

Single-field inflation in models with an R^2 term

Ignatios Antoniadis

LPTHE, Sorbonne Universite, CNRS, 4 Place Jussieu, 75005 Paris, France;
Albert Einstein Center, Institute of Theoretical Physics, University of Bern, Sidlerstrasse 5,
CH-3012, Bern, Switzerland
E-mail: antoniad@lpthe.jussieu.fr

Alexandros Karam*

National Institute of Chemical Physics and Biophysics, Ravala 10, 10143 Tallinn, Estonia;
Physics Department, University of Ioannina, GR-45110 Ioannina, Greece
E-mail: alexandros.karam@kbfi.ee

Angelos Lykkas

Physics Department, University of Ioannina, GR-45110 Ioannina, Greece
E-mail: alykkas@cc.uoi.gr

Thomas Pappas

Physics Department, University of Ioannina, GR-45110 Ioannina, Greece
E-mail: thomasdpappas@gmail.com

Kyriakos Tamvakis

Physics Department, University of Ioannina, GR-45110 Ioannina, Greece
E-mail: tamvakis@uoi.gr

We present two cases where the addition of the R^2 term to an inflationary model leads to single-field inflation instead of two-field inflation as is usually the case. In both cases we find that the effect of the R^2 term is to reduce the value of the tensor-to-scalar ratio r .

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*Speaker.

1. Introduction

The theory of the exponential expansion of the Universe, better known as cosmic inflation, was originally proposed as a solution to the horizon and flatness problems [1, 2]. It was soon after realized that inflation can also provide a mechanism that explains how primordial inhomogeneities were magnified to cosmic size and became the seeds for the growth of structure in the Universe [3–6].

The Starobinsky model [7] is among the simplest and most successful inflationary models, where an R^2 term is added to the Einstein-Hilbert action

$$S_{\text{Star.}} = \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g} \left(R + \frac{1}{6m^2} R^2 \right). \quad (1.1)$$

This model belongs to the general class of $F(R)$ theories [8–20]¹.

$$S_F = \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g} F(R) \quad (1.2)$$

which are equivalently described by an action of this form

$$S[g_{\mu\nu}, \chi] = \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g} [F'(\chi)(R - \chi) + F(\chi)] . \quad (1.3)$$

after we introduce an auxiliary real scalar field χ . By performing a Weyl rescaling of the metric and a field redefinition we obtain the following action:

$$S[g_{\mu\nu}, \varphi] = \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g} R - \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + V(\varphi) \right], \quad (1.4)$$

where χ is minimally coupled to gravity, has a canonical kinetic term and a potential given by the general formula

$$V = \left(\frac{M_{\text{Pl}}^2}{2} \right) \frac{\chi F'(\chi) - F(\chi)}{F'(\chi)^2}, \quad F'(\chi) = \exp\left(\sqrt{\frac{2}{3}} \varphi / M_{\text{Pl}} \right), \quad \varphi = \frac{\sqrt{3} M_{\text{Pl}}}{\sqrt{2}} \ln F'(\chi). \quad (1.5)$$

For the $(R + R^2)$ model we have

$$V(\varphi) = \frac{3}{4} M_{\text{Pl}}^2 m^2 \left[1 - \exp\left(-\sqrt{\frac{2}{3}} \varphi / M_{\text{Pl}} \right) \right]^2. \quad (1.6)$$

Recently, the Starobinsky model has been combined with other popular models such as Higgs inflation [48–70]. As a result, there are two fields that can play the role of the inflaton and the analysis becomes more complicated.

The 2018 results from the Planck satellite [71] in the n_S versus r plane are shown in Fig. 1

¹Which are equivalent to the scalar-tensor theories of gravity [14, 21–47].

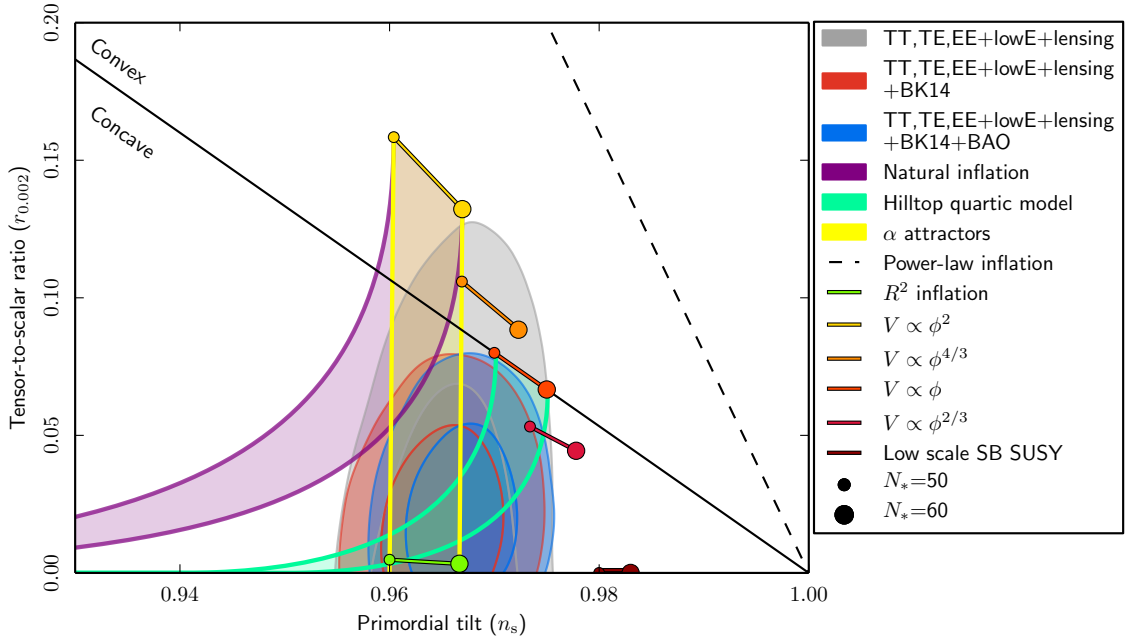


Figure 1: Marginalized joint 68% and 95% CL regions for n_s and r at $k = 0.002 \text{Mpc}^{-1}$ from Planck 2018 [71] compared to the theoretical predictions of selected inflationary models.

The simplest models like ϕ^4 , ϕ^3 and ϕ^2 have already been ruled out by the Planck 2015 data [72], while a little more convoluted models, such as the α -attractors [73], the Starobinsky [7] and some non-minimally coupled models such as Higgs inflation [74, 75] yield predictions that still comply with the observations.

2. Non-minimal Coleman-Weinberg inflation with an R^2 term

In [76], we considered the non-minimal Coleman-Weinberg model where the Planck mass is dynamically generated through the vacuum expectation value (VEV) of a real scalar field ϕ , plus an R^2 term². The action in the Jordan frame has the form

$$S^J = \int d^4x \sqrt{-\bar{g}} \left[\frac{\xi \phi^2}{2} \bar{R} + \frac{\alpha}{2} \bar{R}^2 - \frac{1}{2} \bar{\nabla}^\mu \phi \bar{\nabla}_\mu \phi - \frac{\lambda_\phi}{4} \phi^4 \right]. \quad (2.1)$$

We can eliminate the R^2 term by introducing an auxiliary real scalar field χ

$$S^J = \int d^4x \sqrt{-\bar{g}} \left[\frac{1}{2} (\xi \phi^2 + \alpha \chi^2) \bar{R} - \frac{\alpha}{8} \chi^4 - \frac{1}{2} \bar{\nabla}^\mu \phi \bar{\nabla}_\mu \phi - \frac{\lambda_\phi}{4} \phi^4 \right]. \quad (2.2)$$

Then, by performing a Weyl rescaling of the metric (where we introduced a new field ζ

$$g_{\mu\nu} = \Omega^2 \bar{g}_{\mu\nu}, \quad \Omega^2 = (\alpha \chi^2 + \xi \phi^2) / M_{\text{Pl}}^2 \equiv \frac{\zeta^2}{6M_{\text{Pl}}^2}, \quad (2.3)$$

²See also [37, 38, 42, 77–113] for other models based on scale invariance.

the Einstein frame action takes the form

$$S^E = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{6M_{\text{Pl}}^2}{\zeta^2} \left(\frac{1}{2} \nabla^\mu \phi \nabla_\mu \phi + \frac{1}{2} \nabla^\mu \zeta \nabla_\mu \zeta \right) - V^{(0)E}(\phi, \zeta) \right], \quad (2.4)$$

with the tree-level potential given by the formula

$$V^{(0)E}(\phi, \zeta) = \frac{36M_{\text{Pl}}^4}{\zeta^4} \left[\frac{\lambda_\phi}{4} \phi^4 + \frac{1}{8\alpha} \left(\frac{\zeta^2}{6} - \xi \phi^2 \right)^2 \right]. \quad (2.5)$$

Note that there are two fields in the potential, ϕ and ζ . Usually, scale-invariant beyond the Standard Model theories with multiple scalar fields are studied with the help of the Gildener-Weinberg formalism [114].

