to correspond to the wellknown formula for single string dynamics in the conformal gauge:

$$X^\mu(\sigma, \tau) = X_R^\mu(\tau - \sigma) + X_L^\mu(\tau + \sigma). \quad (1.1)$$

This means that the position and momentum of the string-bit sitting at a pair of objects - a right-mover and a left mover object - is given as the **sum** of the two object momenta and positions.

This means that the objects, if they are considered to be anywhere at all, are at quite different places from the string itself, or the points they correspond to. Completely anti-intuitive for constituents. **Think of Right- and Left mover as distinct degrees of freedom...?**

**A little Problem that say  $X_R(\tau - \sigma)$  not periodic in  $\tau - \sigma$  but only in  $\sigma$ .**

**Technical problem, because  $X_R^\mu$  and  $X_L^\mu$  are not periodic:**

We have to either:

- **Declair** that looking at these right mover and left mover variables  $X_L^\mu$  and  $X_R^\mu$  for the “objects” as *global* functions of  $\tau + \sigma$  and  $\tau - \sigma$  respectively is supposed to lead to ambiguities, so

Closed String : Two cyclically ordered sets of objects

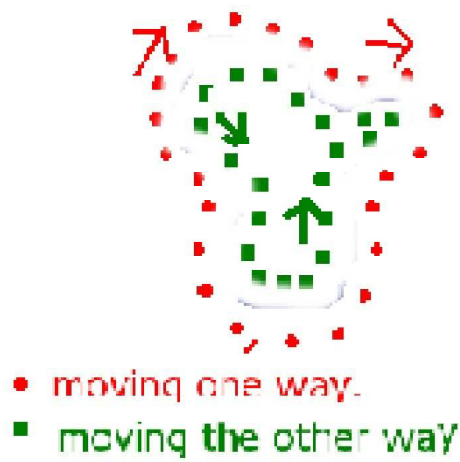


Fig. 3.

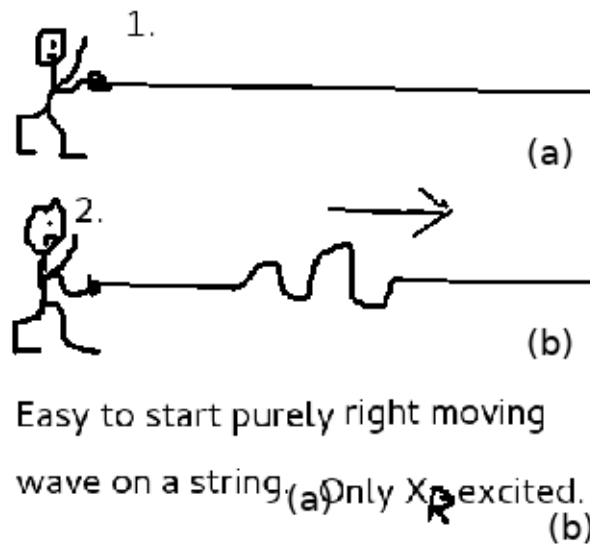
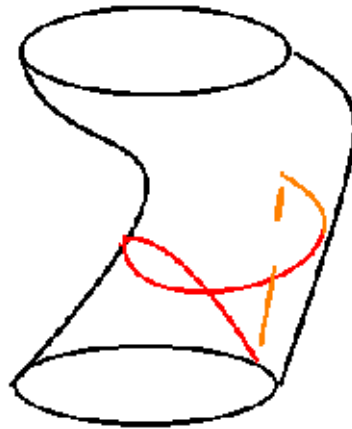


Fig. 4.



Varying only  $\tau \sim \sigma$  but not  $\tau + \sigma$  leads to a spiral, not circle.

Fig. 5

that we only care for local functions in the sense of only letting them be defined on smaller intervals. **only piecewise functions.**

- Or we identify only the derivatives or difference of  $X_L^\mu$  and  $X_R^\mu$ , i.e.

$$J_R^\mu(I) = X_R^\mu(\tau - \sigma''(I) + \Delta/2) - X_R^\mu(\tau - \sigma''(I) - \Delta/2)$$

$$J_L^\mu(I) = X_L^\mu(\tau - \sigma''(I) + \Delta/2) - X_L^\mu(\tau - \sigma''(I) - \Delta/2)$$

Here  $I = \dots, -2, -1, 0, 1, 2, \dots$

and  $\Delta = \text{“object” distance}$ .

