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α_s from the QCD static energy

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We present our latest determination of the strong coupling constant α_s from the Quantum Chromodynamics static energy: $\alpha_s(m_Z) = 0.1166^{+0.0012}_{-0.0008}$, extracted at three loops with leading ultrasoft log resummation. The determination is based on a combination of lattice data on the static energy at small quark-antiquark distance and perturbative high-order calculations of the static energy for small quark-antiquark distance. We discuss further improvements from an upcoming extraction based on new lattice data, at smaller lattice spacings reaching shorter distances, and on lattice data on the singlet free energy at finite temperature at very small distances.

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The QCD static energy $E_0(r)$, i.e. the energy between a static quark and a static antiquark separated by a distance r, is a basic object to understand the behavior of strong interactions [1] and constitutes a fundamental ingredient in the description of many physical processes [2]. The short-distance part of $E_0(r)$ has been computed, in the continuum in the $\overline{\rm MS}$ scheme, using perturbative and effective field theory techniques: it is nowadays known at next-to-next-to-next-to leading-logarithmic (N^{3}LL) accuracy, i.e. including terms up to order $\alpha_s^{4+n}\ln^n\alpha_s$ with $n\geq 0$ [3]. The $\ln\alpha_s$ terms appear due to virtual emissions of ultrasoft gluons, which can change the color state of the quark-antiquark pair [4], and in this context the soft (S) scale is $1/r$ and the ultrasoft (US) scale is α_s/r. $E_0(r)$ is a physical observable (up to an additive constant) and as such it can also be computed on the lattice. It depends only on $\Lambda_{\rm QCD}$ and r. The comparison between the perturbative and the lattice calculations tests our ability to describe the short-distance regime of QCD, provides information on the region of validity of the perturbative weak-coupling approach and allows for an extraction of α_s. In particular, for distances such that $r\Lambda_{\rm QCD}\ll 1$ both the perturbative and the lattice evaluations should agree. Then, one can proceed as follows: fix the scale of the lattice calculation by reproducing a low energy observable[1]; evaluate $E_0(r)$ for small r perturbatively in the $\overline{\rm MS}$ scheme at the needed order; get $\Lambda_{\overline{\rm MS}}$ at a given scale by equating the lattice and the perturbative expressions for E_0; extract α_s from $\Lambda_{\overline{\rm MS}}$ and then run it to the Z mass scale. Notice that in such case no lattice-to-$\overline{\rm MS}$ scheme change is necessary because we deal directly with a physical quantity.

The expression of the static energy in perturbation theory at N^{3}LL is summarized in [9, 7]. It contains a residual mass and it depends on α_s at the scale $1/r$ and on the logs of the US scale, that can be resummed at one (N^{2}LL) or two loop accuracy (N^{3}LL) using renormalization group equations in the effective field theory called potential nonrelativistic QCD [5]. A renormalon ambiguity in the series expansion should be appropriately canceled with the residual mass to leave an object well behaved in perturbation theory. When we compare the perturbative curve for the static energy with the lattice data we need to plot $E_0(r)-E_0(r_{ref})+E_0^{latt}(r_{ref})=E_0(r)+const$ where r_{ref} is the reference distance where we make the perturbative expression coincide with the lattice data and $E_0^{latt}(r_{ref})$ is the value of the static energy computed on the lattice at that distance.

In Ref. [6] we started a program to extract a precise determination of α_s by using lattice data for the short-distance part of the static energy in $2+1$-flavor QCD [8] and comparing them with the perturbative calculation. This allowed us to determine the strong coupling α_s at three-loop accuracy (including resummation of the leading ultrasoft logarithms), in a way that is largely independent from the other determinations that currently enter in the world average. The natural scale where our determination is performed corresponds to the inverse of the typical distance where we have lattice data, i.e. around 1.5 GeV. Therefore, our analysis provided a determination of α_s at a scale smaller than those entering the world average, and constituted in this way an important ingredient to further test asymptotic freedom in QCD. We obtained $r_0\Lambda_{\overline{\rm MS}}=0.70\pm0.07$, which, using $r_0=0.468\pm0.004$ fm [8] gave $\alpha_s(1.5{\rm GeV},n_f=3)=0.326\pm0.019$ corresponding to $\alpha_s(m_Z,n_f=5)=0.1156^{+0.0021}_{-0.0022}$. The error is dominated by the perturbative uncertainty and could be reduced by using lattice data at shorter quark-antiquark distance.

[1] Conventionally in these calculations the scale is fixed through the scale parameters r_0 or r_1 defined by the condition: $r^2\frac{dE_0(r)}{dr}|_{r=r_0}=1.65$, $r^2\frac{dE_0(r)}{dr}|_{r=r_1}=1$. The values of r_0 and r_1 are extracted from a lattice calculation of a low energy observable.