In this approach, the perturbative minimization happens in two steps. First, the tree-level potential is minimized with respect to its field content. This minimization takes place at a specific energy scale, due to the running of the couplings in the full quantum theory, and defines a flat direction among the scalar fields. Then, one computes the one-loop corrections only along this flat direction, since this is where the corrections play the dominant role, remove the flatness, and determine the physical vacuum.

Now, the tree-level minimization conditions in our theory give this relation:

$$\left. \frac{dV^{(0)E}}{d\phi} \right|_{\phi=v_\phi} = \left. \frac{dV^{(0)E}}{d\zeta} \right|_{\zeta=v_\zeta} = 0, \quad \Rightarrow \quad v_\phi^2 = \frac{\xi}{6(\xi^2 + 2\alpha\lambda_\phi)} v_\zeta^2. \quad (2.6)$$

By using an orthogonal rotation of the form

$$\begin{pmatrix} \phi \\ \zeta \end{pmatrix} = \begin{pmatrix} \cos \omega & -\sin \omega \\ \sin \omega & \cos \omega \end{pmatrix} \begin{pmatrix} s \\ \sigma \end{pmatrix}, \quad (2.7)$$

we can diagonalize the mass matrix of the two scalars and we parametrize the mixing angle in this way

$$\tan \omega = \frac{v_\zeta}{v_\phi}, \quad v_s = \frac{v_\phi}{\cos \omega} = \frac{v_\zeta}{\sin \omega}, \quad (2.8)$$

with respect to the VEVs. The tree-level mass eigenstates are

$$m_s^2 = 0, \quad m_\sigma^2 = \frac{\xi(\xi + 12\lambda_\phi\alpha + 6\xi^2)}{6\alpha(2\lambda_\phi\alpha + \xi^2)} M_{\text{Pl}}^2. \quad (2.9)$$

Notice that the mass eigenvalue of s is exactly zero at tree level since it is the pseudo-Goldstone boson of broken classical scale invariance, while σ is the orthogonal eigenstate.

The 1-loop correction along the flat direction takes the form

$$V^{(1)} = \frac{m_\sigma^4}{64\pi^2 v_s^4} s^4 \left[\log \left(\frac{s^2}{v_s^2} \right) - \frac{1}{2} \right], \quad v_s^2 = v_\phi^2 + v_\zeta^2, \quad v_\zeta^2 = 6M_{\text{Pl}}^2. \quad (2.10)$$

At this point we demand that the full one-loop potential vanishes at the minimum³

$$V(v_s) \equiv V^{(0)E}(v_s) + V^{(1)}(v_s) = 0. \quad (2.11)$$

³This implies that the cosmological constant is zero at the 1-loop level.

Finally, we arrive at the following expression for the effective potential along the flat direction

$$V(s) = \frac{m_\sigma^4}{128\pi^2} \left[\frac{\sin^2 \omega}{36M_{\text{Pl}}^4} s^4 \left(2 \ln \left[\frac{s^2 \sin^2 \omega}{6M_{\text{Pl}}^2} \right] - 1 \right) + 1 \right], \quad (2.12)$$

which is shown in Fig. 2. From (2.12) we obtain the radiatively generated mass of the s boson

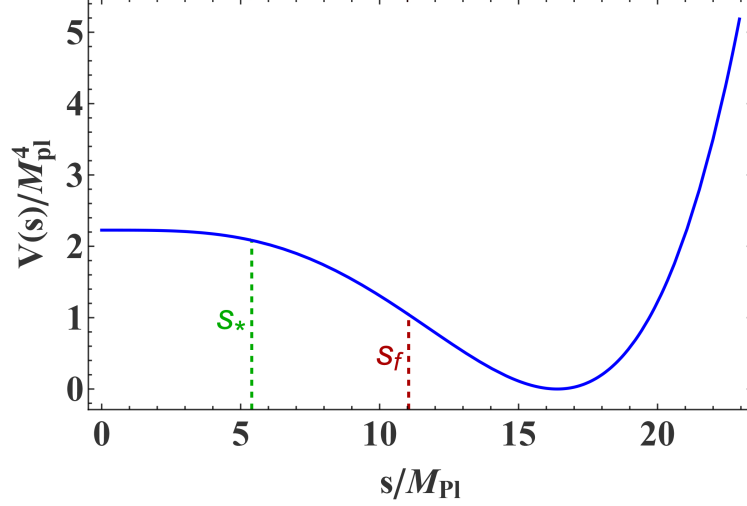


Figure 2: An inflaton excursion that yields $N = 59.3$ when $\sin \omega = 0.15$. Inflation starts(ends) at the green(red)-dashed line. The values of the observables in this case are $(r, n_s) = (0.014, 0.963)$.

$$m_s^2 = \frac{\sin^2 \omega}{48\pi^2} \frac{m_\sigma^4}{M_{\text{Pl}}^2}. \quad (2.13)$$

Notice that it is loop-suppressed with respect to the mass of σ . This means that the orthogonal state σ effectively decouples and only s plays the role of the inflaton along the flat direction.

The predictions of the model in the $n_s - r$ plane are shown in Fig. 3, overlaid with the latest Planck constraints. In the limit of zero mixing angle the potential behaves like the quadratic one, while for larger values we obtain a beautiful banana shape.

3. Palatini inflation in models with an R^2 term

There are two variational principles that one can apply to the Einstein-Hilbert action in order to derive Einstein's equations: the standard metric variation where the metric is the only dynamical degree of freedom and the connection is the Levi-Civita

$$S = \int d^4x \sqrt{-g} \left(\frac{1 + \xi \phi^2}{2} g^{\mu\nu} R_{\mu\nu}(g, \partial g, \partial^2 g) - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \right), \quad (3.1)$$

and the Palatini variation⁴ where the metric and the connection are assumed to be independent variables and one varies the action with respect to both of them

$$S = \int d^4x \sqrt{-g} \left(\frac{1 + \xi \phi^2}{2} g^{\mu\nu} R_{\mu\nu}(\Gamma, \partial \Gamma) - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \right). \quad (3.2)$$

⁴See [115] for a review and [63, 96, 116–153] for various applications.

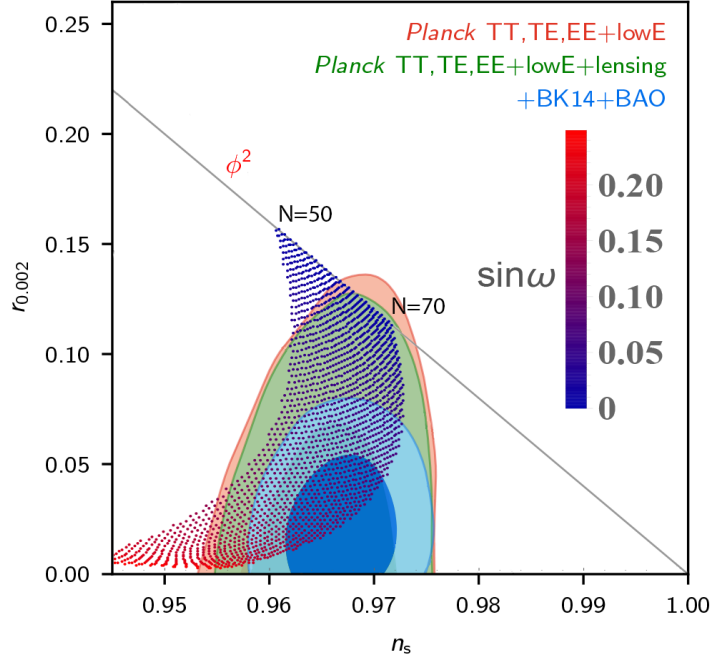


Figure 3: The predictions of the model (2.12) for a wide range of values for the mixing angle ω and for field excursions that yield sufficient amount of inflation i.e. 50 to 70 e-folds.