More detailed: The  $\Delta$  is the “lattice constant” for the variables  $\tau - \sigma$  or  $\tau + \sigma$ , so that we have an “object” for each step of length  $\Delta$  in these variables. Then

$$J_R^\mu(I) \approx \Delta * \dot{X}_R^\mu(\tau - \sigma''(I))$$

$$J_L^\mu(I) \approx \Delta * \dot{X}_L^\mu(\tau - \sigma''(I)).$$

**We then want to Arrange:**

**Each “object” a Full Physical System (a set of variables and their canonically conjugate)**

But have in mind:

The right and left mover variables  $\dot{X}_L^\mu$  and  $\dot{X}_R^\mu$  have in string theory a derivative of delta func-

tion commutation rule with themselves:

$$\begin{aligned} [\dot{X}_L^\mu(\tau_{L1}), \dot{X}_L^\nu(\tau_{L2})] &= -ig^{\mu\nu} \delta'(\tau_{L1} - \tau_{L2}) \\ [\dot{X}_R^\mu(\tau_{R1}), \dot{X}_R^\nu(\tau_{R2})] &= -ig^{\mu\nu} \delta'(\tau_{R1} - \tau_{R2}) \end{aligned}$$

where

$$\begin{aligned} \tau_R &= \tau - \sigma \\ \tau_L &= \tau + \sigma \end{aligned}$$

So one object would at first not commute with its neighbors.

**We move the Information on Oddly Numbered Objects to Neighboring Evenly Numbered Ones to make up Conjugate Variable to say  $\dot{X}_R(I)$ .**

We found a way to achieve the following wishes before identifying (half) the objects with particles in a second quantized field theory. Think of at first having the objects be described just by  $X_R(I)$  and  $X_L(I)$ , then modify them somehow to achieve:

- Those objects, which we end up keeping in the formalism, shall if associated with one variable also be associated with its canonically conjugate one together with it. **Objects have full sets of degrees of freedom.**
- All variables associated with one object shall commute with those associated with an other object. **Different objects commute.**

**Wishes about to construct Object-degrees of freedom:**

- We select a subset of objects still so that all information in  $\dot{X}_R(\tau_R)$  is modulo the discretization in the “kept” “objects”. **The kept right objects carry all the right mover information; and the left carry the left mover information.**

**The Commutator of the  $\dot{X}_R(I)$  of the  $\delta'$  form.**

**Because the  $\delta'(\tau_R)$  discretizes to a function only non-zero in two neighboring points to zero, we could achieve full commutation by leaving out all objects with an odd number.**

So we shall seek to put in all information from  $X_R$  or from  $\dot{X}_R$  in to **only the even numbered objects**. But the odd ones are essentially the conjugate of the even ones. In fact we can and shall choose the odd objects to be written as differences of the conjugate variables for the even neighbors.

**We achieve the wishes by the following ansatz:**

We define for each *even numbered* ( $I$  even) object a variable set  $J_R^\mu(I)$  and the canonically conjugate set  $\Pi_R^\mu(I)$  (here leaving out a quite analogous left set with  $L$  instead of  $R$ ). We call  $\Delta$  the discretization step:

$$\begin{aligned} J_R^\mu(I) &= \dot{X}_R^\mu(I)\Delta = \\ &= X_R^\mu(\tau_R(I+1/2)) - X_R^\mu(\tau_R(I-1/2)) \quad (\text{for } I \text{ even}); \\ &\quad -\pi\Delta * (\Pi_R^\mu(I+1) - \Pi_R^\mu(I-1)) = \dot{X}_R^\mu(I)\Delta = \\ &= X_R^\mu(\tau_R(I+1/2)) - X_R^\mu(\tau_R(I-1/2)) \quad (\text{for } I \text{ odd}) \end{aligned}$$

(1.2)