In our most recent published extraction [9] we therefore used the 2+1 flavor lattice data [15]. The strange-quark mass m_s was fixed to its physical value, while the light-quark masses were chosen to be $m_l = m_s/20$. These correspond to a pion mass of about 160 MeV in the continuum limit, which is very close to the physical value. More precisely, we used lattice QCD data corresponding to the lattice gauge couplings $\beta = 10/g^2 = 7.150,\ 7.280, 7.373, 7.596$ and 7.825. The largest gauge coupling, $\beta = 7.825$, corresponds to lattice spacings of $a = 0.041$ fm[2].

Our extraction [9] was improved in several ways and the central value and the error have been scrutinized with a long list of checks that we briefly describe in the following (the details of all this is described in [9]). Lattice artifacts at small distance r may be significant: such artifacts have been removed and the corresponding systematic error has been estimated. The renormalon subtraction has been optimized. We performed fits to the lattice data for the static energy using the perturbative expression at different orders, starting from tree level up to three-loops and we kept only the range of data in which the fit was improving, confirming that we have reached the perturbative window. We repeated the analysis using both the static energy and the force. We performed the analysis with the ultrasoft resummation at N^{2}LL and at N^{3}LL accuracy as well as with N^{3}LO accuracy plus leading US logarithms. In doing so we found that even if all these analyses turn out to be consistent, the size of the leading US logs appeared to be comparable to the three loops correction, which eventually selected the order at which we extracted α_s. We varied the analysis considering only some subsets of lattice points and/or varying the reference point r_{ref}. We repeated the fits adding r^3 and r^2 monomials to see if the presence of nonperturbative corrections (nonlocal condensates), not accounted in the previous fits, could distort the analysis: we did not find any evidence of nonperturbative corrections. Lastly, we varied the soft scale and consider the size of the next perturbative correction to estimate the perturbative error.

Our final error comes from the sum in quadrature of the statistical error, the perturbative error and the error on the scale r_1. Given all the performed checks, we consider our α_s extraction and the error attached to it pretty solid. We obtained $r_1\Lambda_{\overline{\rm MS}} = 0.495^{+0.028}_{-0.018}$. By converting this result to physical units by using $r_1 = 0.3106 \pm 0.0017$ fm, fixed from the pion decay constant f_π [14], we obtained $\Lambda_{\overline{\rm MS}} = 315^{+18}_{-12}$ MeV. This value of $\Lambda_{\overline{\rm MS}}$ gives $\alpha_s(1.5\ {\rm GeV}, n_f = 3) = 0.336^{+0.012}_{-0.008}$, corresponding to $\alpha_s(m_Z, n_f = 5) = 0.1166^{+0.0012}_{-0.0008}$. This is an extraction of α_s at three loops plus leading US logs resummation and the number is perfectly compatible, but more accurate, with our previous result given above.

In Fig 1 we show the results one obtains when using larger distance ranges in the fits, up to $r < 0.75 r_1$. The distances $r < 0.6 r_1$ are the ones that passed our χ^2 criteria[3], and were therefore deemed as suitable for the α_s extraction. The point of showing here the results from larger distance ranges is to illustrate that nothing dramatic happens beyond that point. Figure 1 shows the results for $r_1\Lambda_{\overline{\rm MS}}$ at three-loop accuracy, in all the distance ranges that we have analyzed in Ref. [9].

As one can see from the figure, the fits that use distances larger than $0.6 r_1$ give results for $r_1\Lambda_{\overline{\rm MS}}$ that are compatible with those used in our main analysis. The error bars, which come from unknown higher-perturbative orders, are larger in the extended distance ranges. This may be

[2] One may worry about the evolution of the topological charge on such fine lattices, but, as it was shown in Ref. [15], the Monte Carlo evolution of the topological charge is acceptable even for $\beta = 7.825$.

[3] We required that the χ^2 should improve or at least stay constant passing from one perturbative order to the subsequent one and by doing so we selected the perturbative window.

