PROCEEDINGS OF SCIENCE

The strong coupling from $e^+e^- \to$ hadrons below charm threshold

Diogo Boito*
Instituto de Física de São Carlos, Universidade de São Paulo, CP 369, 13570-970, São Carlos, SP, Brazil
E-mail: boito@ifsc.usp.br

Maarten Golterman
Department of Physics and Astronomy, San Francisco State University, San Francisco, CA 94132, USA

Alexander Keshavarzi
University of Mississippi - Dept. of Physics & Astronomy, Oxford, Mississippi 38677-1848, USA

Kim Maltman
Department of Mathematics and Statistics, York University, Toronto, ON Canada M3J 1P3
CSSM, University of Adelaide, Adelaide, SA 5005 Australia

Daisuke Nomura
KEK Theory Center, Tsukuba, Ibaraki 305-0801, Japan

Santiago Peris
Dept. of Physics and IFAE-BIST, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona

Thomas Teubner
Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, UK

We present our recent results on the extraction of the strong coupling, α_s, from a new compilation of the hadronic R-ratio obtained from the available data for $e^+e^- \to$ hadrons. The determination uses all data up to $s = 4$ GeV^2, where s is the square of the center-of-mass energy. We obtain competitive values with only a small contamination from non-perturbative effects and with smaller theoretical uncertainties than the extractions from hadronic τ decays.

European Physical Society Conference on High Energy Physics - EPS-HEP2019 -
10-17 July, 2019
Ghent, Belgium

*Speaker.

PoS(EPS-HEP2019)473

1. Introduction and theoretical framework

Extractions of the strong coupling, α_s, at low energies ($\sim$ 2 GeV) provide one of the most stringent tests of asymptotic freedom as predicted by the QCD β function, which at present is known to five loops [1]. These determinations also benefit from a reduction in the relative error once the coupling is evolved to higher energies due to the logarithmic scale dependence, and can therefore be competitive — provided that the non-perturbative contributions are under control. For many years, the main extraction of α_s at low energies was that from the QCD analysis of inclusive $\tau \to \text{hadrons} + \nu_\tau$ decays [2, 3, 4, 5, 6].[1]

Although a competitive extraction of α_s can be achieved from hadronic τ-decay data, non-perturbative contributions, albeit small compared to the perturbative corrections, cannot be neglected. In fact, the treatment of the non-perturbative contributions is still one of the main sources of theoretical uncertainties. The two most recent analyses disagree on how this contribution should be treated, which leads to a certain tension between their results. Concrete evidence [9] supports the notion that the analysis of Ref. [6] underestimates this error, but it remains true that the understanding of non-perturbative corrections in τ decays could be improved with higher-precision data sets, which are unlikely to be available any time soon. A second source of theoretical error stems from the renormalisation-scale setting. A strict fixed order analysis, known as Fixed Order Perturbation Theory (FOPT), leads to smaller values of α_s while the resummation of certain classes of contributions to all orders using the QCD beta fuction, known as Contour Improved Perturbation Theory (CIPT) [10], leads to larger α_s values. In this respect, our understanding of the perturbative series favours the use of the fixed order expansion [11, 12] but, as in the case of non-perturbative contributions, it would be desirable to reduce the theoretical uncertainty that arises from the perturbative series itself.

Non-perturbative contributions as well as the differences related to renormalisation-scale setting should both be smaller at higher energies. The τ-decay data are, of course, kinematically limited by the τ mass. Therefore, a natural way of circumventing this kinematical limitation and attack both issues at the same time is the use of data for the R-ratio defined as

$$R(s) = \frac{3s}{4\pi\alpha^2}\sigma(e^+e^- \to \text{hadrons}(\gamma)) \approx \frac{\sigma(e^+e^- \to \text{hadrons}(\gamma))}{\sigma(e^+e^- \to \mu^+\mu^-)}, \tag{1.1}$$