Note that even though both variational principles lead to the same field equation for an action whose Lagrangian is linear in R and is minimally coupled, this is no longer true for a more general action.

In [63, 154], we considered a real scalar field ϕ , in general non-minimally coupled to gravity plus an R^2 term in the Palatini formalism⁵

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} (M_0^2 + \xi \phi^2) R + \frac{\alpha}{4} R^2 - \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right\}, \quad R = g^{\mu\nu} R^\rho_{\mu\rho\nu}(\Gamma, \partial\Gamma). \quad (3.3)$$

Introducing an auxiliary scalar χ we eliminate the R^2 term

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} (M_0^2 + \alpha \chi^2 + \xi \phi^2) R - \frac{1}{2} (\nabla \phi)^2 - \frac{\alpha}{4} \chi^4 - V(\phi) \right\}, \quad (3.4)$$

and by performing a Weyl rescaling of the metric

$$\bar{g}_{\mu\nu} = \Omega^2(\phi) g_{\mu\nu} \quad \text{with} \quad \Omega^2(\phi) = \frac{M_0^2 + \xi \phi^2 + \alpha \chi^2}{M_P^2}, \quad (3.5)$$

we bring the action to this simpler form:

$$S = \int d^4x \sqrt{-\bar{g}} \left\{ \frac{1}{2} M_P^2 \bar{R} - \frac{1}{2} \frac{(\bar{\nabla} \phi)^2}{\Omega^2} - \bar{V} \right\}, \quad \bar{V}(\phi, \chi) = \frac{1}{\Omega^4} \left(V(\phi) + \frac{\alpha}{4} \chi^4 \right). \quad (3.6)$$

⁵See also [131] where the authors consider a similar setting.

Notice that no kinetic term has been generated for χ , which is usually called the scalaron in the metric formalism, therefore its equation of motion reduces to just a constraint. This means that ϕ is the only propagating degree of freedom which can play the role of the inflaton, contrary to the metric case where we would have two-field inflation.

Using the constraint equation for χ

$$\delta_\chi S = 0 \rightarrow \chi^2 = \frac{\frac{4V(\phi)}{(M_0^2 + \xi\phi^2)} + \frac{(\nabla\phi)^2}{M_P^2}}{\left[1 - \frac{\alpha(\nabla\phi)^2}{M_P^2(M_0^2 + \xi\phi^2)}\right]} \quad (3.7)$$

we can eliminate it altogether and arrive at the action

$$S \approx \int d^4x \sqrt{-g} \left\{ \frac{M_P^2}{2} R - \frac{1}{2} \frac{(\nabla\phi)^2}{\Omega_0^2} \left(\frac{1}{1 + \frac{4\tilde{\alpha}V}{\Omega_0^4}} \right) - \frac{V}{\Omega_0^4} \left(\frac{1}{1 + \frac{4\tilde{\alpha}V}{\Omega_0^4}} \right) + \mathcal{O}((\nabla\phi)^4) \right\}, \quad (3.8)$$

where $\tilde{\alpha} = \alpha/M_P^4$ and $\Omega_0^2 = (M_0^2 + \xi\phi^2)/M_P^2$.

The effect of the R^2 term is that it decreases the height of the effective potential, which always has a plateau at high field values. Of course, one should also take into account that the rate of change of the field is modified and should perform a field redefinition in order to bring the kinetic term into its canonical form.

As a first example, let us consider natural inflation, given by the potential [155, 156]

$$V(\phi) = M^4 \left(1 + \cos\left(\frac{\phi}{f}\right) \right). \quad (3.9)$$

In Fig. 4, the blue curve is the potential in the metric formalism. The red dashed curve is the effective potential with the R^2 in the Palatini formalism. As can be seen, the potential has a lower height and has been flattened. In practice, this means that we can obtain lower values for the tensor-

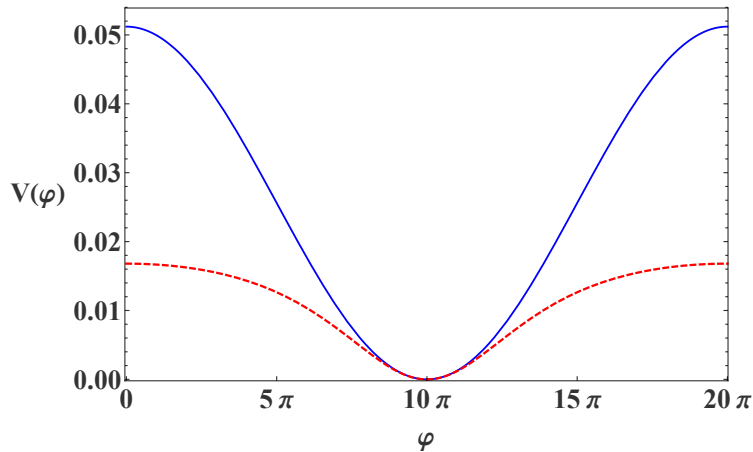


Figure 4: Blue curve: The original natural inflation potential $V(\phi)$. Red dashed curve: The potential $\bar{V}(\zeta(\phi))$ in the Palatini formalism. We have chosen $M = 0.4$, $\alpha = 10$, and $f = 10$ (in natural units).

to-scalar ratio r (see Fig. 5) and essentially save some models that were previously excluded by the Planck constraints. Similarly, for minimal quadratic inflation,

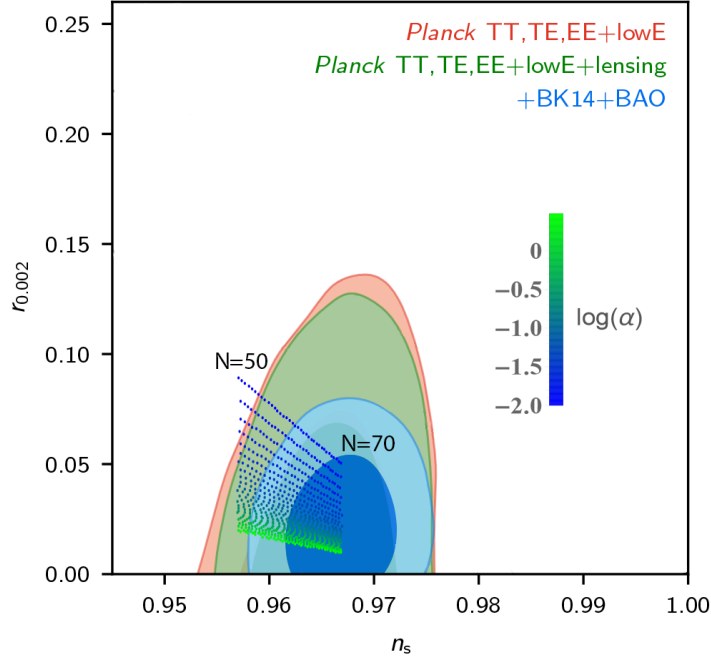


Figure 5: $r - n_s$ plot for $M = 0.7, f = 7$ and $\alpha \in [0.01, 3]$ in natural units.

$$V(\phi) = \frac{1}{2}m^2\phi^2 \quad (3.10)$$

which was excluded, we find that relatively small values of α can significantly lower the tensor-to-scalar ratio r (see Fig. 6)⁶. The same holds for non-minimally coupled models. For example, in the model considered in [157],

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} (\xi \phi^2 + \alpha \chi^2) R - \frac{1}{2} (\nabla \phi)^2 - \frac{\lambda(\phi)}{4} \phi^4 - \frac{\alpha}{4} \chi^4 + \Lambda^4 \right\}, \quad (3.11)$$

$$V_1(\phi) = \frac{1}{4} \lambda(\phi) \phi^4 + \Lambda^4 = \Lambda^4 \left(1 + \frac{\xi^2 \phi^4}{M_P^4} (2 \ln(\xi \phi^2 / M_P^2) - 1) \right), \quad \langle \phi \rangle^2 = M_P^2 / \xi \quad (3.12)$$

where the Planck scale is dynamically generated through the Coleman-Weinberg mechanism, depending on the value of the non-minimal coupling ξ , interpolates between the quadratic and linear inflation limits, both of which are now excluded by Planck. However, in the Palatini formalism with the R^2 term, the model can be easily made viable (see Fig. 7).

