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Statistical Methods for the Neutrino Mass Hierarchy Determination

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In the next years, several experiments will try to perform a precise measurement of the neutrino mass hierarchy, among these there are also reactor neutrino experiments, such as JUNO and RENO 50. One of the challenges that must be overcome in this kind of experiment is the uncertainty on the reactor neutrino spectrum: as became clear with the discovery of the "5 MeV bump", the theoretical models that describe the reactor neutrino spectrum are not very reliable, and tension between the expected and measured spectrum is significantly larger than the difference between the two hierarchies; moreover the only experimental data currently available on the reactor neutrino spectrum are obtained with a worse energy resolution than the one needed in order to determine the hierarchy: I will discuss a model-independent way to treat the errors due to the uncertainty on the spectrum. The uncertainty on the spectrum is one of the reason why the JUNO collaboration announced that a near detector will be build (JUNO-TAO). However there is an additional complication: the far detector will receive neutrinos from two power plants of different model and generation, each of them with multiple reactors, while the near detector can see only the spectrum from one core; this means that the (unoscillated) spectrum at the near and far detector will be different. I will show that, by studying the time evolution of the spectrum at the near detector, it will be possible to reconstruct the spectrum at the far; I will discuss how it is possible to take into account this kind of systematic error in the measurement of the mass hierarchy, how it will affect the sensitivity and the effects of different methods of reconstructing the spectrum.

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1. Mass Hierarchy From Reactor Neutrinos

One of the goals of JUNO is to determine the neutrino mass hierarchy by measuring the oscillation probability of reactor neutrinos at intermediate baseline: in this energy range the matter effects are negligible, however it is still possible to find the hierarchy by looking at the interference between 1-3 and 2-3 oscillations. The main problem in this kind of experiment (which can be related to most if not all the challenges that must be faced) is that there is a strong degeneracy between a change of hierarchy and a shift in Δm^2_{32}: for this reason the signal we are looking for (*i.e.* the difference between the hierarchies, once the degeneracy is taken into account) is quite small, of the order of 1-4%. With the measurement of the "5 MeV bump", it become clear that the theoretical models used for the reactor neutrino spectrum are not in agreement with the experimental data: any uncertainty in the unoscillated spectrum could affect the final results; moreover, since all the experimental data are obtained with an energy resolution considerably lower than what is required for the hierarchy determination ($\Delta E/E \simeq 6\%/\sqrt{E}$, while the accuracy goal in JUNO is $3\%/\sqrt{E}$), it is possible that there is a currently undetected "fine structure" in the reactor neutrino spectrum, which would not be observable with the currently energy resolution but could affect the mass hierarchy determination [1] (some hints in this sense come also from *ab initio* calculations [2, 3]). This is one of the reasons why it was recently announced that JUNO will have a near detector, JUNO-TAO (Taishan Antineutrino Observatory). In this presentation, based on the results published on [4], we will see how the uncertainty on the spectrum can be constrained using the data from the near detector, even if the unoscillated spectra at near and far are different, and how it will affect the sensitivity to the mass hierarchy. In this work we will use $\Delta\chi^2$ as test statistic. It was argued in [5] that using a Fourier transform analysis the effect of the fine structure on the mass hierarchy determination is negligible, however there are two observations that must be made. First of all, it is important to underline that the impact of a certain systematic error depends on the choice of the test statistic: even if a systematic error can be neglected using the test statistic A, it still could still be relevant if we use the test statistic B instead. Second, there are many other problems related to the use of the Fourier transform for the mass hierarchy determination: among the others we should mention that it require a much better control over systematic errors such as non-linearity than the χ^2 analysis, moreover it depends also on the high-energy tail of the spectrum, where the neutrino flux is lower and the background higher [6]; for these reason this approach will not be taken into account here. An additional complication comes from the fact that the JUNO detector will be able to see neutrinos from two power plants, of different model and generation (Taishan, EPR Gen. III, and Yangjiang, CPR-1000 Gen. II), each one with several cores, while JUNO-TAO will only see neutrinos from one of the Taishan's cores. It is extremely unlikely that the average chemical composition of the fuel in the two power plants will be the same: this means that (since the reactor spectrum depends on the chemical composition) the unoscillated spectrum seen at the two sites will be different; however we will show that, by studying the time evolution of the spectrum at the near detector (since the chemical composition changes with time), it will be possible to reconstruct the spectrum at far. It is worth pointing out that since the main goal of this work is to examine how the mass hierarchy determination is affected by the uncertainty of the spectrum and not to calculate precisely the expected sensitivity of a particular experiment, we will ignore several effect that could influence the sensitivity, but not the issue discussed here. For example, one the of main