from which one can extract the imaginary part of the electromagnetic (EM) current-current correlator, $\Pi_{\text{EM}}(s)$, through the optical theorem

$$\text{Im}\,\Pi_{\text{EM}}(s) = \pi\rho_{\text{EM}}(s) = \frac{1}{12\pi}R(s), \tag{1.2}$$

where $\rho_{\text{EM}}(s)$ is the EM spectral function. The γ within parentheses in Eq. (1.1) indicates that the hadronic final states are inclusive of final-state radiation. Here we summarise our recent work [13] in which we employ a recent compilation of R data [14] below the charm threshold in order to extract the strong coupling with $n_f = 3$.

In order to fully profit from the data for $R(s)$ we employ finite-energy sum rules FESRs — the same technique used in the analysis of hadronic τ decay data. Since the EM correlator $\Pi_{\text{EM}}(z)$ is

[1] With lattice data one can also obtain information about α_s at similar energy scales from the $c\bar{c}$ pseudo-scalar correlator as well as from the the $q\bar{q}$ static potential [7, 8].

analytic everywhere in the complex plane except along the real axis, one can relate integrals over $\mathrm{Im}\Pi_{\mathrm{EM}}(s+i0)$ to integrals over $\Pi_{\mathrm{EM}}(z)$ along a closed contour in the complex plane as

$$I^{(w)}(s_0) \equiv \frac{1}{12\pi^2 s_0}\int_0^{s_0} ds\, w\left(\frac{s}{s_0}\right) R(s) = -\frac{1}{2\pi i\, s_0}\oint_{|z|=s_0} dz\, w\left(\frac{z}{s_0}\right)\Pi_{\mathrm{EM}}(z)\,, \tag{1.3}$$

where the weight function $w(y)$ must be analytic and we used Eq. (1.2) to write $\mathrm{Im}\,\Pi_{\mathrm{EM}}(s+i0)$ in terms of the experimental measurements of $R(s)$. The theory predictions for the correlator $\Pi_{\mathrm{EM}}(z)$ enter the contour integral performed along a circle of radius s_0 on the right-hand side of Eq. (1.3). The compelling reason for using FESRs in the case of our analysis is that weighted integrals over the data, which take advantage of the full data set from threshold up to s_0, have a significantly smaller relative error than local $R(s)$ measurements. With a judicious choice of weight functions $w(s)$ and the presently available data for $R(s)$ the FESRs of Eq. (1.3) lead to a competitive extraction of α_s.

The main contribution to the theoretical description of Π_{EM} are the perturbative QCD corrections which are known at present up to order α_s^4 (5-loops) [15]. The perturbative contribution is calculated in the chiral limit. Corrections due to the masses of the quarks u and d can safely be neglected. We add the contributions due to the strange-quark mass, perturbatively, truncating the series at three-loops. We consider the leading EM correction as well, which in the end is responsible for a small shift of about -0.001 in the value of $\alpha_s^{(n_f=3)}(m_\tau^2)$. One must also consider the non-perturbative contributions from the condensates in the Operator Product Expansion (OPE) of $\Pi_{\mathrm{EM}}(z)$ which can be written as

$$\Pi_{\mathrm{OPE}}^{(\mathrm{cond})}(z) = \sum_{k=2}^{\infty}\frac{C_{2k}(z)}{(-z)^k}\,. \tag{1.4}$$

Our investigations in the case of the analysis of hadronic τ data have shown that it is safe to neglect the energy dependence of $C_{2k}(z)$ that arises from the α_s-suppressed logarithms; the condensates are then represented by effective constants C_{2k} to be extracted from the data. Finally, one must consider the contributions that go beyond the OPE and are known as (quark-hadron) duality violations DVs [16]. A reliable parametrisation for the DVs has been developed [17, 18] and it was used to show that their contribution is sizeable for $s \lesssim m_\tau^2$. In the case of the present work, thanks to the fact that the analysis of $R(s)$ can be done for energies higher than m_τ^2, we were able to show, quantitatively, that it is safe to neglect the DVs in the determination of our central values [13]. The DVs are then included, in the end, just as an additional source of theoretical uncertainty.