4. Conclusions

In conclusion, while in the metric formalism adding an R^2 term to our action results in two-field (an exception being the Coleman-Weinberg model we considered in Sec. 2 using the Gildener-Weinberg formalism), the same does not hold in the Palatini formalism. We obtain a single-field effective potential which is asymptotically flat and has a lower value, which means that we can have a smaller tensor-to-scalar ratio.

⁶However, accordance with the measured value of power spectrum requires larger values for α [144].

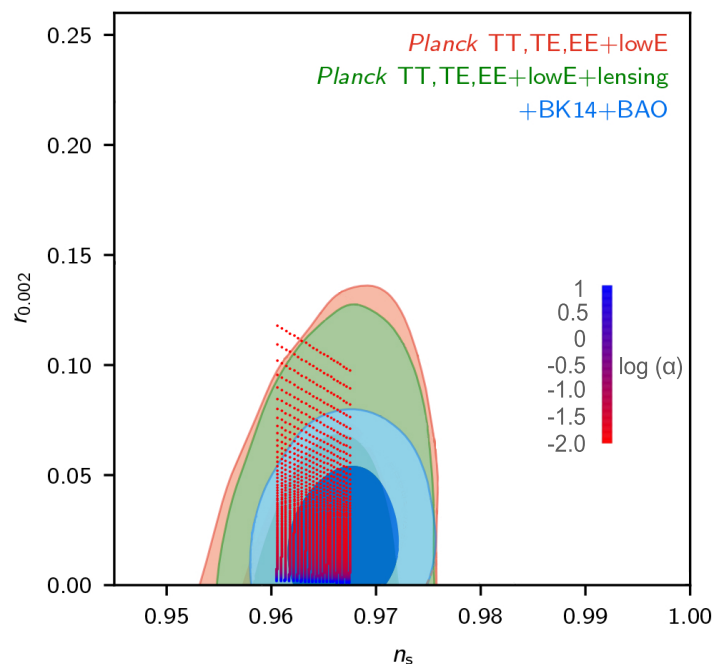
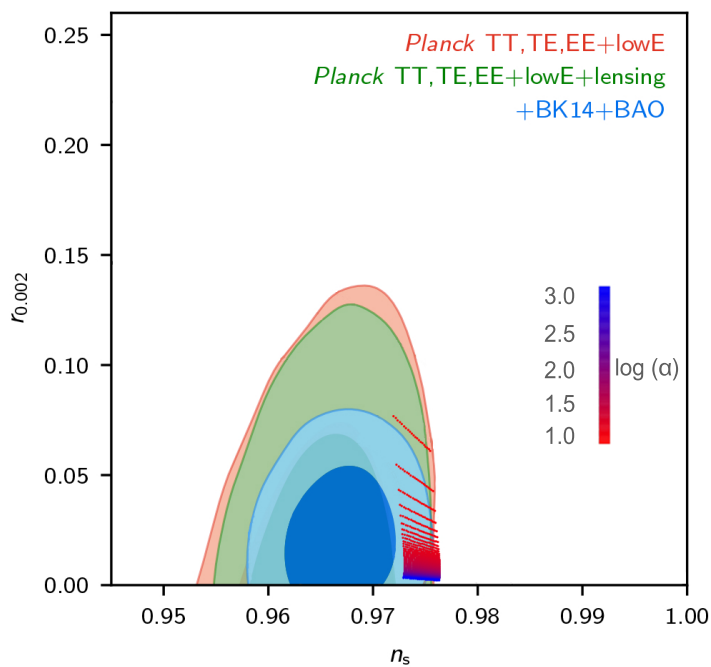


Figure 6: The predictions for the inflationary observables in the $n_s - r$ plane for the minimally-coupled quadratic model for $N = 50 - 60$ e-folds. We have set $m = 0.1$ and have varied α between 0.01 and 10.



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Figure 7: The predictions for the inflationary observables in the $n_s - r$ plane for the nonminimal Coleman-Weinberg model and for $N = 50 - 60$ e-folds. We have set $\Lambda = 0.1$ and $\xi = 0.1$ (in Planck units).

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References

- [1] A. H. Guth, “The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems,” *Phys. Rev.* **D23** (1981) 347–356.
- [2] A. D. Linde, “A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems,” *Phys. Lett.* **108B** (1982) 389–393.
- [3] S. W. Hawking, “The Development of Irregularities in a Single Bubble Inflationary Universe,” *Phys. Lett.* **115B** (1982) 295.
- [4] A. A. Starobinsky, “Dynamics of Phase Transition in the New Inflationary Universe Scenario and Generation of Perturbations,” *Phys. Lett.* **117B** (1982) 175–178.
- [5] A. H. Guth and S. Y. Pi, “Fluctuations in the New Inflationary Universe,” *Phys. Rev. Lett.* **49** (1982) 1110–1113.
- [6] A. D. Linde, “Chaotic Inflation,” *Phys. Lett.* **129B** (1983) 177–181.
- [7] A. A. Starobinsky, “A New Type of Isotropic Cosmological Models Without Singularity,” *Phys. Lett.* **91B** (1980) 99–102.
- [8] S. Capozziello, S.-i. Nojiri, S. Odintsov, and A. Troisi, “Cosmological viability of $f(r)$ -gravity as an ideal fluid and its compatibility with a matter dominated phase,” *Physics Letters B* **639** no. 3, (2006) 135–143.
- [9] F. Briscese, E. Elizalde, S. Nojiri, and S. Odintsov, “Phantom scalar dark energy as modified gravity: Understanding the origin of the big rip singularity,” *Physics Letters B* **646** no. 2, (2007) 105–111.
- [10] S. Nojiri and S. D. Odintsov, “Dark energy, inflation and dark matter from modified $f(r)$ gravity,” 0807.0685v1.
- [11] S. Nojiri and S. D. Odintsov, “Can $f(r)$ -gravity be a viable model: the universal unification scenario for inflation, dark energy and dark matter,” 0801.4843v1.
- [12] A. De Felice and S. Tsujikawa, “ $f(R)$ theories,” *Living Rev. Rel.* **13** (2010) 3, arXiv:1002.4928 [gr-qc].
- [13] S. Capozziello and M. De Laurentis, “Extended Theories of Gravity,” *Phys. Rept.* **509** (2011) 167–321, arXiv:1108.6266 [gr-qc].
- [14] T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, “Modified Gravity and Cosmology,” *Phys. Rept.* **513** (2012) 1–189, arXiv:1106.2476 [astro-ph.CO].
- [15] M. Rinaldi, G. Cognola, L. Vanzo, and S. Zerbini, “Reconstructing the inflationary $f(R)$ from observations,” *JCAP* **1408** (2014) 015, arXiv:1406.1096 [gr-qc].