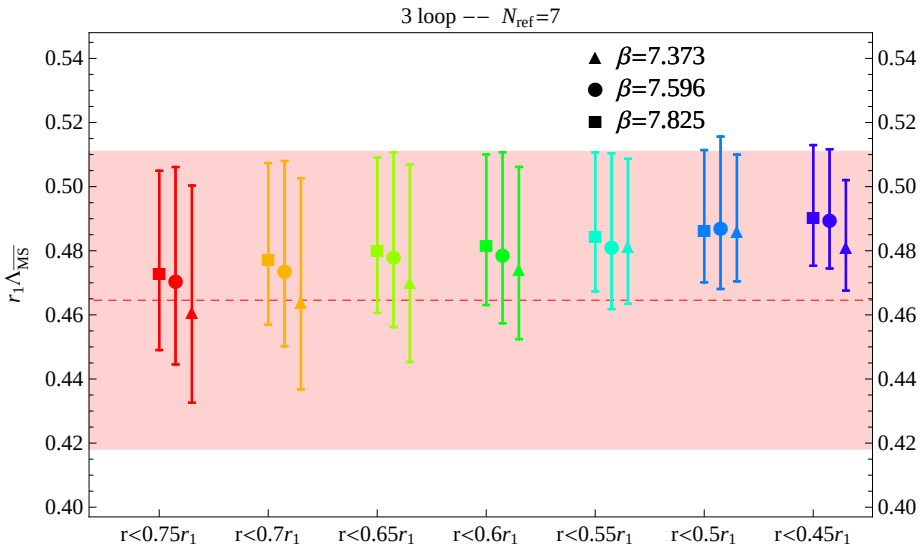

Figure 1: Results for $r_1\Lambda_{\overline{\rm MS}}$ at three-loop accuracy, also showing the outcome of analyses with extended distance ranges. For reference and comparison, the band shows our previous result in Ref. [6]. This figure is taken from [9].

attributed to the fact that those fits involve lower-energy scales and therefore larger values of α_s.

In Fig. 2 we put together the data for all the lattice spacings we have, including those used in Ref. [6], i.e. from $\beta = 6.664$ to $\beta = 7.825$, and compare them with the perturbative expressions at different orders of accuracy. The uncertainties due to the normalization of the lattice data to a common scale are now included in the error bars, as it is appropriate when putting together data from different lattice spacings. One can see that the lattice data are perfectly reproduced by perturbation theory and the different perturbative orders converge to the lattice data.

We would like to further reduce our error: this would entail to get lattice data at smaller spacing and smaller r. At the present day, these lattices still pose a major challenge due to critical slowing down, topological freezing, and the need to maintain a sufficiently large volume (in units of the inverse pion mass). In an upcoming paper [10], we use lattices [11] with extraordinarily fine lattice spacing ($a = 0.0246\,\text{fm}$) to achieve a systematically improved extraction of α_s. Additionally, we exploit a new idea. One reason for which it is challenging to reach such fine lattice spacings is that one has to simultaneously maintain the control over finite volume effects from the propagation of the lightest hadronic modes, namely, the Goldstone bosons, at the pion scale. A lattice simulation at high enough temperature avoids this infrared problem, and thus enables reaching much finer lattice spacings using smaller volumes. We use finite temperature lattices with unprecedentedly fine lattice spacing ($a = 0.00848\,\text{fm}$) [12]. The singlet static free energy is again a function of the static quark-antiquark distance and has been calculated on the lattice [12] and perturbatively using finite temperature effective field theory methods [13]. The comparison between the two offers a novel and independent method to get a precise determination of α_s. The results that we are obtaining in these two ways in [10] confirm our 2014 determination of α_s [9] with smaller errors.

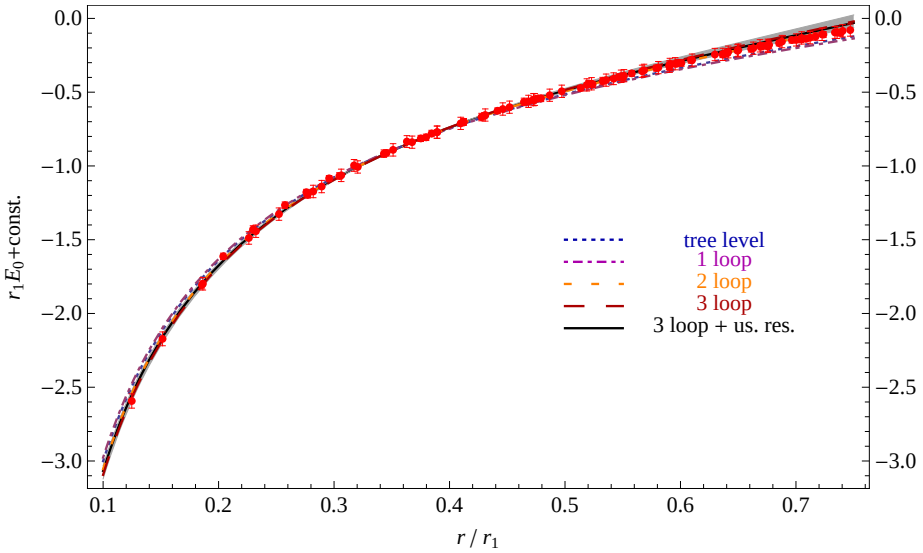

Figure 2: Comparison of the lattice data for the static energy with perturbative expressions at different orders of accuracy. $r_1\Lambda_{\overline{\rm MS}} = 0.495$ is used for all the curves. The grey band corresponds to the variation $r_1\Lambda_{\overline{\rm MS}} = 0.495^{+0.028}_{-0.018}$ for the three-loop plus leading-ultrasoft-resummation accuracy curve. Figure taken from [9].

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