The weight functions that we considered in the FESRs of our analysis are

$$\begin{aligned} w_0(y) &= 1\,, \\ w_2(y) &= 1-y^2\,, \\ w_3(y) &= (1-y)^2(1+2y)\,, \\ w_4(y) &= (1-y^2)^2. \end{aligned} \tag{1.5}$$

This choice of weight functions is based on a few guiding principles. First, it is important to restrict the weight functions to lower powers of y since a monomial of degree y^n is maximally sensitive to the condensate of dimension $2n+2$. It is dangerous to probe the OPE at high orders [19] since

PoS(EPS-HEP2019)473

very little is known about the $C_{2k}(z)$ in these cases and the series is expected to be divergent (if asymptotic). It is also convenient to include moments that are "pinched," meaning moments that have a zero at $z = s_0$. The zero in these moments suppresses DVs since they are expected to be more prominent close to the real axis. Finally, we do not include moments with a term linear in y which would bring contributions of dimension $D = 4$ in the OPE, linked to a more unstable perturbative series [12].

The central results from our analysis come from FESRs with $s_0 \leq 4.0$ GeV^2, although in principle we could use data at energies up to the charm threshold. The data up to 4.0 GeV^2 are obtained from sums over all exclusive hadronic channels,[2] using results from many different experiments, taking into account all the available correlated uncertainties [14]. There is a wealth of data and a very fine binning can be achieved in this procedure. For $s > 4.0$ GeV^2, on the other hand, the data is obtained from the available inclusive measurements of $R(s)$ which leads to a coarse binning. In short, the inclusion of data for $s > 4.0$ GeV^2 does not lead to a significant gain in information (although compatible results are obtained when we extend the analysis beyond 4.0 GeV^2).

In Sec.2 we discuss the results of our analysis and present our final values for the strong coupling.

2. Analysis and results

The main results from our analysis come from fits performed with $m_\tau^2 \leq s_0 \leq 4.0$ GeV^2. In Fig. 1 we show a visual depiction of the results obtained from fitting to the spectral integrals with the weight functions of Eq. (1.5) in the interval $3.25 \leq s \leq 4.0$ GeV^2. We have carefully checked that the results are consistent between the different moments and are stable with respect to the fit window — as long as the window is restricted to the higher-energy portion of the available interval, in order to minimise the influence of DVs. Our final results for α_s from each moment include a theoretical error that arises from the variation of the fit window. In Tab. 1, we quote the final results for each of the weight functions employed in our work, with an uncertainty that combines the error from the data with an estimate for the error due to the choice of fit window. We choose to quote results for $\alpha_s^{(n_f=3)}(m_\tau^2)$ in order to facilitate the comparison with those obtained from hadronic τ-decay data. The results for the condensate C_6 (not shown here) which contributes to moments using weight functions w_2, w_3, and w_4 also comes out perfectly consistent between the fits to the different moments [13], which indicates that our treatment of the condensate contributions is under good control.

We have performed additional consistency checks on our results. The first one concerns the importance of DVs. If the fit window is expanded to include s_0 values below $\sim m_\tau^2$ the value of α_s becomes lower and the fit quality worsens. For these fits, consistent and stable results can be obtained with the inclusion of a contribution from the DVs, which is modelled incorporating our previous knowledge from the analysis of the isovector channel in hadronic τ-decay data. When the DVs are included in this way, the values of α_s obtained from the different fit windows, including or not values of s_0 below the τ mass, are fully compatible again. We therefore conclude that it is safe to perform analyses restricting the fit windows to higher energies without the inclusion of

[2]In this respect our $R(s)$ data set differs from the original analysis of Ref. [14] where the exclusive data were used up to 3.75 GeV^2