- [16] K. Bamba, S. Nojiri, S. D. Odintsov, and D. Sáez-Gómez, “Inflationary universe from perfect fluid and $F(R)$ gravity and its comparison with observational data,” *Phys. Rev.* **D90** (2014) 124061, [arXiv:1410.3993 \[hep-th\]](#).
- [17] B. J. Broy, F. G. Pedro, and A. Westphal, “Disentangling the $f(R)$ - Duality,” *JCAP* **1503** no.~03, (2015) 029, [arXiv:1411.6010 \[hep-th\]](#).
- [18] S. D. Odintsov and V. K. Oikonomou, “Inflationary α -attractors from $F(R)$ gravity,” *Phys. Rev.* **D94** no.~12, (2016) 124026, [arXiv:1612.01126 \[gr-qc\]](#).
- [19] C. Guzzetti, M., N. Bartolo, M. Liguori, and S. Matarrese, “Gravitational waves from inflation,” *Riv. Nuovo Cim.* **39** no.~9, (2016) 399–495, [arXiv:1605.01615 \[astro-ph.CO\]](#).
- [20] S. Yu. Vernov, V. R. Ivanov, and E. O. Pozdeeva, “Superpotential method for $F(R)$ cosmological models,” [arXiv:1912.07049 \[gr-qc\]](#).
- [21] R. V. Wagoner, “Scalar tensor theory and gravitational waves,” *Phys. Rev.* **D1** (1970) 3209–3216.
- [22] T. Damour and G. Esposito-Farese, “Tensor-multi-scalar theories of gravitation,” *Classical and Quantum Gravity* **9** no. 9, (1992) 2093.
- [23] T. Damour and K. Nordtvedt, “Tensor-scalar cosmological models and their relaxation toward general relativity,” *Physical Review D* **48** no. 8, (1993) 3436.
- [24] J. D. Barrow, “Slow-roll inflation in scalar-tensor theories,” *Physical Review D* **51** no. 6, (1995) 2729.
- [25] J. Garcia-Bellido and D. Wands, “Constraints from inflation on scalar - tensor gravity theories,” *Phys. Rev.* **D52** (1995) 6739–6749, [arXiv:gr-qc/9506050 \[gr-qc\]](#).
- [26] H. Boutaleb-Joutei and A. L. Marrakchi, “General scalar-tensor theories for induced gravity inflation,” *Nuovo Cim.* **B112** (1997) 1605–1624.
- [27] B. Boisseau, G. Esposito-Farese, D. Polarski, and A. A. Starobinsky, “Reconstruction of a scalar tensor theory of gravity in an accelerating universe,” *Phys. Rev. Lett.* **85** (2000) 2236, [arXiv:gr-qc/0001066 \[gr-qc\]](#).
- [28] J. Morris, “Generalized slow-roll conditions and the possibility of intermediate scale inflation in scalar-tensor theory,” *Classical and Quantum Gravity* **18** no. 15, (2001) 2977.
- [29] G. Esposito-Farese and D. Polarski, “Scalar tensor gravity in an accelerating universe,” *Phys. Rev.* **D63** (2001) 063504, [arXiv:gr-qc/0009034 \[gr-qc\]](#).
- [30] T. Chiba, “ $1/r$ gravity and scalar-tensor gravity,” *Physics Letters B* **575** no. 1, (2003) 1–3.
- [31] A. Stabile, A. Stabile, and S. Capozziello, “Conformal transformations and weak field limit of scalar-tensor gravity,” *Physical Review D* **88** no. 12, (2013) 124011.
- [32] T. Chiba and M. Yamaguchi, “Conformal-Frame (In)dependence of Cosmological Observations in Scalar-Tensor Theory,” *JCAP* **1310** (2013) 040, [arXiv:1308.1142 \[gr-qc\]](#).
- [33] Y. N. Obukhov and D. Puetzfeld, “Equations of motion in scalar-tensor theories of gravity: A covariant multipolar approach,” *Physical Review D* **90** no. 10, (2014) 104041.
- [34] L. Järv, P. Kuusk, M. Saal, and O. Wilson, “Transformation properties and general relativity regime in scalar–tensor theories,” *Classical and Quantum Gravity* **32** no.~23, (2015) 235013, [arXiv:1504.02686 \[gr-qc\]](#).

- [35] P. Kuusk, L. Järv, and O. Wilson, “Invariant quantities in the multiscalar-tensor theories of gravitation,” *Int. J. Mod. Phys. A* **31** no.~02n03, (2016) 1641003, [arXiv:1509.02903 \[gr-qc\]](#).
- [36] O. Wilson, “Some remarks concerning invariant quantities in scalar-tensor gravity,” *Adv. Appl. Clifford Algebras* **27** no.~1, (2017) 321–332, [arXiv:1509.02481 \[gr-qc\]](#).
- [37] G. Tambalo and M. Rinaldi, “Inflation and reheating in scale-invariant scalar-tensor gravity,” *Gen. Rel. Grav.* **49** no.~4, (2017) 52, [arXiv:1610.06478 \[gr-qc\]](#).
- [38] M. Artymowski and A. Racioppi, “Scalar-tensor linear inflation,” *JCAP* **1704** no.~04, (2017) 007, [arXiv:1610.09120 \[astro-ph.CO\]](#).
- [39] S. Bhattacharya, K. Das, and K. Dutta, “Attractor Models in Scalar-Tensor Theories of Inflation,” [arXiv:1706.07934 \[gr-qc\]](#).
- [40] K. Bhattacharya and B. R. Majhi, “Fresh look at the scalar-tensor theory of gravity in Jordan and Einstein frames from undiscussed standpoints,” *Phys. Rev. D* **95** no.~6, (2017) 064026, [arXiv:1702.07166 \[gr-qc\]](#).
- [41] D. Burns, S. Karamitsos, and A. Pilaftsis, “Frame-Covariant Formulation of Inflation in Scalar-Curvature Theories,” *Nucl. Phys. B* **907** (2016) 785–819, [arXiv:1603.03730 \[hep-ph\]](#).
- [42] A. Karam, T. Pappas, and K. Tamvakis, “Frame-dependence of higher-order inflationary observables in scalar-tensor theories,” *Phys. Rev. D* **96** no.~6, (2017) 064036, [arXiv:1707.00984 \[gr-qc\]](#).
- [43] S. Karamitsos and A. Pilaftsis, “Frame Covariant Nonminimal Multifield Inflation,” [arXiv:1706.07011 \[hep-ph\]](#).
- [44] S. Karamitsos and A. Pilaftsis, “On the Cosmological Frame Problem,” *PoS CORFU2017* (2018) 036, [arXiv:1801.07151 \[hep-th\]](#).
- [45] A. Karam, A. Lykkas, and K. Tamvakis, “Frame-invariant approach to higher-dimensional scalar-tensor gravity,” *Phys. Rev. D* **97** no.~12, (2018) 124036, [arXiv:1803.04960 \[gr-qc\]](#).
- [46] A. Karam, T. Pappas, and K. Tamvakis, “Frame-dependence of inflationary observables in scalar-tensor gravity,” *PoS CORFU2018* (2019) 064, [arXiv:1903.03548 \[gr-qc\]](#).
- [47] C. F. Steinwachs and M. L. van der Wild, “Quantum gravitational corrections to the inflationary power spectra in scalar-tensor theories,” *Class. Quant. Grav.* **36** no.~24, (2019) 245015, [arXiv:1904.12861 \[gr-qc\]](#).
- [48] M. Artymowski, Z. Lalak, and M. Lewicki, “Inflationary scenarios in Starobinsky model with higher order corrections,” *JCAP* **1506** (2015) 032, [arXiv:1502.01371 \[hep-th\]](#).
- [49] C. van de Bruck and L. E. Paduraru, “Simplest extension of Starobinsky inflation,” *Phys. Rev. D* **92** (2015) 083513, [arXiv:1505.01727 \[hep-th\]](#).
- [50] T. Asaka, S. Iso, H. Kawai, K. Kohri, T. Noumi, and T. Terada, “Reinterpretation of the Starobinsky model,” *PTEP* **2016** no.~12, (2016) 123E01, [arXiv:1507.04344 \[hep-th\]](#).
- [51] M. Artymowski, Z. Lalak, and M. Lewicki, “Saddle point inflation from higher order corrections to Higgs/Starobinsky inflation,” *Phys. Rev. D* **93** no.~4, (2016) 043514, [arXiv:1509.00031 \[hep-th\]](#).