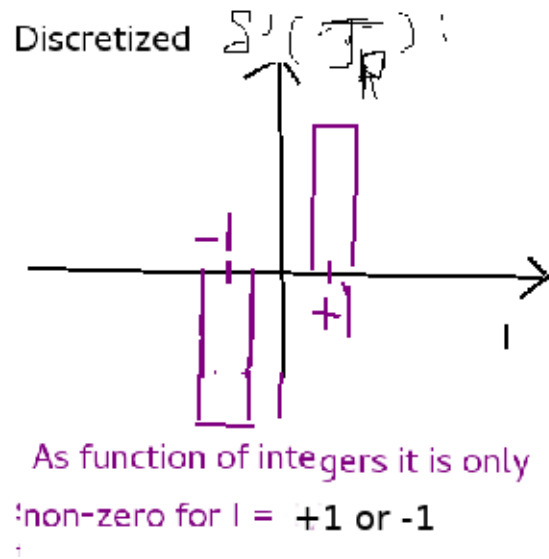


Fig. 6

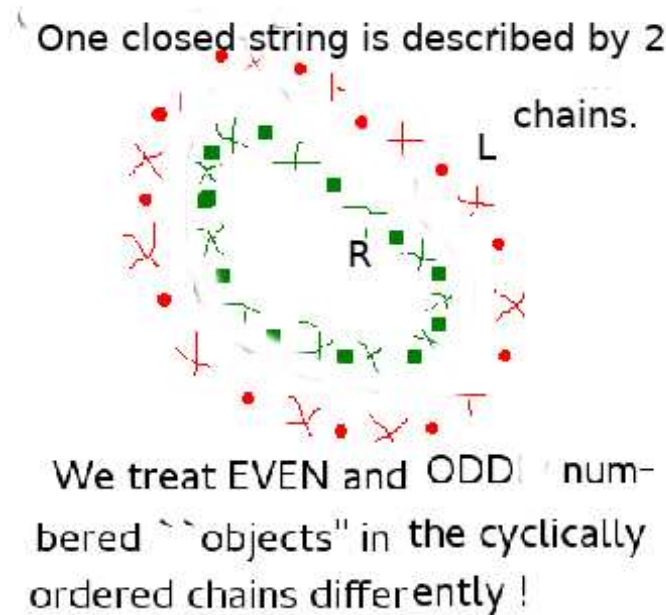


Fig. 7

Note: We only use  $\Pi_R^\mu(I)$  and  $J_R^\mu(I)$  here for *EVEN*  $I$ .

We got a way to put the information of a free closed string solution of equations of motion into that two sets of (infinite) numbers of “even objects” with their  $(J_R^\mu(I), \Pi_R^\mu(I))$  ( $I = \dots, -4, -2, 0, 2, 4, \dots$ ).

Summary: We can describe **one** closed string by two sets,  $R$  and  $L$ , of cyclically ordered “even numbered objects”.



Each “object” has in the bosonic string theory one degree of freedom - a  $J_R^\mu(I)$  and the canonically conjugate  $\Pi_R^\mu(I)$  one - for each of the 25+1 dimensions. (modulo some troubles with gauge choosing ...)

**Main Point: We Second Quantize the Objects; and There can be Many Strings in the Same Quantum Field Theory**

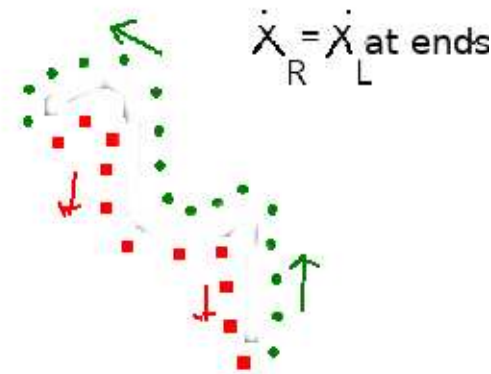
- First describing **one** string you let it correspond to a **quantum field theory** in which you have the two cyclically ordered chains of “objects” (= the particles in the quantum field theory (with massless non-interacting spin zero particles)).
- This gives the obvious idea to describe **several strings**, namely by just putting more couples of cyclically ordered chains up in the same quantum field theory Hilbert space(s). I.e. you got a “**string field theory**” (= second quantized string theory like Kaku and Kikkawa[2] or Witten[2]...) **for free!**

**1.3 Scattering of Strings is a Fake.**

We managed - though with some technical difficulties - to obtain string scattering as a **fake**, meaning that the objects themselves do **not** do anything during the scattering, but we/the physicists **reclassify the objects into new cyclically ordered chains**, and then we have a formal scattering, although in the physics of our model for non-interacting “objects” nothing can happen.

**In String Theories with Open Strings Only One type of “Objects”; while only closed ones have  $R$  and  $L$ .**

At the ends  $R \leftrightarrow L$  in open string.