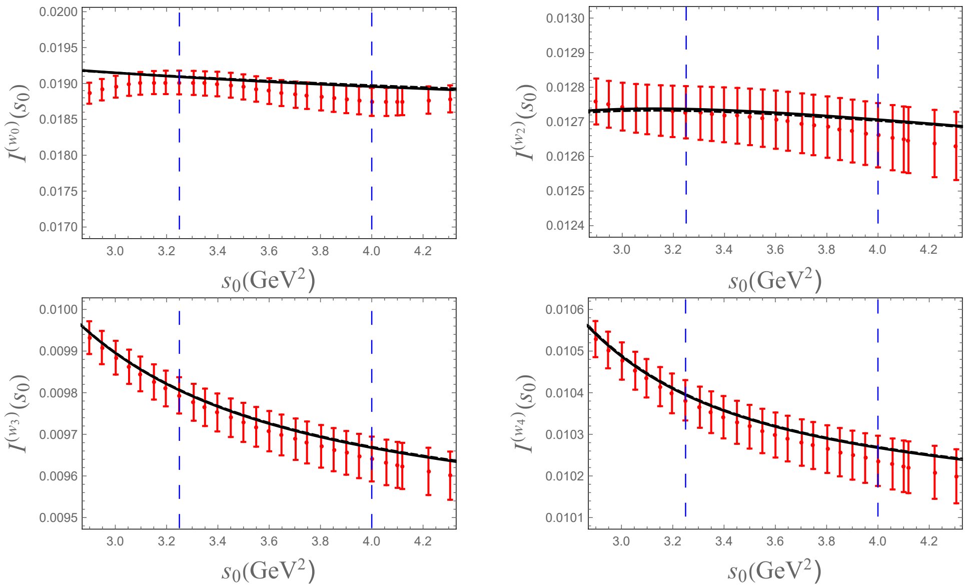

Figure 1: Comparison of the data for $I^{(w_i)}(s_0)$ with the fits on the interval $s_0^{\rm min} = 3.25$ to 4 GeV^2, for $i = 0$ (upper left panel), $i = 2$ (upper right panel), $i = 3$ (lower left panel), and $i = 4$ (lower right panel). Solid black curves indicate FOPT fits, dashed curves CIPT. The fit window is indicated by the dashed vertical lines. Further details can be found in the original publication [13].

Table 1: Values for $\alpha_s^{(n_f=3)}(m_\tau^2)$ using FOPT and CIPT from the four weights of Eq. (1.5). Uncertainties include the error from the fit as well as an estimate of the error due to the variation of the fit window.

weight	$\alpha_s^{(n_f=3)}(m_\tau^2)$ (FOPT)	$\alpha_s^{(n_f=3)}(m_\tau^2)$ (CIPT)
w_0	0.299(16)	0.308(19)
w_2	0.298(17)	0.305(19)
w_3	0.298(18)	0.303(20)
w_4	0.297(18)	0.303(20)

a DV contribution, but we do estimate their impact on our final results and we enlarge the final errors to encompass their effects. We have also checked that the inclusion of points in the inclusive region, beyond 4.0 GeV^2, produce compatible results, even though little is gained since very little information is added due to the coarse binning in that region. The details of these additional tests can be found in the original publication [13].

3. Final results and conclusions

Our final results, combining the fits to the different moments, for $\alpha_s^{(n_f=3)}(m_\tau^2)$ are

$$\alpha_s^{(n_f=3)}(m_\tau^2) = \begin{cases} 0.298 \pm (0.016)_{\rm data} \pm (0.005)_{\rm DVs} \pm (0.003)_{\rm pt} = 0.298 \pm 0.017 & \text{(FOPT)}\,, \\ 0.304 \pm (0.018)_{\rm data} \pm (0.005_{\rm DVs} \pm (0.003)_{\rm pt} = 0.304 \pm 0.019 & \text{(CIPT)}\,, \end{cases} \tag{3.1}$$