- [52] S. Kaneda and S. V. Ketov, “Starobinsky-like two-field inflation,” *Eur. Phys. J.* **C76** no.~1, (2016) 26, [arXiv:1510.03524 \[hep-th\]](#).
- [53] X. Calmet and I. Kuntz, “Higgs Starobinsky Inflation,” *Eur. Phys. J.* **C76** no.~5, (2016) 289, [arXiv:1605.02236 \[hep-th\]](#).
- [54] C. van de Bruck, P. Dunsby, and L. E. Paduraru, “Reheating and preheating in the simplest extension of Starobinsky inflation,” [arXiv:1606.04346 \[gr-qc\]](#).
- [55] Y.-C. Wang and T. Wang, “Primordial perturbations generated by Higgs field and R^2 operator,” *Phys. Rev.* **D96** no.~12, (2017) 123506, [arXiv:1701.06636 \[gr-qc\]](#).
- [56] Y. Ema, “Higgs Scalon Mixed Inflation,” *Phys. Lett.* **B770** (2017) 403–411, [arXiv:1701.07665 \[hep-ph\]](#).
- [57] T. Mori, K. Kohri, and J. White, “Multi-field effects in a simple extension of R^2 inflation,” *JCAP* **1710** no.~10, (2017) 044, [arXiv:1705.05638 \[astro-ph.CO\]](#).
- [58] S. Pi, Y.-l. Zhang, Q.-G. Huang, and M. Sasaki, “Scaloron from R^2 -gravity as a heavy field,” *JCAP* **1805** no.~05, (2018) 042, [arXiv:1712.09896 \[astro-ph.CO\]](#).
- [59] M. He, A. A. Starobinsky, and J. Yokoyama, “Inflation in the mixed Higgs- R^2 model,” *JCAP* **1805** no.~05, (2018) 064, [arXiv:1804.00409 \[astro-ph.CO\]](#).
- [60] D. Gorbunov and A. Tokareva, “Scaloron the healer: removing the strong-coupling in the Higgs- and Higgs-dilaton inflations,” [arXiv:1807.02392 \[hep-ph\]](#).
- [61] D. M. Ghilencea, “Two-loop corrections to Starobinsky-Higgs inflation,” [arXiv:1807.06900 \[hep-ph\]](#).
- [62] S.-J. Wang, “Quintessential Starobinsky inflation and swampland criteria,” [arXiv:1810.06445 \[hep-th\]](#).
- [63] I. Antoniadis, A. Karam, A. Lykkas, and K. Tamvakis, “Palatini inflation in models with an R^2 term,” *JCAP* **1811** no.~11, (2018) 028, [arXiv:1810.10418 \[gr-qc\]](#).
- [64] A. Gundhi and C. F. Steinwachs, “Scaloron-Higgs inflation,” [arXiv:1810.10546 \[hep-th\]](#).
- [65] V.-M. Enckell, K. Enqvist, S. Rasanen, and L.-P. Wahlman, “Higgs- R^2 inflation - full slow-roll study at tree-level,” [arXiv:1812.08754 \[astro-ph.CO\]](#).
- [66] M. He, R. Jinno, K. Kamada, S. C. Park, A. A. Starobinsky, and J. Yokoyama, “On the violent preheating in the mixed Higgs- R^2 inflationary model,” *Phys. Lett.* **B791** (2019) 36–42, [arXiv:1812.10099 \[hep-ph\]](#).
- [67] F. Bezrukov, D. Gorbunov, C. Shepherd, and A. Tokareva, “Some like it hot: R^2 heals Higgs inflation, but does not cool it,” *Phys. Lett.* **B795** (2019) 657–665, [arXiv:1904.04737 \[hep-ph\]](#).
- [68] D. D. Canko, I. D. Gialamas, and G. P. Kodaxis, “A simple $F(\mathcal{R}, \phi)$ deformation of Starobinsky inflationary model,” [arXiv:1901.06296 \[hep-th\]](#).
- [69] D. Samart and P. Channuie, “Unification of inflation and dark matter in the Higgs–Starobinsky model,” *Eur. Phys. J.* **C79** no.~4, (2019) 347, [arXiv:1812.11180 \[gr-qc\]](#).
- [70] Y. Ema, “Dynamical Emergence of Scalon in Higgs Inflation,” *JCAP* **1909** no.~09, (2019) 027, [arXiv:1907.00993 \[hep-ph\]](#).

- [71] **Planck** Collaboration, Y. Akrami *et al.*, “Planck 2018 results. X. Constraints on inflation,” [arXiv:1807.06211 \[astro-ph.CO\]](#).
- [72] **Planck** Collaboration, P. A. R. Ade *et al.*, “Planck 2015 results. XIII. Cosmological parameters,” *Astron. Astrophys.* **594** (2015) A13, [arXiv:1502.01589 \[astro-ph.CO\]](#).
- [73] R. Kallosh and A. Linde, “Multi-field conformal cosmological attractors,” *Journal of Cosmology and Astroparticle Physics* **2013** no. 12, (2013) 006.
- [74] F. L. Bezrukov and M. Shaposhnikov, “The Standard Model Higgs boson as the inflaton,” *Phys. Lett. B* **659** (2008) 703–706, [arXiv:0710.3755 \[hep-th\]](#).
- [75] F. Bezrukov and M. Shaposhnikov, “Standard Model Higgs boson mass from inflation: Two loop analysis,” *JHEP* **07** (2009) 089, [arXiv:0904.1537 \[hep-ph\]](#).
- [76] A. Karam, T. Pappas, and K. Tamvakis, “Nonminimal Coleman–Weinberg Inflation with an R^2 term,” *JCAP* **1902** (2019) 006, [arXiv:1810.12884 \[gr-qc\]](#).
- [77] F. Cooper and G. Venturi, “Cosmology and Broken Scale Invariance,” *Phys. Rev.* **D24** (1981) 3338.
- [78] M. Shaposhnikov and D. Zenhausern, “Scale invariance, unimodular gravity and dark energy,” *Phys. Lett. B* **671** (2009) 187–192, [arXiv:0809.3395 \[hep-th\]](#).
- [79] J. Garcia-Bellido, J. Rubio, M. Shaposhnikov, and D. Zenhausern, “Higgs-Dilaton Cosmology: From the Early to the Late Universe,” *Phys. Rev.* **D84** (2011) 123504, [arXiv:1107.2163 \[hep-ph\]](#).
- [80] V. V. Khoze, “Inflation and Dark Matter in the Higgs Portal of Classically Scale Invariant Standard Model,” *JHEP* **1311** (2013) 215, [arXiv:1308.6338 \[hep-ph\]](#).
- [81] F. Bezrukov, G. K. Karananas, J. Rubio, and M. Shaposhnikov, “Higgs-Dilaton Cosmology: an effective field theory approach,” *Phys. Rev.* **D87** no.~9, (2013) 096001, [arXiv:1212.4148 \[hep-ph\]](#).
- [82] E. Gabrielli, M. Heikinheimo, K. Kannike, A. Racioppi, M. Raidal, *et al.*, “Towards Completing the Standard Model: Vacuum Stability, EWSB and Dark Matter,” *Phys.Rev.* **D89** no.~1, (2014) 015017, [arXiv:1309.6632 \[hep-ph\]](#).
- [83] A. Salvio and A. Strumia, “Agravity,” *Journal of High Energy Physics* **2014** no. 6, (2014) 1–26.
- [84] C. Csaki, N. Kaloper, J. Serra, and J. Terning, “Inflation from Broken Scale Invariance,” *Phys. Rev. Lett.* **113** (2014) 161302, [arXiv:1406.5192 \[hep-th\]](#).
- [85] K. Kannike, G. Hütsi, L. Pizza, A. Racioppi, M. Raidal, A. Salvio, and A. Strumia, “Dynamically induced planck scale and inflation,” *Journal of High Energy Physics* **2015** no. 5, (2015) 1–31.
- [86] A. O. Barvinsky, A. Yu. Kamenshchik, and D. V. Nesterov, “Origin of inflation in CFT driven cosmology: R^2 -gravity and non-minimally coupled inflaton models,” *Eur. Phys. J.* **C75** no.~12, (2015) 584, [arXiv:1510.06858 \[hep-th\]](#).
- [87] L. Marzola and A. Racioppi, “Minimal but non-minimal inflation and electroweak symmetry breaking,” *JCAP* **1610** no.~10, (2016) 010, [arXiv:1606.06887 \[hep-ph\]](#).
- [88] N. D. Barrie, A. Kobakhidze, and S. Liang, “Natural Inflation with Hidden Scale Invariance,” *Phys. Lett. B* **756** (2016) 390–393, [arXiv:1602.04901 \[gr-qc\]](#).
- [89] L. Marzola, A. Racioppi, M. Raidal, F. R. Urban, and H. Veermäe, “Non-minimal CW inflation, electroweak symmetry breaking and the 750 GeV anomaly,” *JHEP* **03** (2016) 190, [arXiv:1512.09136 \[hep-ph\]](#).