One open string  
Fig. 8

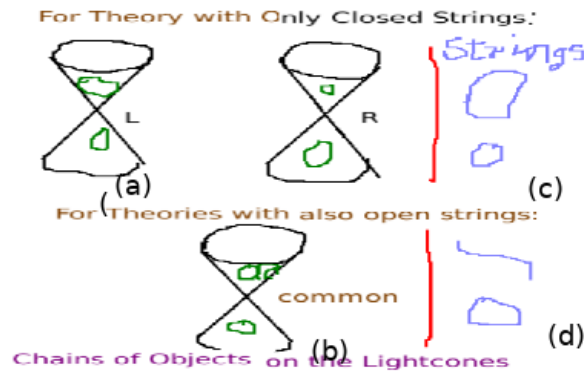
**The Constraints  $\dot{X}^2 + (X')^2 \approx 0$  and  $\dot{X} \cdot X' \approx 0$  look nice in “objects”.**

The usual constraints that  $\dot{X}^2 + (X')^2$  and  $\dot{X} \cdot X'$  should be “weakly” 0 take a nice form in the variables used for our “objects”:

$$\begin{aligned} \dot{X}_R^2 &= (\dot{X}_R^\mu)^2 \approx 0 \\ \dot{X}_L^2 &= (\dot{X}_L^\mu)^2 \approx 0 \end{aligned} \tag{1.3}$$

Here we used  $\approx$  to denote the “weak” equality.

### Detail of Open Versus Closed Strings in Our Model



String in terms of objects:  
 closed (a),  
 open and closed (b)  
 The string of  $X_\mu$  in 25+1 dimensions:  
 closed (c),  
 and open and closed (d) respectively.

Fig 9

#### 1.3.1 Stringiness (one-dimensionally touching of objects) is put in via the States of the “Objects”

Since we have no interaction between the “objects” such an interaction cannot be used to keep or make the objects sitting in a chain. Since they only move trivially though they will continue to “sit” (or better run) as forming a continuous chain, provided they have started forming such a chain. In this sense the only way to assume the objects to form continuous chains would be to just assume it as an initial condition, being true - say in the beginning - and thus continuing being true forever (from the trivial development).

Our “String Field Theory” model is basically the second quantized theory described as particles (best in a timeless universe), and then with the constraints  $\dot{X}^2 + (X')^2 \approx 0$  and  $\dot{X} \cdot X' \approx 0$  imposed.

In that model, a quantum field theory of just non-interacting massless particles, there is no stringiness at all! **It is only by the (initial) states of the objects considered that there is any allusion**

### Translation from strings to “Cyclic Chains”

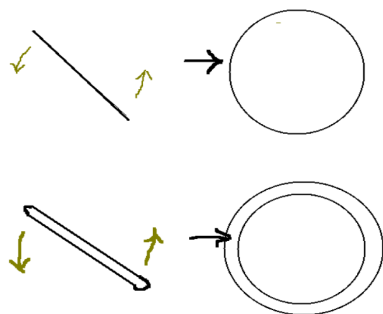


Fig. 10

to the hanging together to one-dimensional chains. Something that at the end leads to the stringy hanging together.

The string is in the initial - and also final conditions we need - conditions only!

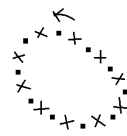
(When we derive our Veneziano model amplitudes from our Novel SFT[3], then it is crucial for reaching the Veneziano model, that also the ansatz states for the strings in the final state - the outgoing strings - are chains with the same Gaussian wave function and associated continuity behavior having at least some stringiness in it. Thus we assume basically something about the state of the future, namely a certain stringiness.)

#### 1.3.2 A single Open String is described by just one Cyclically ordered chain of “even objects”:

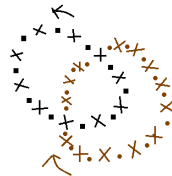
Now Add One more Open String described also as a Cyclically Ordered Chain of Objects.

If the two cyclically ordered chains cross in two places, we can interpret it as corresponding to two open strings in different ways. Indeed we can first interpret it as a couple of cyclically ordered chains in Minkowsky space in two ways: Indeed we symbolically show in fig 11 (b) two cyclically ordered chains, a black and a brown say, having two points in common. By using just the points, objects, one can see that we extract from the combination of these objects the two combinations of objects shown on Fig 12 (a) and fig 12 (b) can be extracted. These cyclically ordered chains fig 12(a) and 12 (b) are each of them along some piece at least different from the two illustrated in fig 11 (b). Thus one can out of two in two points crossing cyclic chains extract quite different ones. But now there correspond to different cyclically ordered chains open strings with different shape and time development.

The Rest of the Objects also form a Cyclically ordered chain meaning it gives a String. Without Anything happening except in Phantasy two strings scattered to two other strings!



(a)



(b)

Fig 11-(a) and (b)

On (b) cyclically ordered chains of objects for two open strings.