challenge is to constrain the systematic errors in the energy reconstruction (*i.e.* "non linearity"), because it could strongly affect the final result [7, 8, 9]: in this work, however, we will not take into account this systematic error; for the same reason the background will not be considered as well. Since the JUNO detector will receive neutrinos from multiple cores, whose baselines will be slightly different, there will also be an interference effect, that will decrease the sensitivity to the mass hierarchy [10]. In this presentation we will assume all the baselines to be the same, however the generalization is quite straightforward and can be found in [4].

2. Approximations

In order to obtain analytical results we will use some approximations. Since the mass hierarchy is determined from the differences between very fast oscillations, the energy bins must be very small; in this work we will divide the region between 1.5 and 8.5 MeV into 700 10-keV energy bins . This means that it is reasonable to use a discrete approximation, replacing all the integrals with sums. The expected number of events at the far detector in the bin i is given by

$$N_{F,i} = \int G(E,E')P(E')n(E')\mathrm{d}E' \to \sum_j G_{ij}P'_j n'_j$$

Where, in the last step, the discrete approximation was used; G_{ij} is a matrix related to the Gaussian distribution that describe the smearing due to the finite energy resolution (we assumed the energy resolution to be the same at the near and far detector), P'_j is the oscillation probability and n'_j is the unoscillated spectrum. The spectrum uncertainty now can be easily parametrized by introducing a set of pull parameters β'_i, one for each energy bin; β'_i and β'_j are completely uncorrelated if $i \neq j$, however, for a given i, the same β'_i must be used at near and far detector. Considering a χ^2 test, the minimization over β'_i would be trivial with perfect energy resolution, *i.e.* if $G_{ij} = \delta_{ij}$, otherwise the energy smearing will mix the β'_i's from different energy bins, and any analytical solution would require the computation of G^{-1}, leading to large numerical errors.

It is still possible to obtain an analytical result if we perform the Gaussian convolution separately for the oscillation probability and the unoscillated spectrum, namely

$$(GP'n') \to (GP')(Gn')$$

The reason why this approximation is reasonable is that in order to determine the mass hierarchy an excellent energy resolution is needed: in the correct calculation, the oscillation probability should be "weighted" during the convolution, with a weight proportional to the unoscillated spectrum, however since the Gaussian is very peaked, the spectrum will not change so much in the allowed region. The error in this approximation is less than 0.5% (except for the very low energy part of the spectrum, *i.e.* $E < 2$ MeV, which however is not important for the hierarchy determination), but the signal we are looking for is of the same order of magnitude, so its effect could be non-negligible. This approximation, however, can be used safely in numerical simulations employing the Asimov data set if it is used to compute both the fit and the Asimov data set, as it can be seen in Tab. 1: in this way, the 0.5% error would be on the **difference** between the spectra (which, as we have seen, it is very small), and it would be a second order effect that can be neglected. We can redefine

$$GP' \to P \qquad Gn' \to n \qquad G\beta' \to \beta$$

Asimov\Fit	$(GP'n')$	$(GP')(Gn')$
$(GP'n')$	12.60	13.77
$(GP')(Gn')$	-	12.53

Table 1: $\Delta\chi^2$ calculated with and without the approximation

Now it is possible to minimize analytically over β_i. Assuming the same chemical composition at far and near detector we have

$$\Delta\chi^2 = \min_{\Delta m^2_{32}} \sum_i \frac{(\Delta P_i n_i)^2}{P_{NH,i} n_i + \mathscr{R} P^2_{IH,i} n_i} + P.T.$$

where $\mathscr{R}$ is a normalization factor between the near and far detectors (which takes into account the different masses, baselines, etc...), $P_{NH(IH),i}$ is the oscillation probability given the normal (inverted) hierarchy, $\Delta P_i = P_{NH,i} - P_{IH,i}$. and $P.T.$ indicates the penalty term for the eventual pull parameters. No penalty terms for the β_i's were considered because the theoretical constraints would be negligible with respect to the data from the near detector. We used only one additional pull parameter, for Δm^2_{32}, since the uncertainty on the other oscillation parameters would not affect strongly the mass hierarchy determination: $P.T.$ contains the penalty term for that parameter .