- [90] M. Rinaldi and L. Vanzo, “Inflation and reheating in theories with spontaneous scale invariance symmetry breaking,” *Phys. Rev.* **D94** no.~2, (2016) 024009, [arXiv:1512.07186 \[gr-qc\]](#).
- [91] A. Farzinnia and S. Kounn, “Classically scale invariant inflation, supermassive WIMPs, and adimensional gravity,” *Phys. Rev.* **D93** no.~6, (2016) 063528, [arXiv:1512.05890 \[hep-ph\]](#).
- [92] K. Kannike, “Vacuum stability of a general scalar potential of a few fields,” *Eur.Phys.J. C* **76** (Mar., 2016) 324, [1603.02680](#).
- [93] G. K. Karananas and J. Rubio, “On the geometrical interpretation of scale-invariant models of inflation,” *Phys. Lett.* **B761** (2016) 223–228, [arXiv:1606.08848 \[hep-ph\]](#).
- [94] P. G. Ferreira, C. T. Hill, and G. G. Ross, “Scale-Independent Inflation and Hierarchy Generation,” *Submitted to: Phys. Rev. Lett.* (2016), [arXiv:1603.05983 \[hep-th\]](#).
- [95] K. Kannike, M. Raidal, C. Spethmann, and H. Veermäe, “The evolving Planck mass in classically scale-invariant theories,” *JHEP* **04** (2017) 026, [arXiv:1610.06571 \[hep-ph\]](#).
- [96] A. Racioppi, “Coleman-Weinberg linear inflation: metric vs. Palatini formulation,” *JCAP* **1712** no.~12, (2017) 041, [arXiv:1710.04853 \[astro-ph.CO\]](#).
- [97] P. G. Ferreira, C. T. Hill, and G. G. Ross, “Weyl Current, Scale-Invariant Inflation and Planck Scale Generation,” *Phys. Rev.* **D95** no.~4, (2017) 043507, [arXiv:1610.09243 \[hep-th\]](#).
- [98] A. Salvio, “Inflationary Perturbations in No-Scale Theories,” *Eur. Phys. J. C* **77** no.~4, (2017) 267, [arXiv:1703.08012 \[astro-ph.CO\]](#).
- [99] K. Kannike, A. Racioppi, and M. Raidal, “Super-heavy dark matter – Towards predictive scenarios from inflation,” *Nucl. Phys.* **B918** (2017) 162–177, [arXiv:1605.09378 \[hep-ph\]](#).
- [100] A. Barnaveli, S. Lucat, and T. Prokopec, “Inflation as a spontaneous symmetry breaking of Weyl symmetry,” [arXiv:1809.10586 \[gr-qc\]](#).
- [101] P. G. Ferreira, C. T. Hill, and G. G. Ross, “Inertial Spontaneous Symmetry Breaking and Quantum Scale Invariance,” [arXiv:1801.07676 \[hep-th\]](#).
- [102] A. Racioppi, “A new universal attractor: linear inflation,” [arXiv:1801.08810 \[astro-ph.CO\]](#).
- [103] A. Karam, L. Marzola, T. Pappas, A. Racioppi, and K. Tamvakis, “Constant-Roll (Quasi-)Linear Inflation,” *JCAP* **1805** no.~05, (2018) 011, [arXiv:1711.09861 \[astro-ph.CO\]](#).
- [104] S. Casas, G. K. Karananas, M. Pauly, and J. Rubio, “Scale-invariant alternatives to general relativity. III. The inflation-dark energy connection,” *Phys. Rev.* **D99** no.~6, (2019) 063512, [arXiv:1811.05984 \[astro-ph.CO\]](#).
- [105] C. Wetterich, “Quantum scale symmetry,” [arXiv:1901.04741 \[hep-th\]](#).
- [106] I. Oda, “Planck Scale from Broken Local Conformal Invariance in Weyl Geometry,” [arXiv:1909.09889 \[hep-th\]](#).
- [107] Y. Tang and Y.-L. Wu, “Weyl Symmetry Inspired Inflation and Dark Matter,” [arXiv:1904.04493 \[hep-ph\]](#).
- [108] J. Kubo, M. Lindner, K. Schmitz, and M. Yamada, “Planck mass and inflation as consequences of dynamically broken scale invariance,” *Phys. Rev.* **D100** no.~1, (2019) 015037, [arXiv:1811.05950 \[hep-ph\]](#).

- [109] D. M. Ghilencea, “Weyl R^2 inflation with an emergent Planck scale,” *JHEP* **10** (2019) 209, [arXiv:1906.11572 \[gr-qc\]](#).
- [110] A. Salvio, “Quasi-Conformal Models and the Early Universe,” *Eur. Phys. J.* **C79** no.~9, (2019) 750, [arXiv:1907.00983 \[hep-ph\]](#).
- [111] S. Vicentini, L. Vanzo, and M. Rinaldi, “Scale-invariant inflation with one-loop quantum corrections,” *Phys. Rev.* **D99** no.~10, (2019) 103516, [arXiv:1902.04434 \[gr-qc\]](#).
- [112] P. G. Ferreira, C. T. Hill, J. Noller, and G. G. Ross, “Scale Independent R^2 Inflation,” *Phys. Rev.* **D100** no.~12, (2019) 123516, [arXiv:1906.03415 \[gr-qc\]](#).
- [113] P. G. Ferreira and O. J. Tattersall, “Scale Invariant Gravity and Black Hole Ringdown,” [arXiv:1910.04480 \[gr-qc\]](#).
- [114] E. Gildener and S. Weinberg, “Symmetry Breaking and Scalar Bosons,” *Phys.Rev.* **D13** (1976) 3333.
- [115] T. P. Sotiriou and V. Faraoni, “f(R) Theories Of Gravity,” *Rev. Mod. Phys.* **82** (2010) 451–497, [arXiv:0805.1726 \[gr-qc\]](#).
- [116] F. Bauer and D. A. Demir, “Inflation with Non-Minimal Coupling: Metric versus Palatini Formulations,” *Phys. Lett.* **B665** (2008) 222–226, [arXiv:0803.2664 \[hep-ph\]](#).
- [117] M. Borunda, B. Janssen, and M. Bastero-Gil, “Palatini versus metric formulation in higher curvature gravity,” *JCAP* **0811** (2008) 008, [arXiv:0804.4440 \[hep-th\]](#).
- [118] G. J. Olmo, “Palatini Approach to Modified Gravity: f(R) Theories and Beyond,” *Int. J. Mod. Phys.* **D20** (2011) 413–462, [arXiv:1101.3864 \[gr-qc\]](#).
- [119] F. Bauer, “Filtering out the cosmological constant in the Palatini formalism of modified gravity,” *Gen. Rel. Grav.* **43** (2011) 1733–1757, [arXiv:1007.2546 \[gr-qc\]](#).
- [120] N. Tamanini and C. R. Contaldi, “Inflationary Perturbations in Palatini Generalised Gravity,” *Phys. Rev.* **D83** (2011) 044018, [arXiv:1010.0689 \[gr-qc\]](#).
- [121] K. Enqvist, T. Koivisto, and G. Rigopoulos, “Non-metric chaotic inflation,” *JCAP* **1205** (2012) 023, [arXiv:1107.3739 \[astro-ph.CO\]](#).
- [122] A. Borowiec, M. Kamionka, A. Kurek, and M. Szydlowski, “Cosmic acceleration from modified gravity with Palatini formalism,” *JCAP* **1202** (2012) 027, [arXiv:1109.3420 \[gr-qc\]](#).
- [123] S. Rasanen and P. Wahlman, “Higgs inflation with loop corrections in the Palatini formulation,” [arXiv:1709.07853 \[astro-ph.CO\]](#).
- [124] C. Fu, P. Wu, and H. Yu, “Inflationary dynamics and preheating of the nonminimally coupled inflaton field in the metric and Palatini formalisms,” *Phys. Rev.* **D96** no.~10, (2017) 103542, [arXiv:1801.04089 \[gr-qc\]](#).
- [125] A. Stachowski, M. Szydlowski, and A. Borowiec, “Starobinsky cosmological model in Palatini formalism,” *Eur. Phys. J.* **C77** no.~6, (2017) 406, [arXiv:1608.03196 \[gr-qc\]](#).
- [126] M. Szydlowski, A. Stachowski, and A. Borowiec, “Emergence of running dark energy from polynomial f(R) theory in Palatini formalism,” *Eur. Phys. J.* **C77** no.~9, (2017) 603, [arXiv:1707.01948 \[gr-qc\]](#).
- [127] T. Tenkanen, “Resurrecting Quadratic Inflation with a non-minimal coupling to gravity,” [arXiv:1710.02758 \[astro-ph.CO\]](#).