(a)

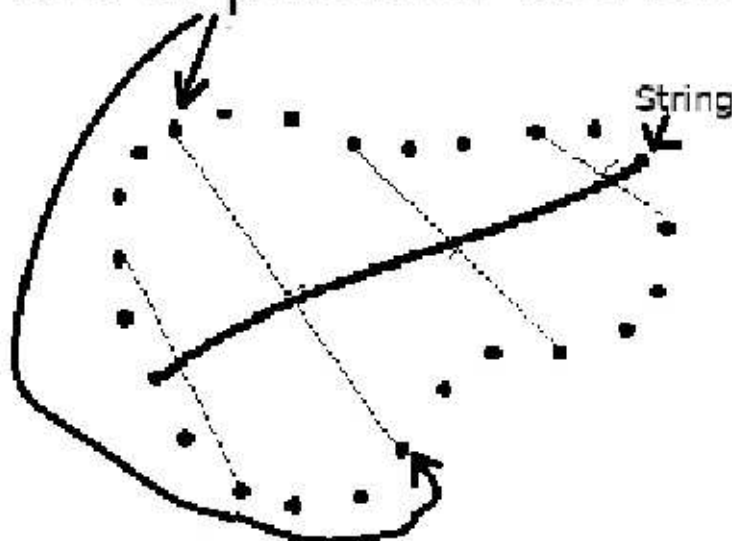


(b)

Fig 12 (a) and (b)

The reader may keep in mind the way one obtains the open string from a cyclically ordered chain of objects. In fact it is so that sum the space-time positions of any pair of the objects to get that of a space-time point for a point on the (open) string. Whether we sum such a pair of point-coordinates or we take the average of them is of course only a convention of a factor 2. So you could then say the events through which the the string passes is the collection of all the averages of pairs of points on the cyclically ordered chain. Thinking of the situation in a moment of time we can claim as drawn in the figure to give at least the crude idea that the string is gotten by a series of

## Pairs of points of the chain



The points on the string is in a certain moment apart from a factor 2 the average of a series of pairs of points from the chain.

Fig. 13

pairs of objects - paired with each other by a series of locally approximate parallel lines. The string goes through the middle points of all the pairing pieces of lines.

If one has couple of double crossing cyclically ordered chains, one can expect that in some Lorentz frame you can find the piece of line connecting the two crossing points and on the middle of that connection line piece will be the crossing point of the open strings for the appropriate moment: The idea of “faked scattering” now is that we split the combined sets of the two cyclically ordered chains into a couple of cyclically ordered chains **in a new way!** We take part of the chain corresponding to the first of the two initial strings and combine it with a part of the one that corresponds to the second string.

## 2. Achievements

### 2.1 In principle we rewrote String Theory, but still would like to check if it is true.

To argue that our string theory really is correct, one would first see that essentially from locality one can see that indeed the system of “objects” will **not** change in a true scattering of dual strings.