3. Different Chemical Composition

As mentioned before, the reactor neutrino spectrum depends on the chemical composition of the fuel; at JUNO it is likely that the unoscillated spectra at near and far detectors will be different. However the chemical composition will change in time: we will show here that by studying the time evolution of the spectrum at the near detector it is possible to reconstruct the spectrum at far. We consider only two isotopes, ^{239}Pu and ^{235}U, we now have $n_{Pu,i}$ and $n_{Ur,i}$, hence we must introduce two set of pull parameters, $\beta_{Pu,i}$ and $\beta_{Ur,i}$. Calling ρ the fraction of fissions generated from Pu, the unoscillated spectrum at the far detector is given by

$$n_{F,i} = \rho(n_{Pu,i} + \beta_{Pu,i}) + (1-\rho)(n_{Ur,i} + \beta_{Ur,i})$$

We divide the data collected at the near detector into N_t time bins, sampling different chemical compositions; in the time bin γ the fission fraction is ρ_γ and

$$n_{N,\gamma,i} = 1/\mathscr{R}(\rho_\gamma(n_{Pu,i} + \beta_{Pi,i}) + (1-\rho_\gamma)(n_{Ur,i} + \beta_{Ur,i}))$$

We can now minimize analytically over the β's; $\Delta\chi^2$ can be written as

$$\Delta\chi^2 = \min_{\Delta m^2_{32}} \sum_i \frac{(n_i \Delta P_i)^2 (C_{Ur,i} C_{Pu,i} - C^2_{Mix,i})}{(C_{Ur,i} C_{Pu,i} - C^2_{Mix,i})\sigma^2_{Exp,i} + (C_{Pu,i} + C_{Ur,i} - 2C_{Mix,i})} + P.T.$$

$$C_{Pu,i} = \sum_\gamma \frac{\rho^2_\gamma}{\rho^2 \mathscr{R}^2 P^2_{IH,i} n_{N,\gamma,i}} \qquad C_{Ur,i} = \sum_\gamma \frac{(1-\rho_\gamma)^2}{(1-\rho)^2 \mathscr{R}^2 P^2_{IH,i} n_{N,\gamma,i}}$$

$$C_{Mix,i} = \sum_\gamma \frac{(1-\rho_\gamma)\rho_\gamma}{(1-\rho)\rho \mathscr{R}^2 P^2_{IH,i} n_{N,\gamma,i}} \qquad \sigma^2_{Exp,i} = P_{NH,i} n_i$$

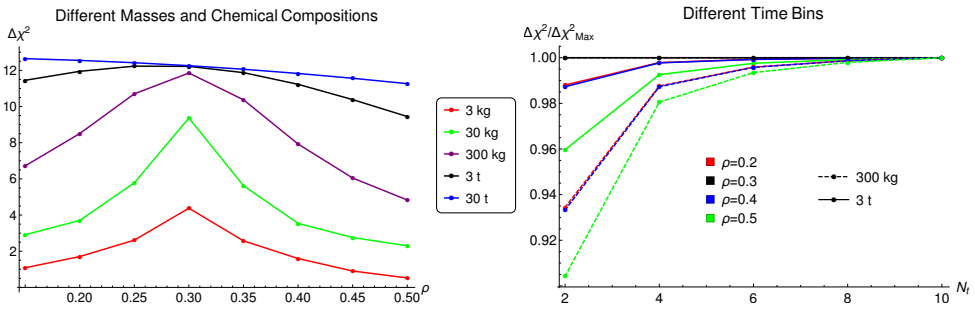

Figure 1: Left Panel: $\Delta\chi^2$ as a function of the chemical composition, for different masses of the near detector. Right Panel: $\Delta\chi^2$ as a function of the number of time bins

In Fig. 1 (left panel) it is shown $\Delta\chi^2$ as a function of the average chemical composition seen at the far detector; for the near detector we considered two time bins, with chemical composition 0.275 and 0.325, different curves correspond to different masses of the near detector. It is possible to see that if the mass of the near detector is of the order of tons, the effect of this uncertainty is very small. From the right panel we can see that if we increase the number of time bins the accuracy can be improved, however if the near detector is sufficiently large this effect is relatively small. Finally, it is worth noticing that, even if in this presentation we discussed only the mass hierarchy determination, the same approach can be applied also to different problems: other experiments could use the data collected at JUNO-TAO, where the spectrum is measured with an excellent energy resolution, to extrapolate the spectrum that will be seen in their detectors, even if the chemical composition of the fuel is not the same.

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