- [128] T. Markkanen, T. Tenkanen, V. Vaskonen, and H. Veermäe, “Quantum corrections to quartic inflation with a non-minimal coupling: metric vs. Palatini,” *JCAP* **1803** no.~03, (2018) 029, [arXiv:1712.04874 \[gr-qc\]](#).
- [129] P. Carrilho, D. Mulryne, J. Ronayne, and T. Tenkanen, “Attractor Behaviour in Multifield Inflation,” *JCAP* **1806** no.~06, (2018) 032, [arXiv:1804.10489 \[astro-ph.CO\]](#).
- [130] V.-M. Enckell, K. Enqvist, S. Rasanen, and E. Tomberg, “Higgs inflation at the hilltop,” *JCAP* **1806** no.~06, (2018) 005, [arXiv:1802.09299 \[astro-ph.CO\]](#).
- [131] V.-M. Enckell, K. Enqvist, S. Rasanen, and L.-P. Wahlman, “Inflation with R^2 term in the Palatini formalism,” *JCAP* **1902** (2019) 022, [arXiv:1810.05536 \[gr-qc\]](#).
- [132] A. Kozak and A. Borowiec, “Palatini frames in scalar-tensor theories of gravity,” [arXiv:1808.05598 \[hep-th\]](#).
- [133] L. Järv, A. Racioppi, and T. Tenkanen, “Palatini side of inflationary attractors,” *Phys. Rev.* **D97** no.~8, (2018) 083513, [arXiv:1712.08471 \[gr-qc\]](#).
- [134] Z. Wang, P. Wu, and H. Yu, “Stability analysis for non-minimally coupled dark energy models in the Palatini formalism,” *Astrophys. Space Sci.* **363** no.~6, (2018) 120.
- [135] J. Wu, G. Li, T. Harko, and S.-D. Liang, “Palatini formulation of $f(R, T)$ gravity theory, and its cosmological implications,” *Eur. Phys. J.* **C78** no.~5, (2018) 430, [arXiv:1805.07419 \[gr-qc\]](#).
- [136] M. Szydlowski and A. Stachowski, “Polynomial $f(R)$ Palatini cosmology – dynamical system approach,” *Phys. Rev.* **D97** no.~10, (2018) 103524, [arXiv:1712.00822 \[gr-qc\]](#).
- [137] S. Rasanen and E. Tomberg, “Planck scale black hole dark matter from Higgs inflation,” [arXiv:1810.12608 \[astro-ph.CO\]](#).
- [138] S. Rasanen, “Higgs inflation in the Palatini formulation with kinetic terms for the metric,” [arXiv:1811.09514 \[gr-qc\]](#).
- [139] J. P. B. Almeida, N. Bernal, J. Rubio, and T. Tenkanen, “Hidden Inflaton Dark Matter,” [arXiv:1811.09640 \[hep-ph\]](#).
- [140] J. Rubio and E. S. Tomberg, “Preheating in Palatini Higgs inflation,” *JCAP* **1904** no.~04, (2019) 021, [arXiv:1902.10148 \[hep-ph\]](#).
- [141] K. Shimada, K. Aoki, and K.-i. Maeda, “Metric-affine Gravity and Inflation,” *Phys. Rev.* **D99** no.~10, (2019) 104020, [arXiv:1812.03420 \[gr-qc\]](#).
- [142] T. Takahashi and T. Tenkanen, “Towards distinguishing variants of non-minimal inflation,” *JCAP* **1904** (2019) 035, [arXiv:1812.08492 \[astro-ph.CO\]](#).
- [143] R. Jinno, K. Kaneta, K.-y. Oda, and S. C. Park, “Hillclimbing inflation in metric and Palatini formulations,” *Phys. Lett.* **B791** (2019) 396–402, [arXiv:1812.11077 \[gr-qc\]](#).
- [144] T. Tenkanen, “Minimal Higgs inflation with an R^2 term in Palatini gravity,” *Phys. Rev.* **D99** no.~6, (2019) 063528, [arXiv:1901.01794 \[astro-ph.CO\]](#).
- [145] A. Edery and Y. Nakayama, “Palatini formulation of pure R^2 gravity yields Einstein gravity with no massless scalar,” *Phys. Rev.* **D99** no.~12, (2019) 124018, [arXiv:1902.07876 \[hep-th\]](#).
- [146] R. Jinno, M. Kubota, K.-y. Oda, and S. C. Park, “Higgs inflation in metric and Palatini formalisms: Required suppression of higher dimensional operators,” [arXiv:1904.05699 \[hep-ph\]](#).

- [147] K. Aoki and K. Shimada, “Scalar-metric-affine theories: Can we get ghost-free theories from symmetry?,” *Phys. Rev.* **D100** no.~4, (2019) 044037, [arXiv:1904.10175 \[hep-th\]](#).
- [148] M. Giovannini, “Post-inflationary phases stiffer than radiation and Palatini formulation,” *Class. Quant. Grav.* **36** no.~23, (2019) 235017, [arXiv:1905.06182 \[gr-qc\]](#).
- [149] N. Bostan, “Non-minimally coupled quartic inflation with Coleman-Weinberg one-loop corrections in the Palatini formulation,” [arXiv:1907.13235 \[gr-qc\]](#).
- [150] N. Bostan, “Quadratic, Higgs and hilltop potentials in the Palatini gravity,” [arXiv:1908.09674 \[astro-ph.CO\]](#).
- [151] T. Tenkanen, “Trans-Planckian Censorship, Inflation and Dark Matter,” [arXiv:1910.00521 \[astro-ph.CO\]](#).
- [152] I. D. Gialamas and A. B. Lahanas, “Reheating in R^2 Palatini inflationary models,” [arXiv:1911.11513 \[gr-qc\]](#).
- [153] A. Racioppi, “Non-minimal (self-)running inflation: metric vs. Palatini formulation,” [arXiv:1912.10038 \[hep-ph\]](#).
- [154] I. Antoniadis, A. Karam, A. Lykkas, T. Pappas, and K. Tamvakis, “Rescuing Quartic and Natural Inflation in the Palatini Formalism,” *JCAP* **1903** no.~03, (2019) 005, [arXiv:1812.00847 \[gr-qc\]](#).
- [155] K. Freese, J. A. Frieman, and A. V. Olinto, “Natural inflation with pseudo - Nambu-Goldstone bosons,” *Phys. Rev. Lett.* **65** (1990) 3233–3236.
- [156] F. C. Adams, J. R. Bond, K. Freese, J. A. Frieman, and A. V. Olinto, “Natural inflation: Particle physics models, power law spectra for large scale structure, and constraints from COBE,” *Phys. Rev.* **D47** (1993) 426–455, [arXiv:hep-ph/9207245 \[hep-ph\]](#).
- [157] K. Kannike, A. Racioppi, and M. Raidal, “Linear inflation from quartic potential,” *JHEP* **01** (2016) 035, [arXiv:1509.05423 \[hep-ph\]](#).