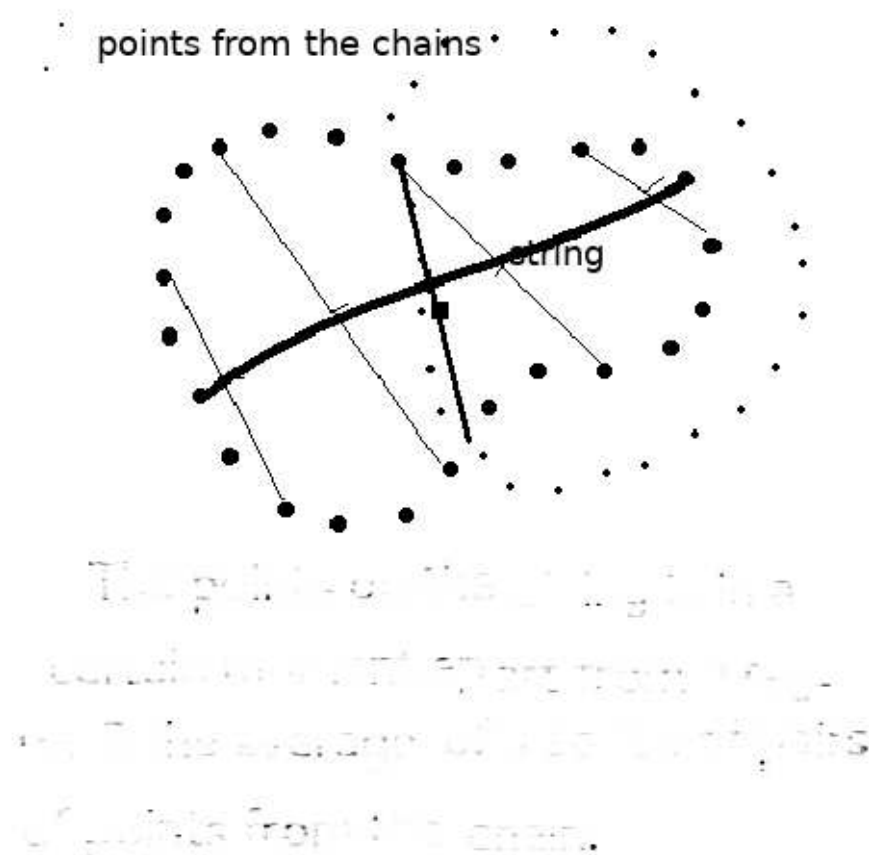


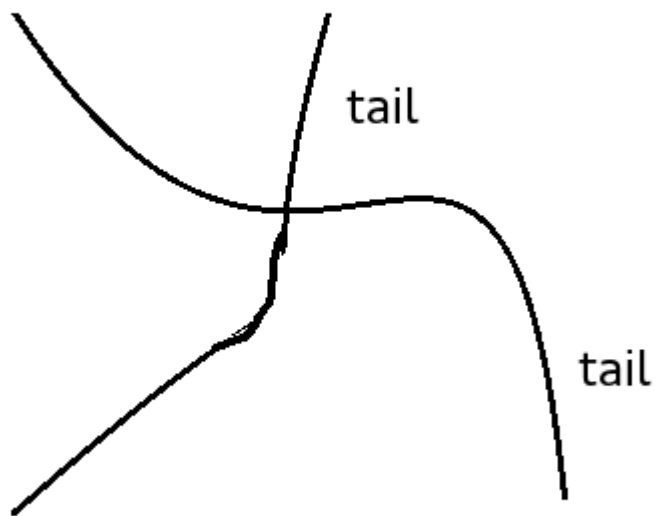
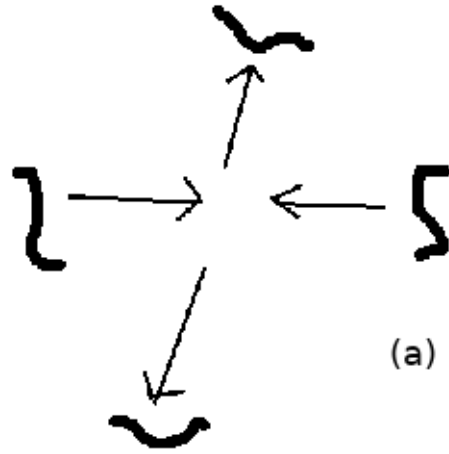
Fig 14.

Think say of a couple of strings scattering classically by “exchange of tails”. Then the image of the  $\hat{X}_R^\mu$  will only change on a nullset. By locality one argued the image of the  $\hat{X}_{R/L}^\mu$  remain the same ( mod. nullset) In fact we calculated:

- Spectrum of a string using our Novel String Field Theory Model.[3, 6, 10]
- The (“faked”) scattering amplitude, expecting to get the Veneziano model; but got only one out of three terms in the infinite momentum frame gauge choice.
- Including the possibility of negative energies for the constituents(objects) - like in Nambu-Bethe-Salpeter[8] equation - we at least glimps a way to get all three terms of the Veneziano amplitude.

**2.2 The Veneziano Model Derivation Quickly comes to Technical String Calculation[3]:**

In spite of the fact that our model being only a quantum field theory formalism having no stringiness proper in itself, but only in initial and final state input, the derivation of the Veneziano model from our model quickly goes into the track of string theory:



Two open strings hit in a point, and exchange tails.  
Fig 15 (a) and (b)

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- We begin the calculation by representing the external string by wave functions in terms of “objects” being derived from a functional integral through an imaginary time.
- Next we must take the overlap between the initial state - of some strings giving some cyclically ordered chains - and the final state.
- This leads to gluing together the space-time or rather  $(\tau, \sigma)$  regions from the various imaginary time developments.
- The glued together regions for definition of the functions over which to functionally integrate leads to complex surfaces which becomes just like the corresponding functional integral regions in usual string theory.
- The counting of the various ways to identify the systems of objects in initial and final states leads to precise integration measures for the Veneziano model integrals.