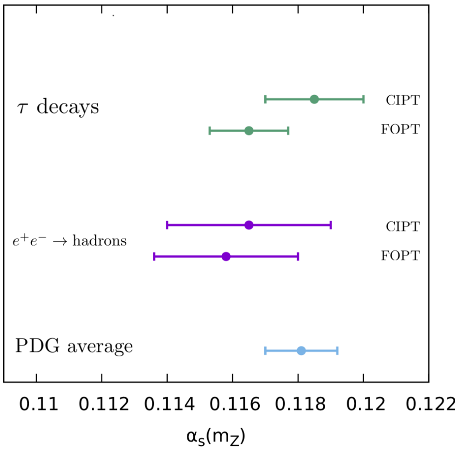

Figure 2: $\alpha_s^{(n_f=5)}(m_Z^2)$ from hadronic τ-decay data [5], $R(s)$ data [13], and the PDG world average [20].

where, as indicated, the first error is from the data, the second due to the residual DV contribution, and the third error is an estimate for the error due to the truncation of the perturbative series. It is remarkable that the difference between FOPT and CIPT, ~ 0.006, is rather small compared to the results from τ decays where this difference reaches ~ 0.016.

Evolving our final values to m_Z we obtain for $\alpha_s^{(n_f=5)}(m_Z^2)$ (in $\overline{\rm MS}$) the following results

$$\alpha_s^{(n_f=5)}(m_Z^2) = \begin{cases} 0.1158 \pm 0.0022 & \text{(FOPT)}\,, \\ 0.1165 \pm 0.0025 & \text{(CIPT)}\,. \end{cases} \tag{3.2}$$

These results are somewhat lower than, but fully compatible with, the PDG world average. We remark that the smallness of the residual theory error due to the different prescriptions for the treatment of perturbation theory makes this determination competitive with that from τ decays. In the latter, the nominal error of the values from FOPT and CIPT are smaller by a factor of about two but their difference, which is a remaining theoretical uncertainty that in the end must be taken into account, is larger by a factor of about three. A visual comparison of the results from τ decays and from $R(s)$ is shown in Fig. 2. In conclusion, the extraction of α_s from $R(s)$ data below charm threshold is sound and the errors are dominated by the data uncertainties, with good prospects for improvements in the future.

Acknowledgments

The work of DB is supported by the São Paulo Research Foundation (FAPESP) Grant No. 2015/20689-9 and by CNPq (309847/20184). The work of MG is supported by the U.S. Department of Energy, Office of Science, Office of High Energy Physics, under Award Number DE-FG03-92ER40711. The work of AK is supported by STFC under the consolidated grant ST/N504130/1. KM is supported by a grant from the Natural Sciences and Engineering Research Council of Canada. The work of DN is supported by JSPS KAKENHI grant number JP17H01133. SP is supported by FPA2017-86989-P and 2017 SGR 1069. The work of TT is supported by STFC under the consolidated grant ST/P000290/1.

PoS(EPS-HEP2019)473

References

[1] P. A. Baikov, K. G. Chetyrkin and J. H. Kühn, Phys. Rev. Lett. **118**, no. 8, 082002 (2017) [arXiv:1606.08659 [hep-ph]]; F. Herzog, B. Ruijl, T. Ueda, J. A. M. Vermaseren and A. Vogt, JHEP **1702**, 090 (2017) [arXiv:1701.01404 [hep-ph]].

[2] E. Braaten, S. Narison and A. Pich, Nucl. Phys. B **373**, 581 (1992).

[3] D. Boito, M. Golterman, M. Jamin, A. Mahdavi, K. Maltman, J. Osborne and S. Peris, Phys. Rev. D **85**, 093015 (2012) [arXiv:1203.3146 [hep-ph]].

[4] M. Davier, A. Hoecker, B. Malaescu, C. Z. Yuan and Z. Zhang, Eur. Phys. J. C **74**, 2803 (2014) [arXiv:1312.1501 [hep-ex]].

[5] D. Boito, M. Golterman, K. Maltman, J. Osborne and S. Peris, Phys. Rev. D **91**, 034003 (2015) [arXiv:1410.3528 [hep-ph]].

[6] A. Pich and A. Rodríguez-Sánchez, Phys. Rev. D **94**, 034027 (2016) [arXiv:1605.06830 [hep-ph]].

[7] A. Bazavov, N. Brambilla, X. Garcia i Tormo, P. Petreczky, J. Soto and A. Vairo, Phys. Rev. D **90**, no. 7, 074038 (2014) [arXiv:1407.8437 [hep-ph]].

[8] P. Petreczky and J. H. Weber, Phys. Rev. D **100**, no. 3, 034519 (2019) [arXiv:1901.06424 [hep-lat]].

[9] D. Boito, M. Golterman, K. Maltman and S. Peris, Phys. Rev. D **95**, 034024 (2017) [arXiv:1611.03457 [hep-ph]].

[10] A. A. Pivovarov, Z. Phys. C **53**, 461 (1992) [Sov. J. Nucl. Phys. **54**, 676 (1991)] [Yad. Fiz. **54** (1991) 1114] [arXiv:hep-ph/0302003]; F. Le Diberder and A. Pich, Phys. Lett. B **286**, 147 (1992).

[11] M. Beneke and M. Jamin, JHEP **0809**, 044 (2008) [arXiv:0806.3156 [hep-ph]].

[12] M. Beneke, D. Boito and M. Jamin, JHEP **1301**, 125 (2013) [arXiv:1210.8038 [hep-ph]].

[13] D. Boito, M. Golterman, A. Keshavarzi, K. Maltman, D. Nomura, S. Peris and T. Teubner, Phys. Rev. D **98**, no. 7, 074030 (2018) [arXiv:1805.08176 [hep-ph]].

[14] A. Keshavarzi, D. Nomura and T. Teubner, Phys. Rev. D **97**, no. 11, 114025 (2018) [arXiv:1802.02995 [hep-ph]].

[15] P. A. Baikov, K. G. Chetyrkin and J. H. Kuhn, Phys. Rev. Lett. **101**, 012002 (2008) [arXiv:0801.1821 [hep-ph]]; F. Herzog, B. Ruijl, T. Ueda, J. A. M. Vermaseren and A. Vogt, JHEP **1708**, 113 (2017) doi:10.1007/JHEP08(2017)113 [arXiv:1707.01044 [hep-ph]].

[16] B. Blok, M. A. Shifman and D. X. Zhang, Phys. Rev. D **57**, 2691 (1998) [Erratum-ibid. D **59**, 019901 (1999)] [arXiv:hep-ph/9709333]; I. I. Y. Bigi, M. A. Shifman, N. Uraltsev and A. I. Vainshtein, Phys. Rev. D **59**, 054011 (1999) [hep-ph/9805241]; M. A. Shifman, [hep-ph/0009131]; O. Cata, M. Golterman and S. Peris, JHEP **0508**, 076 (2005) [hep-ph/0506004]; O. Cata, M. Golterman and S. Peris, Phys. Rev. D **77**, 093006 (2008) [arXiv:0803.0246 [hep-ph]].

[17] O. Catà, M. Golterman and S. Peris, Phys. Rev. D **79**, 053002 (2009) [arXiv:0812.2285 [hep-ph]].

[18] D. Boito, I. Caprini, M. Golterman, K. Maltman and S. Peris, Phys. Rev. D **97**, 054007 (2018) [arXiv:1711.10316 [hep-ph]].

[19] D. Boito, M. Golterman, K. Maltman and S. Peris, Phys. Rev. D **100**, no. 7, 074009 (2019) [arXiv:1907.03360 [hep-ph]].

[20] M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D **98**, no. 3, 030001 (2018).

PoS(EPS-HEP2019)473