2.3 The “usual surfaces” in string theory.

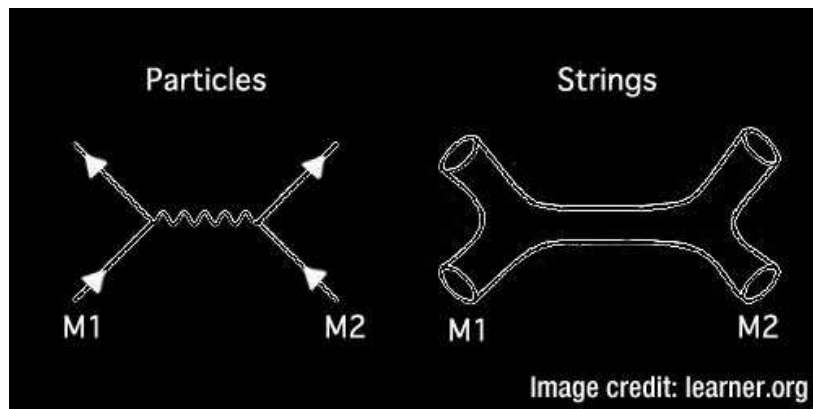


Fig 16

The basic second quantized Hilbert space for our string field theory is only particle second quantized space  $\mathcal{H}$ .

On this ordinary free particle second quantized Hilbert space  $\mathcal{H}$  one can create particles (free and massless) with creation operators denoted  $a^\dagger(J^\mu)$ , where we can think of the 26-vector  $J^\mu$  (which is the main variable for one of our objects/bits) as say the 26-momentum (really we may only need 24 components in infinite momentum frame) for the free massless particle that can be created in our basic model Fig 2.

**Formulation of Second quantized “strings”**

- The string states are created by acting on this space  $\mathcal{H}$  with an in the limit infinite number of creation operators  $a^\dagger(J^\mu)$  associated with a series  $J^\mu$ -values (26-vectors) as taken on by our type “objects” and weighted with a wave function  $\Psi(J_0^\mu, J_2^\mu, \dots, J_{N-4}^\mu, J_{N-2}^\mu)$  for a single



string described as consisting of a cyclically ordered chain of even numbered “objects”:

$$\begin{aligned}
& |string_{\Psi} \rangle = \\
& = \int \dots \int \mathcal{D}J^{\mu} * \\
& \quad * \Psi(J_0^{\mu}, J_2^{\mu}, \dots, J_{N-4}^{\mu}, J_{N-2}^{\mu}) * \\
& \quad * a^{\dagger}(J_0^{\mu}) a^{\dagger}(J_2^{\mu}) \dots a^{\dagger}(J_{N-4}^{\mu}) a^{\dagger}(J_{N-2}^{\mu}) |“vac” \rangle .
\end{aligned}$$

The state  $|“vac” \rangle \in \mathcal{H}$  is a priori the empty state, meaning without any of the scalar free particles ( $\sim$  our objects).

## 2.4 Really the vacuum $|“vac” \rangle$ should be Rough

We ran into troubles of only obtaining one out of three Euler Beta-functions for the scattering amplitude by using infinite momentum naturally combined with such a no particle vacuum used for  $|“vac” \rangle$ . This problem is supposed to be cured by a more complicated vacuum state (the rough Dirac sea for Boson [7]), so that one can both add and subtract energy to the back ground state  $|“vac” \rangle$  used. **Cruz: We can make many string states trivially:**

A single string creation operator in our scheme in terms of object-creation operators  $a^{\dagger}(J^{\mu})$  has the form

$$\begin{aligned}
& string_{\Psi} = \\
& = \int \dots \int \mathcal{D}J^{\mu} * \\
& \quad * \Psi(J_0^{\mu}, J_2^{\mu}, \dots, J_{N-4}^{\mu}, J_{N-2}^{\mu}) * \\
& \quad * a^{\dagger}(J_0^{\mu}) a^{\dagger}(J_2^{\mu}) \dots a^{\dagger}(J_{N-4}^{\mu}) a^{\dagger}(J_{N-2}^{\mu}) .
\end{aligned}$$

With that we can simply make a several string state by acting with several of these operators on a vacuum  $|“vac” \rangle$ :

$$|1, 2, \dots, n \rangle = \tag{2.1}$$

$$= string_1^{\dagger} string_2^{\dagger} \dots string_n^{\dagger} |“vac” \rangle \tag{2.2}$$

## 2.5 Got Veneziano model from Fake scattering!

When we used infinite momentum frame (for gauge choice) we got, but interestingly enough only quite right for 25+1 dimensions, (for our bosonic string constituent SFT) for the scattering amplitude  $A(s, t, u)$ :

$$\begin{aligned}
& A(s, t, u) = C * B(-\alpha(t), -\alpha(u)) \\
& \text{where e. g. } \alpha(t) = \alpha(0) + \alpha' * t \\
& \text{and } \alpha(0) = -(-1/\alpha') = 1/\alpha'
\end{aligned}$$

We did not determine the overall coefficient  $C$  in the mentioned calculation.

### 3. Perspective

**Perspective: String theory may be considered as a mathematical construction on a free quantum field theory (for massless spin zero particles) Remarkable** with our non-interacting constituents (=objects= our string bits) string field theory model:

- The basic SFT Hilbert space is just that of free massless **particles** and carries no signal of strings a priori in it. Rather **the strings come in only via the objects being inserted in (quantum fluctuating)chains!**
- **Nothing happens to the objects** under the scattering! It is **only**, that one has a **non-zero overlap between the object states corresponding to the strings in the “incoming” state of strings and in the “outgoing” one!** Rather as if the strings are just a clever( $\sim$  **artificial**) construction in a fundamental world, most naturally interpreted as a particle theory.

### 4. Generalizations

#### Generalization of non-interacting constituent idea

Let us suppose we **seek a theory that is finite** by looking e.g. at hadrons with their **rapidly falling off** amplitude for **large transverse momenta**.

If they really **fell off** exponentially they would in loops give rise to **convergent loop** corrections and there would be **no ultraviolet divergence problem; but** we now know that the hadrons contain **partons** and because of that emit **less strongly falling off** particles (hadrons).

**But if all the partons had Bjorken variable[9]  $x = 0$ , then there would be no energy for the partons to scatter on each other and the exponential fall off would be preserved!**

**How the soft/exponential cut off of the Veneziano model comes in say  $p_{transverse}$  in our formulation.**

#### Object-picture from the wave function of the string in terms of constituents.

Since the main thing - to get soft cut off to **avoid divergences** hopefully - comes from the wave function of the “string” in terms of our objects(=bits, but in  $X_R$  and  $X_L$  separately), we could replace the ‘string’ by thought upon structure provided we take its internal **wave function** in terms of the objects to **fall off** exponentially (as Gaussians).

The suppression of large transverse momenta in scattering in the faked way is rather easily to understand, when one has assumed the wave function of the “string” in terms of objects as being a Gaussian falling off one. Then the large momentum faked scattering cannot be achieved, because it would require some constituents  $\sim$  “objects” to have large transverse momenta, but they do not have that. Or rather it is only exponentially few objects with large transverse momentum compared to the ones with small momenta.

### 5. Conclusion

**Conclusion** We have presented a “novel string field theory” or perhaps better to say “A solution of several-strings-string-theory”:

- Our model is string-bit-theory deviating in important way from Charles Thorn’s one.

- It is a string field theory in the sense of describing possibly an arbitrary number of strings. But it is very different in the spirit of the formalism from usual string field theories, such as Kaku and Kikkawa or Wittens string field theories[2].
- Our formulation can be considered a **solution** of string theory, in the sense that there is no more time development left in our object formulation, meaning that we solved the equations of motion.
- All the time development is **fake** especially the **scattering**.
- There is so little “string” in basic formulation only being a massless second quantized particle, that one would say: The string is a mathematical construction from rather trivial starting formalism.

### Conclusion on Generalization to other Finite theories (hope)

Concerning the general looking for finite and therefore meaningful potential theories for all of physics:

- By analogy with a hadronic bound state, we argued that the ultraviolet causing high (transverse) momenta should be avoided by making the **Bjorken  $x$  for all constituent zero!** (then there would even be no energy for the partons to scatter at all.
- A model with only zero Bjorken  $x = 0$  of course would need an infinite number of constituents (since the Bjorken  $x$ 's must add up to unity.
- The string can with our objects be considered such an infinite number of constituents model.

### Conclusion on Generalization continued.

- Could one generalize the string to another pattern for the constituents (than our cyclically ordered sets the doubled strings) ?
- The p-adic string[4, 5] is the obvious candidate for such generalization[10].

### Technical Resume Conclusion.

- The basic formalism is a massless quantum field theory without interactions.
- These particles are called “even objects” and are identified with the even numbered constituents/string-bits, when one first discretize the string after having divided it into right mover and left mover d.o.f.

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